1 Introduction

Composites made of silicon carbide matrix reinforced by carbon fibers can be used up to temperature of 1500 °C and are characterized by low thermal expansion coefficients, low density and high toughness compared to traditional ceramics. For such reasons, they are optimal candidates for high temperature applications, such as gas turbine engines, rocket engines or thermal protection systems. In many cases reinforcement is constituted by long carbon fibers, which are often used in the form of fabric plies with different textile architectures. However, short fiber reinforced CMC can also be produced by using a hybrid process based on polymer infiltration and pyrolysis (PIP) and melt infiltration (MI). Such a process, also known as Liquid Silicon Infiltration (LSI), has found increasing application for the production of brake disks in high performance automobiles and in airplanes [1-3]. This type of materials, which represents the focus of this paper, is typically reinforced by bundles made of thousands of carbon fibers with a length of the order of ten millimeters. Hence, from the meso-scale standpoint, the material can be seen a composite made by an isotropic matrix, nominally composed by silicon carbide (SiC), reinforced by cylindrical inclusions represented by bundles of fibers. It should be noted that the dimensions of bundles are relatively large compared to disk geometrical details and that volumetric fractions of reinforcement can exceed 0.40. Moreover, in some cases, bundles can be split during the preparation of the precursor. As a consequence, the material exhibits preferential directions of reinforcement orientation, thus leading to anisotropic properties, and local variations of composition in terms of volumetric fraction, size and orientation distribution of reinforcing bundles. Such features complicates the evaluation of material properties by experimental testing, since tests in different load conditions and samples taken from different regions of the disk are required. Hence, the availability of an approach for the prediction of material properties is considered a fundamental issue for optimizing material characteristics, speeding up the design process and applying well-defined margins of safety in combined load conditions. Nevertheless, the development of a material model have to face severe difficulties, which are inherent to the nature of the material and of the technological process used in manufacturing. One of the difficulties is the uncertainty of the material properties referred to the constituent phases. Such problem is also recognized in the development of models for SiC matrix composites with textile reinforcements, produced by means of an hybrid CVI (chemical vapor infiltration)/MI process [4,5]. In the short fiber-reinforced material produced through LSI processes, the microstructure of the material is even more complex due to the different sizes and orientation of the reinforcement, a more irregular configuration of the matrix-fiber interface, to the presence of voids, unreacted silicon and micro-cracks originated in the cooling phase. From a methodological standpoint, application of micromechanical analytical approaches to such materials is difficult due to the high volumetric content and the non-uniform size and orientation distribution of bundles [6]. A numerical approach based on FE models of representative volume elements (RVEs) can be regarded as a possible solution, but the generation of FE meshes to model the material meso-structure is not straightforward, because algorithms based on positioning fibers...
according to a random or a specifically selected distributions are not successful for cylindrical inclusions at high volumetric content [7].

A promising approach to overcome the aforementioned difficulties and to develop a meso-scale model of the material has been devised by generating the FE meshes with the use of a packing simulation and by applying a peculiar strategy for the identification of the elastic properties of the constituent phases [8]. The numerical tools applied and the identification of phase properties made possible a prediction of the elastic constants of the short fiber reinforced composite material. Moving from such results, the work presented is aimed at extending the methodology and to develop a non-linear numerical material model with potential for the identification of damage threshold and failure criteria in combined stress states.

In the following sections of the paper the meso-scale structure of the material and the macroscopic response obtained in experimental tests are described. Thereafter, the characteristic of the numerical approach are discussed. Finally, the selected constitutive laws, the identification of material properties and the correlation with experimental tests is presented.

2 CMC material structure and experimental response

2.1 Technological process and material structure

The LSI process for the production of a short fiber-reinforced CMC component starts with the molding of a polymeric precursor reinforced by bundles of carbon fibers. A tile is sketched in Fig. 1-A, which includes a photograph referred to the top view of the precursor, on a plane perpendicular to the packing direction. Carbon fibers are aligned on such a plane due to the packing action and bundles are split during mixing and packing process, so that a non-uniform size and orientation distribution of the inclusions is obtained.

After pyrolysis and silicon infiltration, the internal meso-scale structure in the through-the-thickness direction of the tile is presented in Fig. 1-B, which is taken from a computer tomography analysis of a tile. The bundles of carbon fibers can be recognized, but it is apparent that they are infiltrated by dendrites made of SiC and unreacted silicon. The tomogram clearly confirms that bundles are oriented in the direction perpendicular to the packing direction.

![Fig. 1. Packing and molding of polymeric precursor (A), meso-scale structure of final material (B) an material micro-structure (C)](image-url)

The description of the material as an isotropic matrix reinforced by inclusions, which are represented by fiber bundles, is a simplification of the material structure, but it can be still considered an interesting standpoint for the development of a material model. Indeed, the microstructure of the material is characterized by irregular and complex features, such as the infiltration of silicon and SiC in the bundles, the presence of porosities and micro-cracks due to the cooling process and residual of unreacted carbon, which derives from the pyrolysis of polymeric precursor. An example is provided in Fig. 1-C, where two adjacent bundles can be distinguished. The irregularities in the microstructure can be homogenized at the meso-scale level by defining a isotropic matrix phase and a transversely isotropic reinforcement phase. In Fig. 1-
C, the arrangement of fibers in the bundles can also be appreciated. It is worth observing that a carbon volumetric content of 40% involves a much higher volumetric content of the inclusions representing the bundles in the meso-scale description, which can exceed 50%.

The adoption of a meso-scale standpoint absolutely requires the identification of the effective properties of the constituent phases in the final material. The application of the properties referred to the nominal constituents, SiC matrix and carbon fibers, is unfeasible, as it can be proved by simply comparing the properties of the CMC materials with the typical elastic properties of the nominal constitutive phases [2, 8].

2.2 Experimental response

The presence of different phases, initial porosities and micro-cracking, the development of local plasticity and the growth of defects into macro-cracks at increasing loads are known sources of non-linear behavior in ceramic matrix composites [9].

Three types of test were carried out to provide data for the development of a numerical approach with the capability of modeling the accumulation of permanent strain, damage and the failure of the material. Such experiments were performed on specimens cut from CMC tiles, with a length of 70 mm ÷ 80 mm, and a section of 5 mm x 10 mm.

A first set of specimen was subjected to a thermal cycle consisting in heating the specimen in an oven at 800 °C for more than 10 hours. As a consequence, the carbon reinforcement was burnt and the bending response of such coupons can be potentially used to identify the properties of the matrix phase in the numerical model. Three-point bending tests were performed on such specimens under displacement control at a rate of 0.1 mm/min.

The same type of bending test was performed on a second set of specimens, representing the standard CMC material used in brake production. Finally, a third set of CMC standard specimen was endowed with glass-fiber reinforced tabs and tested in tension, by using an Instron MTS-647 electro-mechanic test system at a loading rate of 0.1 mm/min. The specimens subjected to tensile tests and the lay-out of the three-point bending experiments are shown in Fig. 2.

Equivalent stress-strain curves were obtained from the force vs. displacement responses measured in all the tests. In particular, the following formulation was applied to bending data:

$$\sigma = \frac{3FL}{2bh^2}; \quad \varepsilon = \frac{6\delta h}{L^2} \quad (1)$$

All test were performed by applying a series of loading-unloading cycles and by recording the permanent deformation at end of each unloading step. Such procedure was applied to measure at increasing loading levels both permanent deformations accumulated by the specimens and degradation of secant modulus.

The results of the three-point bending tests performed on burnt specimens are summarized in Fig. 3. All stress-strain curves were interpolated and a mean curve was calculated. The stress and strain values reported in Fig. 3 are normalized with respect to the maximum stress and to the corresponding in the mean curves.

The procedure based on loading-unloading cycles makes possible a distinction between the non-linearity induced by the accumulation of permanent deformation and damage. Indeed, in each cycle, a
maximum strain $\varepsilon_i$ corresponding to a stress $\sigma_i$ is obtained and the permanent deformation at unloading, $\varepsilon_i^p$, is measured. Hence, damage could be evaluated by applying:

$$d(\varepsilon_i) = \frac{\sigma_i}{E_0(\varepsilon_i - \varepsilon_i^p)}$$  \hspace{1cm} (2)

The effect of damage was removed from the response by calculating the effective stress level without modulus degradation:

$$\sigma_i^{\text{undamaged}} = E_0(\varepsilon_i - \varepsilon_i^p)$$  \hspace{1cm} (3)

The undamaged curve for burnt specimen in three-point bending condition is represented in Fig 3 by the dashed line.

![Fig. 3. Normalized stress-strain response obtained in the three-point bending tests on CMC burnt specimens](image)

The same elaboration procedure was performed on the bending tests performed on CMC standard specimens and on CMC tensile specimens. The results are summarized in Figs. 4 and 5, respectively, where stress and strain levels are normalized with respect to the same values considered Fig. 3, referred to the burnt specimens. The main results of the experimental tests are reported in Tab. 1. Young modulus was taken considering the tangent slope of the stress vs. strain curve in the first loading cycle, between 0.2 and 0.6 of the maximum strain reached.

It can be observed that the responses in the different tests are quite similar from a qualitative standpoint. Initial deviation from linearity occurs well before maximum stress and can be initially attributed only to the development of permanent deformation. Reduction of secant modulus, which can be taken as overall damage index, accumulates slowly before maximum loads until a critical value, where macroscopic fractures develop and lead to a quite steep reduction of load carrying capability.

![Fig. 4. Normalized stress-strain response obtained in the three-point bending tests on CMC specimens](image)

![Fig. 5. Normalized stress-strain response obtained in the tensile tests on CMC specimens](image)

<table>
<thead>
<tr>
<th>Tests performed</th>
<th>Burnt CMC 3-pt bend.</th>
<th>Standard CMC 3-pt bend.</th>
<th>Standard CMC – tension</th>
</tr>
</thead>
<tbody>
<tr>
<td>E (GPa)</td>
<td>$10.5 \pm 1.0$</td>
<td>$19.3 \pm 1.1$</td>
<td>$25.8 \pm 1.9$</td>
</tr>
<tr>
<td>$\sigma_i^{\text{undamaged}} / \sigma_i^{\text{burnt}}$</td>
<td>$1.0 \pm 0.1$</td>
<td>$14.2 \pm 1.3$</td>
<td>$4.7 \pm 0.4$</td>
</tr>
<tr>
<td>$\varepsilon(\sigma_i^{\text{undamaged}}) / \varepsilon(\sigma_i^{\text{burnt}})$</td>
<td>$1.0 \pm 0.1$</td>
<td>$7.4 \pm 0.6$</td>
<td>$2.5 \pm 0.4$</td>
</tr>
</tbody>
</table>
Quantitative comparison between the 3-point bending tests performed on the burnt and on the standard CMC specimens provides an indication of the effects of reinforcement fibers: Young modulus is more than doubled and maximum strength is about 14 times higher. Comparison between bending and tensile tests shows that material non-linearity and non-homogeneity lead to bending maximum stress which is 3 times higher than the corresponding tensile ultimate strength. The capability of capturing such qualitative and quantitative effects of the macroscopic response is a very important goal for the non-linear meso-scale material model to be developed.

3 Generation of meso-scale material models

3.1 Packing simulations

The development of a numerical approach at the meso-scale level is based on representative FE models. The capability of generating FE meshes could be used to predict material properties before physical manufacturing. Moreover, the reconstruction of the meso-scale configuration after manufacturing is quite complicated for the material considered, because the identification of bundle boundaries is complicated by silicon infiltration, as it is apparent from Fig. 1-B.

Unfortunately, the generation of FE meshes by means of a sequential placement of cylindrical inclusions, based on a given size and orientation distribution, is not feasible at high volumetric contents, as it is shown in [7] and confirmed in [8]. Such difficulties motivated the development of a packing simulation process in [8]. First of all, the distribution of size and section shapes obtained in the mixing process were experimentally measured. A population of inclusions was generated considering such distribution and placed, by using a random sequential absorption algorithm, in a loosely packed configuration, which occupies a relatively large volume.

Then, data referred to inclusion shape and positions in the pre-packed configuration were used to automatically generate a model for packing simulation. In the present work, the pre-packed configuration is obtained by applying the same approach and using the same size distributions described in [8], but new tools are developed for packing simulation and mesh generation.

Fig. 6. Packed configuration of the inclusions in a tile obtained by using Bullet Physics Library

Indeed, the simulation of packing is aimed at obtaining realistic and feasible configurations of highly packed inclusions rather than modeling the molding process from a physical standpoint and one of the most critical aspect of such simulation is management of mutual contact interactions among a very large number of bodies. Multi-physic software libraries used in the development of computer games are endowed with very efficient methods for collision detection among bodies, such as GFK algorithm [10]. Moving from such considerations, the open source Bullet Physic Library is adopted to simulate the packing of inclusions, which are modeled as rigid bodies. In the procedure devised to generate the meso-scale configuration, the pre-packed configuration is first generated in a volume bounded by rigid wall which are then moved in the packing direction until the required volumetric
fraction is obtained. The final packed configuration of a 50 mm x 50 mm x 12 mm tile including 1340 inclusions with elliptical sections is presented in Fig. 6. Final volumetric fraction of inclusion in this tile is 0.52.

The distributions of orientation can be described by the orientation tensor $a_{ij}$, which can be calculated by knowing the distribution function $ψ(ϑ, φ)$:

$$ a_{ij} = \int_{0}^{2\pi} \int_{0}^{\pi} p_{i} p_{j} ψ(ϑ, φ) \sin ϑ \, dθ \, dφ $$

where $ϑ$ is the angles of the inclusion axis with the packing direction $x$, $φ$ defines the orientation in the $y$-$z$ plane and $p_{ij}$ are the direction cosines of the inclusion axis.

The orientation tensor obtained in the packing of the tile shown in Fig. 6 is hereby reported:

$$ a_{ij} = \begin{bmatrix}
0.0893 & -0.001 & 0.0011 \\
-0.001 & 0.4753 & 0.3135 \\
0.0011 & 0.3135 & 0.4719
\end{bmatrix} $$

According to the results, the packed configuration is characterized by a quite uniform orientation in the plane perpendicular to packing direction. Only few inclusions exhibit significant orientation out of this plane, in a good qualitative agreement with the experimental evidences. The packing simulation of a tile can be performed in a short computational time (about 10 minutes by using a workstation endowed with 2.6 GHz AMD Opteron processors). Hence, from a computational standpoint, a significant improvement in the efficiency of packing simulation has been obtained with respect to results reported in [8].

3.2 Modeling techniques

Automatic meshing of packed meso-scale configurations at high reinforcement content represents a challenging task due to extremely small gaps that can exist between adjacent inclusions. In this work the problem is complicated by the need of performing non-linear analyses and to model the development and propagation of macro-cracks. Since a possible approach is based on the adoption of explicit analyses, a mesh based on 8-noded brick elements with a reduced integration scheme is recommended [11], but automatic meshing of solid volumes occupied by matrix and inclusions is unfeasible if brick elements are adopted.

A modeling approach is proposed in this work, based pre-meshing the volume occupied by the material with the use of a regular mesh of solid elements. The boundaries of the inclusions are not defined in this mesh, but elements are subsequently attributed to the matrix phase or to an inclusion by a script that compares the position of each element with the results of the packing simulation. Such procedure is used to prepare the model of the matrix phase, as it is shown in Fig. 7.
a matrix/inclusion interface that is geometrically representative of the true boundaries between the phases in the meso-scale idealization.

Fig. 8. Reference model of fiber pull-out (A) and modeling technique based on dissimilar meshes (B)

Fig. 9. Comparison of stress-state in the models of a pull-out test.

A numerical study representing a pull-out test of a fiber has been conducted to compare the stress states modeled by adopting the proposed modeling technique with the results of a more traditional modeling approach (Fig. 8). In the traditional model the meshes of the matrix and of the fiber are connected at nodes. The interface layer visible in Fig. 8-A is characterized with the same material properties of the matrix. In the modeling technique proposed such interface is substituted by cohesive elements regularly connected at nodes of the fiber. The external surface of this cohesive layer is involved in the TIE connection with the irregular matrix boundary (Fig. 8-B).

According to the results shown in Fig. 9, the stress state in the fiber is identical in both models and the maximum values of Von Mises stress in the matrix are only slightly overestimated (about 10%) in the approach based on dissimilar meshes. Therefore, the representation of the stress state in the proposed technique can be considered acceptable.

3.3 Models of experimental tests

The technique based on packing simulation, mapping of matrix phase on a pre-meshed volume and on the introduction of geometrically-based meshes of inclusions, which are tied to the matrix elements, is applied to model the experimental tests performed on the CMC material.

Fig. 10. Specimen model (A), meso-scale model of the matrix (B) and of CMC material (C)

The expected computational times required for non-linear analyses are very high and this suggests to limit the meso-scale description of the material to
the central zones of the specimens used in the experimental tests. The lateral parts are modeled at the homogenized level, as it shown in Fig. 10-A, by using a regular scheme of 36000 bricks with sides of 0.3 mm ÷ 0.5 mm. The central meso-scale mesh is connected to the homogenized parts by means of TIE algorithms. The matrix phase is mapped on a mesh of bricks with a side of 0.1 mm and is represented in Fig. 10-B. The mesh of the matrix alone is used to model the burnt specimens, where reinforcement content is eliminated. The model of the standard CMC material is obtained by introducing the meshes of 210 inclusions, which are modeled by brick elements having a typical size of about 0.1 mm (Fig. 10-C). The volumetric fraction of the inclusion in the meso-scale part of the FE model is 0.48. Total number of brick elements in the meso-scale part of the model is about 1 million. All numerical analyses are performed by using the Abaqus/Explicit code, by applying appropriate velocity boundary conditions to sets of nodes, with a time history tuned to avoid the onset of numerical oscillations so to perform quasi-static analyses [11]. Computational time cost is very high, although it depends on the number of processors used. The analysis of the tensile test on the CMC standard specimen can require up to 5 days by using a workstation endowed with 2.6 GHz AMD Opteron processors.

It is worth noting that the application of the hybrid models require, for the representation of non-linear responses, the attribution of non-linear constitutive laws to the elements belonging both to homogenized and to the meso-scale parts of the FE scheme.

4 Constitutive laws and identification strategies for the elastic-plastic response

4.1 Elastic material properties

In the meso-scale standpoint adopted in this work, matrix is considered an isotropic material and fiber bundles are modeled by using a transversely isotropic material model. The properties of the matrix in the meso-scale model are identified by considering the experimental results referred to burnt specimens. This a fundamental characteristic of the general strategy followed for the identification of the properties in the models and is applied both to the linear and non-linear aspects of the constitutive law.

Accordingly, an hybrid model with a meso-scale part constituted only by the mesh of the matrix phase is used to analyze the bending test performed on the burnt specimens. The elastic modulus attributed to the matrix is varied until an acceptable correlation is obtained. The Poisson’s ratio of the material is set to 0.1, following the considerations reported in [8]. Considering the experimental results, an elastic modulus of 10.5 GPa is attributed to the homogenized material at the side of the hybrid model. A good correlation between the experimental and numerical stiffness is obtained with a matrix Young modulus $E_m = 56.8$ GPa.

The identification of the effective elastic properties of the bundles is indeed a complicated task that was carried out in [8] by using results referred to long fiber-reinforced ceramic specimens. In this work, a set of material properties obtained in a different identification activity is used, which was based on CMC specimens at different volumetric contents. Elastic parameters are reported in Tab. 2. It can be observed that the value of the Young Modulus $E_a$ in the axial direction is very close to the one identified in [8] but that the elastic transverse modulus $E_t$ and the shear moduli $G_t$ and $G_{ta}$ are higher, though they remain one order of magnitude lower with respect to $E_a$.

<table>
<thead>
<tr>
<th>$E_a$ (GPa)</th>
<th>$E_t$ (GPa)</th>
<th>$G_t$ (GPa)</th>
<th>$G_{ta}$ (GPa)</th>
<th>$v_{ta}$ (–)</th>
<th>$v_t$ (–)</th>
</tr>
</thead>
<tbody>
<tr>
<td>81.25</td>
<td>10.0</td>
<td>4.2</td>
<td>3.9</td>
<td>0.1</td>
<td>0.3</td>
</tr>
</tbody>
</table>

4.2 Drucker-Prager plastic flow rule

The non-linear response originated by the accumulation of permanent deformation in the matrix of the ceramic material is modeled by using an elastic-plastic constitutive law associated to Drucker-Prager criterion, which is also proposed in [4] for the characterization of the matrix in a binary model of a ceramic matrix composite. Drucker-Prager criterion is attributed both to the homogenized materials for the lateral parts of the hybrid models and for the matrix in the central part modeled at the meso-scale level. According to the criterion, yielding depends on Von Mises stress (deviatoric stress) and on the applied pressure stress. In the plane represented in Fig. 11 a yield surface
based on Von Mises criterion would be represented by an horizontal line, whereas Drucker-Prager yield surface is characterized by the angle $\beta$, which defines the dependency on the applied pressure and determines different yield stresses in tensile and compressive conditions.

After yielding, strain hardening in the material is represented by the following two-parameters law:

$$\sigma = \sigma^Y + K\varepsilon^p N$$  \hspace{1cm} (5)

The identification of the material parameters is based on the correlation with the undamaged curves obtained in the three types of tests performed on CMC standard and burnt specimens, which are reported in Figs. 3, 4 and 5. Initially, identification of the parameters of homogenized material has been carried out by using completely homogenized FE models. The obtained parameters provided a basis for the identification of the properties of the matrix in the meso-scale part of the model. Simplifying assumptions have also been introduced for the identification of $\beta$ angles, since tensile tests are not available for burnt specimens. Table 3 reports the material parameters defined for all the three material models involved in the numerical analyses. Stress values are normalized to the experimental average ultimate stress of burnt specimens.

Quasi-static explicit analyses are performed by using the hybrid models represented in Fig. 10 and adopting the elastic-plastic constitutive laws with the parameters reported in Tab. 3. Final numerical results are reported and correlated with experiments in Fig. 12. In such models, fiber bundles are characterized with the linear elastic material properties reported in Tab. 2 and no damage is activated in the cohesive elements connecting such bundles to the matrix mesh. Overall, results prove that the introduction of elastic inclusions accounts for the difference in the initial slope between the
bending tests performed on burnt and standard CMC. Moreover, the adoption of Drucker-Prager associated plasticity for the matrix material can capture the non-linear response both in tensile and in bending curves. A contour describing the development of permanent deformation in the analyses of bending and tensile test of CMC specimens is presented in Fig. 13. Stress concentrations in the numerical models lead to the localization of plastic strains in the matrix of the meso-scale model. Plastic strains in bending are mainly developed on the tensile surface (Fig. 13-A), whereas they are more diffused throughout the volume in the tensile test analysis (Fig. 13-B).

5 Modeling of damage and failure

A scalar damage variable can be included in the elastic-plastic material model based on Drucker-Prager criterion [11]. Damage threshold is set at a given value of plastic strain, \( \varepsilon_P^0 \), whereas damage evolution depends on the plastic displacement in the element, \( u_p \), which is introduced to regularize the damage law on the basis of a typical element size, \( L \):

\[
u_p = L \varepsilon_p^0 \quad (6)
\]

In the case considered, the regular size of the matrix mesh in the meso-scale model makes possible a direct conversion of the plastic displacement into the plastic strain. The following exponential form is chosen for the damage evolution law:

\[
d(1-e^{-\alpha (u_p/u_p^f)}) = L \varepsilon_p^{0_N} \quad (7)
\]

where two new parameters are introduced: the final plastic displacement corresponding to unit damage, \( u_p^f \), and the exponent \( \alpha \). A final plastic strain, \( \varepsilon_p^f \), can be defined by applying Eq. 6.

By adopting the general identification strategy defined in this work, a damage law is first identified for the burnt material. The analysis of plastic strain evolution in the elastic-plastic and the comparison with experiments leads to identify the damage threshold and then the values reported in Tab. 4 are selected after a sensitivity study. Plastic strains are normalized with respect to the average experimental strain at failure of the bending tests on the burnt specimens.

Tab. 4 – Parameters of damage law for burnt specimens

<table>
<thead>
<tr>
<th>( \varepsilon_0^p / \varepsilon(\sigma^U)_\text{burnt} )</th>
<th>( \alpha )</th>
<th>( \varepsilon_f^p / \varepsilon(\sigma^U)_\text{burnt} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.06</td>
<td>0.5</td>
<td>12.3</td>
</tr>
</tbody>
</table>

Fig. 14. Plastic strain contour at the onset of a transverse fracture (A) and at failure (B) in the analysis of the bending test on the burnt specimen.

Fig. 15. Sensitivity to damage threshold in the analyses of burnt specimens in bending (A) and CMC specimen in tension (B).

The introduction of the damage law in the hybrid model of the burnt specimen in bending test conditions leads to the localization of plastic strain and damage along a fracture that originates from the tensile surface (Fig. 14-A) and propagates towards
the compressed surface (Fig. 14-B). The numerical-experimental correlation reported in Fig. 15-A shows that damage development leads to the failure of the specimen at a load level that is in good quantitative agreement with experimental results. The damage law identified considering the burnt material is then applied in the analysis of the tensile test on CMC standard specimen. Although the inclusion are modeled as linear elastic cylinders and damage is not activated in the cohesive elements, the introduction of damage in the elastic-plastic constitutive response of the matrix leads to the failure of the specimen at a load level that is close to the average experimental values, as it is reported in Fig. 15-B. The correlation referred to the predicted strength in the CMC tensile specimen is improved by using a damage threshold of $1.3 \cdot \varepsilon_P^0$. However, it can be observed in Fig. 15-A that the correlation of the response referred to the burnt specimen can be still considered acceptable even if a damage threshold of $2.16 \varepsilon_P^0$ is adopted. The development of damage in the analysis of the tensile test is still characterized by a transverse fracture, though the effects of the elastic fiber bundles can be noticed in Fig. 16, where only the element of the matrix phase are shown.

The analysis of the bending test on the CMC specimen is carried out by using the damage law with $1.3 \cdot \varepsilon_P^0$. Damage develops quite early in the numerical response, but the effect on the equivalent stress vs. strain response is negligible until the development a fracture, which is apparent in Fig. 17, that leads to the failure of the specimen. It can be observed that damage tends also to spread along the boundary on the inclusions on the tensed surface of the specimen.

The stress and the strain levels at failure in the bending test are in acceptable agreement with the experimental average values, as it shown in Fig. 18, which reports the overall numerical-experimental correlation for all the three tests considered. It is remarked that overall correlation is obtained by using a single set of material parameters for the meso-scale part of the model in all the three analyses, with the inclusion elastic properties reported in Tab. 2, the elastic-plastic parameters reported in Tab. 3 and a damage law based on Tab. 4, with a damage threshold set at $1.3 \cdot \varepsilon_P^0$.
Concluding remarks

The development of a numerical meso-scale model for a short fiber–reinforced ceramic matrix composite has been presented. The main critical issues for the generation of meso-scale finite element models have been addressed by developing effective methodologies for simulating the packing of fiber bundles during manufacturing and for meshing the obtained meso-scale configurations. The experiments performed proved to be adequate to set up an approach aimed at modeling permanent strain accumulation, damage onset and failure in the material considered. In particular, the tests performed on burnt specimens allowed the identification of the material parameters for the matrix phase, which have been successfully applied to the analyses of the standard CMC material. The selection of an elastic-plastic material model based on Drucker-Prager yield criterion for the matrix phase can be considered a key issue for the achievement of a good numerical-experimental correlation in all the three type of tests considered in the activity. Finally, it has to be remarked that the damage law identified considering the tests on the burnt specimens was applied to model the maximum load carrying capability and the failure mode of the standard CMC in tension and failure with appreciable results. In particular, the introduction of the reinforcement bundle in the numerical model of the matrix leads to almost the same increment of strength that was recorded between the bending tests performed in the burnt material and in the standard CMC specimens. The introduction of damage in the constitutive law of the bundles and in the cohesive bundle/matrix interface could be required to increase the reliability of the model but, considering the results obtained by applying a non-linear model for the matrix material, the developed approach can be already considered a promising tool to support the design of CMC components in engineering application.

References