1. Introduction

The collapse of underwater implodable volumes, for example due to hydrostatic pressure, is a highly dynamic phenomenon and can have catastrophic effects on nearby structures. To achieve accurate numerical modeling of these effects, fluid-structure interactions must be taken into account.

The implosion occurs due to a large pressure differential across the water-gas surface (the bubble). This forces the bubble to collapse and initiates a cyclic motion of expansions and contractions. The dynamic cycle repeats as damping reduces the amplitude of the oscillations. The first pressure wave is strongest and may cause structural damage. The waves also are an acoustic source which is important for the sonar signature. Understanding this phenomenon is crucial in that the collapse of one small pressure housing could trigger a chain reaction of implosions with increasing destructive capability, exemplified by [1].

Lord Rayleigh (1917) [2] and Lamb (1932) [3] were the first to examine the theory for spherical bubbles. Their work provided a framework for later implodable volume analyses. However by assuming an incompressible fluid, their analysis was only applicable for one cycle. The Rayleigh theory was modified to account for inner gas pressure leading to the oscillating expansion-contraction theory.

The Keller-Kolodner [4] model developed for underwater explosions can be easily adopted for underwater spherical implosions. Keller and Kolodner treat the water as a compressible fluid which leads to damped oscillations of the bubble. Epstein and Keller [5] then developed the formulations to handle planar, cylindrical, and spherical bubbles during implosions and explosions.

Many groups of researchers have done work combining bubble collapse theory with structural analysis. For example, Cor and Miller [6, 7] investigated spherical and cylindrical implosions using the Laplace equation, which leads to undamped oscillations and hence is only applicable for the first cycle. They considered the effect of interior structures of brittle, plastic, and elastic shells on implosion dynamics. Kalumuck et al. studied fluid-structure interactions using a three dimensional boundary finite element model [8]. Iakolev’s work on submerged shells subject to a shock wave [9] can be applied to implosion problems.

In terms of experimental research there has been relatively limited work reported in open literature. Vazant et al. imploded 27 aluminum spheres with three interior configurations: vacuum, air at one atm of pressure, and filled with Styrofoam pellets. They found that recorded pressures were lower than theoretical results, oscillation periods were longer than theory predicted, while inertia results matched well with theory [10]. Orr and Schoenberg imploded glass spheres of varying radii and depths. They reported implosion depth and pressure versus glass thickness, as well as the acoustic signature of the spheres [11]. Turner also studied glass spheres imploding due to hydrostatic pressure, and compared results to both his computational model and the Keller-Kolodner model [12].

For naval applications, it is imperative to understand the response and integrity of underwater structures to implosive loads. This understanding should help to deduce methods of tailoring the internal structure to mitigate the emitted pressure pulses.
In the present work, computational models are used to investigate the mitigating effect of structural deformation on the process of underwater implosion dynamics. For this purpose, the problem setup involves a simple cylindrical implosion with a cylindrical shell placed inside the bubble. The focus is to examine the sensitivity of the implosion process to the variation of such parameters as inner shell structural characteristics and configurations. The benchmark for comparison is the peak acoustic pressure in the water at a specified distance from the initial bubble radius.

The objectives of the present study are threefold. First, it is necessary to analyze different structural configurations and compare different designs. Comparisons are made using the peak acoustic pressure. The goal is then to use structural deformation to most effectively absorb energy and mitigate the effect of the implosion pressure peaks on the host or adjacent structures. One important question is if an implodable structure is to fail, can it be designed to minimize its destructive capability on other nearby structures.

Finally, the research will help identify behavioral trends and important parameters which can guide design choices. The focus here is to determine the sensitivity of the implosion process to various parameters. The parameters of particular interest are structural shell thickness, shell location relative to the initial bubble radius, depth, initial air pressure inside the structure, shell material properties (density, Young’s modulus, and Poisson’s ratio), among others. Additionally, for these types of computations, such parameters as the time step, element size, and the domain size must be discussed to determine the necessary numerical resolution for the computational scheme.

2. Problem Formulation and Approach

One-dimensional spherical implosions were analyzed previously in [13, 14]. The present work considers cylindrical implosions and cylindrical shells. The governing equations for fluid motion are the conservation laws of mass, momentum, and energy. These conservation equations relate time derivatives of density, momentum, and energy to appropriate flux terms for each equation.

For the underwater cylindrical implosion problems to be considered in the present work, boundary conditions are assumed to be hydrostatic at the far field, and no mixing at the bubble surface which separates water from gas. Therefore, due to this assumption, there is zero momentum flux at both of the fluid boundaries. As such, the effect of the momentum flux can be assumed to be negligible everywhere in the domain, and the convection term in the momentum equation can be ignored. This leads to the simpler governing law, which is a form of the wave equation:

\[ \rho \frac{\partial}{\partial t} \mathbf{v} + \nabla p = 0, \]  

where \( \mathbf{v} \) is the velocity vector and \( \rho \) is the fluid density. The pressure \( p \) is defined using acoustic Hooke’s law with the bulk modulus \( \kappa \), which relates changes in pressure to changes in volume, as:

\[ p = -\kappa \nabla \cdot \mathbf{u} + p_0, \]  

where \( \mathbf{u} \) is the displacement vector and \( p_0 \) is the pressure at the state where displacement is zero. These equations are expressed in cylindrical coordinates for convenience. Introducing the finite element approximation to the wave equation leads to the following equation.

\[ M_f \ddot{\mathbf{q}} + K_f \mathbf{q} = F_f \]  

In the above equation the variables \( M_f, K_f, F_f \) symbolize the global fluid mass matrix, stiffness matrix, and load vector, while \( \mathbf{q} \) and \( \ddot{\mathbf{q}} \) are the nodal displacement and acceleration vectors.
A pressure boundary condition is applied at the bubble surface from the gas region and a zero displacement (hydrostatic) boundary condition is applied at the far end of the fluid domain. The fluid mesh is updated every time step to account for bubble motion. The fluid mass and stiffness matrices are updated at each time step to correspond to the new configuration of the fluid node points.

The domain is assumed to be large enough that the displacement is set to zero at the very last node point in the fluid domain. Therefore the pressure remains constant at the hydrostatic pressure at this point. Numerical studies [13, 14] show that this assumption is acceptable, as long as the domain outer radius is at least an order of magnitude greater than the initial bubble radius.

The finite element formulation of the equations of elasticity is used to model the response of elastic cylindrical shell. The finite element approximation leads to the following equation:

$$M_S \ddot{d} + K_S d = F_S$$  \hspace{1cm} (4)

The variables $M_S, K_S, F_S, \ddot{d}, d$ are the structural mass matrix, stiffness matrix, load vector, nodal acceleration vector, and nodal displacement vector.

The finite element solver program has been developed and adapted to handle the fluid subsystem. The program is highly readable and adaptable. For the solution of the problem at hand, the explicit fourth order Runge-Kutta method is used for time discretization. The problem is nonlinear due to mesh updates every time step to handle the moving interface between the gas and liquid. To account for the mesh motion, the displacement is reset to zero at every time step, and the velocity and acceleration are interpolated using a spline fit.

The validity of the fluid model has been verified by comparisons with the widely used Keller-Kolodner model for simple spherical bubbles. Errors in the peak acoustic pressure from the Keller model were less than 0.3% for spherical implosions [13, 14].

Fig. 1 shows a section of the idealized model of infinitely long cylindrical bubble with interior cylindrical shells, which is used for all the results presented in the following sections. A similar problem setup was examined by Cor and Miller [6, 7]. The fluid and the inner wall structure are coupled via the gas inside the bubble. The pressure in the outer and inner gas is found using the adiabatic constants $k_{in}, k_{out}$. The following equation shows how the gas pressures are updated at each time step.

$$p_{in} = k_{in}[\pi(r_{in}^2)]^{-\gamma}; \quad p_{out} = k_{out}[\pi(a^2 - r_{out}^2)]^{-\gamma}$$ \hspace{1cm} (5)

In equation (5), $a$ is the bubble radius, the variables $r_{in}, r_{out}$ are the inner and outer radii of the structural domain which remains aligned with the shell as it deforms in time, the variables $p_{in}, p_{out}$ are the inner and outer gas pressures, and $\gamma$ is the specific heat ratio. The adiabatic constants are found using the initial pressures and volumes of the inner and outer gas regions.

3. Numerical results

3.1 No interior structure

First considered is the case where no interior structures are present. The resulting maximum acoustic pressure in the fluid will serve as a benchmark for other cases. The initial bubble radius is 0.5 m and the depth is 100.0 m. Figs. 2-5 present the time histories for bubble radius, bubble velocity, acoustic pressure at 1.0 m from the bubble center, and gas pressure.
For this case, the maximum acoustic pressure at 1.0 m from the bubble center is 41.45 atm, occurring at 36.7 ms. At this time, the bubble achieves its minimum radius at 0.109 m. The pressure in the interior gas region also achieves a maximum pressure of 71.05 atm.

![Graph of bubble radius vs. time with no interior structures](image)

Fig. 2: Bubble radius vs. time with no interior structures

![Graph of bubble velocity vs. time with no interior structures](image)

Fig. 3: Bubble velocity vs. time with no interior structures

![Graph of acoustic pressure at 1.0 m vs. time with no interior structures](image)

Fig. 4: Acoustic pressure at 1.0 m vs. time with no interior structures

![Graph of gas pressure vs. time with no interior structures](image)

Fig. 5: Gas pressure vs. time with no interior structures

### 3.2 Carbon-epoxy composite shells

The goal of this exercise is to examine the effect of carbon-epoxy composite layup configurations to mitigate the maximum pressure peak from the first bubble collapse. For all six of the configurations, the bubble is initially at a radius of 0.5 m, and the shell has an initial outer radius of 0.48 m, with a thickness of 2.5 mm. Material properties of a single carbon-epoxy ply are given in Table 1.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus in fiber direction</td>
<td>142 GPa</td>
</tr>
<tr>
<td>Young’s modulus in transverse direction</td>
<td>10.3 GPa</td>
</tr>
<tr>
<td>Shear modulus (fiber-transverse)</td>
<td>7 GPa</td>
</tr>
<tr>
<td>Poisson’s ratio (fiber-transverse)</td>
<td>0.27</td>
</tr>
<tr>
<td>Poisson’s ratio (transverse-transverse)</td>
<td>0.458</td>
</tr>
<tr>
<td>Density</td>
<td>1580 kg/m³</td>
</tr>
</tbody>
</table>

The six layups have fiber angles of [0/90/0/0/90/0], [90/0/45/45/0/90], [0/-60/60/-60/0/-60] (quasi-isotropic), [0/30/60/90/90/90/30/60], [0/60/45/90/90/45/60/0], and [0/15/30/45/60/75/90]. The results are compared with a rigid wall placed at 0.48 m to investigate the mitigative effect of structural deformation. The bubble radius, shell radius, and acoustic pressure in the water at 1.0 m are plotted with respect to time in Figs. 6-8.

The results demonstrate significant mitigation due to structural deformation. The maximum pressure for no structures is 41.45 atm (Sec. 5.1), while for the rigid wall it is 26.82 atm. The [0/15/30/45/60/75/90] layup is the most effective, lowering the peak pressure to 20.21 atm, a 51.2% reduction from no
structures and a 24.7% from the rigid wall. The peak pressure mitigation is evidently dependent on the amount of deformation experienced by the structure and the smoothness of the transition from collapse to expansion.

Fig. 6: Bubble radius vs. time for six composite laminates compared to rigid wall case

Fig. 7: Outer shell radius vs. time for six composite laminates compared to rigid wall case

Fig. 8: Acoustic pressure at 1.0 m vs. time for six composite laminates compared to rigid wall case

3.3 Varying depth

The next simulation involves varying the depth at which the implosion occurs, while keeping other parameters constant. A quasi-isotropic composite shell is used with a thickness of 2.5 mm. Depth values of 50, 100, and 200 m are used. Figs. 9-12 illustrate the bubble radius, shell radius, and acoustic pressure results. Fig. 12 plots the maximum pressure at 1.0 m vs. the depth.

Peak pressure values for the three depths are 11.58, 21.43, and 38.15 atm, respectively. Fig. 12 shows that the relationship between the maximum pressure and depth is nearly linear.

Fig. 9: Bubble radius vs. time for varying depth

Fig. 10: Shell radius vs. time for varying depth

Fig. 11: Acoustic pressure vs. time for varying depth

Fig. 12: Maximum acoustic pressure at 1.0 m vs. depth
3.4 Varying initial shell radius

This section details the effect of varying the initial location of the shell relative to the initial bubble radius. As the initial shell radius is varied, all other parameters are kept constant. A quasi-isotropic composite shell is used with a thickness of 2.5 mm. The initial radii values are 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.34, 0.345, 0.35, 0.4, 0.45, and 0.48 m. Fig. 13 and 14 present the bubble radius and acoustic pressure time histories for all twelve initial shell radii. Fig. 15 plots the maximum pressure vs. initial shell radius.

It is interesting to note that, for small initial shell radius (less than 0.25 m), the maximum pressure is greater than the case where there is no interior structures. As the initial shell radius approaches the initial bubble radius the maximum acoustic pressure decreases rapidly. By placing an obstacle (the shell) to decrease the gas volume into which the bubble can collapse, the strength of the implosion and acoustic pressure peak decreases.

3.5 Varying shell thickness

In the following simulation, the initial shell radius is kept constant, while the shell thickness is varied from 2.5 to 50.0 mm. The quasi-isotropic shell is placed at an initial radius of 0.48 m. Results are shown in Figs. 16-19.

As the thickness increases the structure behaves more like a rigid wall, and one observes an asymptotic trend in the maximum acoustic pressure vs. thickness plot (Fig. 19). The thinner shells respond to the implosion with greater deformations. Similarly to other simulations, as structural deformation increases, the pressure peak decreases.
EFFECT OF FLUID-STRUCTURE INTERACTIONS ON UNDERWATER IMPLOSION DYNAMICS

3.6 Carbon-epoxy polyethylene sandwich shells

The next case involves a sandwich structure with carbon-epoxy facesheets and a highly deformable polyethylene core section. The same six layups are compared, along with the rigid wall result. Figs. 20-22 show the bubble, shell, and pressure time histories. The bubble is initially placed at 0.5 m, the structure outer edge is initially at 0.48 m. The face sheets of the sandwich structure are 2.5 mm thick, while the ultra-high molecular weight polyethylene (UHMWPE) core thickness is 5.0 mm. The UHMWPE core has isotropic material properties tabulated below in Table 2.

Table 2: Material properties of Ultrahigh Molecular Weight Polyethylene foam [16]

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus</td>
<td>690 MPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.32</td>
</tr>
<tr>
<td>Density</td>
<td>930 kg/m³</td>
</tr>
</tbody>
</table>

Again the [0/15/30/45/60/75/90] layup is the most effective among the six configurations analyzed at mitigation of the peak pressure. This layup achieves a maximum pressure of 21.81 atm, a 47.4% mitigation from no structure and a 18.7% mitigation from the rigid wall. The mitigative effectiveness of the sandwich structure is worse than the composite shell (Sec. 5.2) due to the total structural thickness being four times greater. This increases the in-plane stiffness of the shell, reducing the deformation and increasing pressure. However, the sandwich structure will be more effective if bending deformation is allowed.
3.7 Double hull structures

This section details the results for a double hull structure with an additional air volume separating the two hulls. The two composite hulls are placed to match the sandwich structure with the polyethylene core replaced by air. Therefore, both composite shells have a thickness of 2.5 mm while the air region in between has a thickness of 5.0 mm. Time histories of bubble radius, the two shells’ radii, and acoustic pressure are given in Figs. 23-26.

The peak pressure for the [0/15/30/45/60/75/90] composite shell is 21.33 atm, 2.2% lower than the composite-polyethylene sandwich structure. The shell experiences a deformation of 5.3 mm, compared to 3.5 mm for the corresponding sandwich structure. The other layups have a more significant disparity between double hull and sandwich results. The [0/90/0/90/0] double hull structure, for example, has a peak pressure of 22.9 atm, 5% lower than its value as a sandwich structure.

4. Conclusions

Results for this idealized model demonstrate the influence of fluid-structure interactions on the dynamic problem of underwater implosions. The presence of deformable interior structures can greatly mitigate the peak acoustic pressure in the water, alleviating damage to nearby structures from the implosion event. Composite shells and sandwich structures in particular are shown to be effective energy absorbers. Fluid pressure and structural response are shown to be sensitive to parameters such as composite fiber orientation, depth, initial shell radius, and shell thickness.

Future work involves the use of a new solver, called UMDynamics, which is three-dimensional and allows for more complex and physically relevant simulations. It will also incorporate structural failure modes, including buckling.
References

[1] K. Riesselmann “Totsuka: ‘we will rebuild the detector’ after shattering setback at Super-K”. 
[2] L. Rayleigh “On the pressure developed in a liquid during the collapse of a spherical cavity”. 
Philosophical Magazine, no. 34, pp 94-98.