REALIZING DOMAIN SUPERPOSITION TECHNIQUE IN NASTRAN FOR PREDICTING THE MECHANICAL PROPERTIES OF TEXTILE COMPOSITE

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Keywords: textile composite; mechanical properties; domain superposition technique

1 Introduction

Textile composites have become widely used in the application of light weight structures in aerospace, marine, automobile as well as civil architectural industries in the last decade [1]. It shows a better performance with easily manufactured and shaped ability than traditional unidirectional reinforced materials while keeping the quality of high strength & stiffness versus weight ratio. Textile composite is the most commonly used fabric material in industry. A comprehensive classification was presented by Cox [2]. The geometry modeling of plain weave fabric composite was addressed by Kuhn [3] and Adumitroaie [4]. A review of recent developments and results in predicting mechanical properties of textile composites using finite element analysis and theoretical analysis methods, focusing on elastic behavior for woven fabric composites was presented by Tan and Tong et al [5].

Various traditional methods predicting the mechanical properties of textile composite had been presented including theoretical and numerical models. Ishikawa and Chou presented three models for micromechanical analysis of woven fabric composites, which are the mosaic model, the fiber undulation model and the bridging model [6, 7]. Because the two sets of yarns, usually called warp and fill, are interlaced with each other in mutually orthogonal directions, the deformation within the fabric plate and the out of plane directions are coupled. Traditional methods cannot solve such complicated problems with sufficient accuracy. Some new efforts were also presented to predict the elastic moduli including the nonlinear properties of the plain weave fabric composites [8, 9]. With the development of computer technology, researchers can simulate the mechanical analysis with finite element modes in micro scale. Most of the finite element models of the textile composite were established using the RVE (Represent Volume Element) method. Lomov and his coworkers provided a quick modeling software, named Wise Tex, which can handle modeling of a complex textile structure quickly [10]. However, a detailed modeling for the weave fabrics still requires a huge amount of three dimensional solid elements in the FEM due to the complexity of the geometry parameters. The intersection area of the tows is very singular and the matrix model is difficult to be simulated. Thus, a very fine model to deal with the resin model in the tows intersection position will lead to millions of solid elements. This usually makes the simulation quit because of insufficient computer memory reasons. Other various finite element modeling techniques and assumptions were proposed for simplifying the modeling by lost precision.

To solve the finite element analysis efficiently, Mote provided a “Global–Local” concept for the modeling method of FEM in 1971 [11]. Iarve presented an independent mesh method for three-dimensional stress analysis in composites [12]. In the independent mesh method, the penalty method was used to impose all displacement continuity conditions between fiber and matrix. Fish proposed a multi-scale analytical method, called the s-version finite element method [13]. The s-version increases the resolution by superimposing additional meshes of higher-order hierarchical elements. Fish applied the mesh superposition method to accurately resolve the stress field in the vicinity of free edges and capturing localized phenomena within the multilayered laminates consist of many unidirectional layers [14, 15]. This method was mostly focused on dealing with the crack and stress-concentration problems [16, 17]. The most important point of the mesh superposition method is that it enables the estimation of the mechanical behavior of two models considering their interaction even though they are generated individually.
Park provided a formulation and implemented aspects of the mesh superposition technique for the application to the finite element structural analysis [18]. However, he believed that it was cumbersome to use transition elements or multi-point constraints method to refine the local mesh. Jiang provided a domain superposition technique (DST) for the modeling of woven fabric composites. He presented a coupling technique by using the coordinate interpolations and can solve the interaction displacement of the individual mesh groups [19]. From his method, the fiber mesh and global matrix mesh can be established individually and the displacements are followed the relationships by the interpolation rules with an extra program. In this paper, a new approach for modeling the plain weave textile composite FEM is proposed to predict the mechanical properties. The domain superposition technique is followed in establishing the individual finite element model of tows and matrix while their interrelationships are established with multi-point constraints (MPC) element implemented in MSC.Nastran with special weight factors. This approach is carried out by the pre/post processor software MSC.Patran with its development language, PCL (Patran Command Language). The mechanical properties of the textile composite are investigated. The numerical results are described to compare the results with test results.

2 Domain Superposition Technique

The domain superposition technique is carried out with the “Group” method implemented in MSC.Patran to establish the models of fiber tows and resin matrix. The FEM of fiber tows and resin matrix are generated individually. The two groups mesh overlap around the interface area. The matrix mesh is defined as a global mesh, which is usually established with sparse grid mesh. The fiber mesh is defined as a local mesh, which usually consists of many finer elements. The principle of domain superposition technique and conventional FEM method are illustrated in Fig. 1. Where M-D represents matrix domain, F-D is fiber domain, and G-D is global domain.

![Diagram](image)

(a) Principle of conventional FEM

(b) Principle of domain superposition technique

Fig. 1. Principle of conventional FEM and domain superposition model.

2.1. Approach of Interrelationship between Different Domains

A displacement coupling technique from [19] is implemented here to ensure the same position has the same displacement with C0 continuous in both the global domain and the fiber domain. Because the stress and strain tensors come from displacement in elements, interpolation functions are used for the interrelationship of different domains. From theoretical finite element method, the displacement interpolations of a given coordinate inside the element are addressed by equation (1).

\[
x = \sum_{i=1}^{m} N_i x_i \quad y = \sum_{i=1}^{m} N_i y_i \quad z = \sum_{i=1}^{m} N_i z_i
\]

(1)

where \( m \) is the total number of nodes of the element; \( x_i, y_i, z_i \) are the coordinates of the element nodes in the global coordinate system; the interpolation function \( N_i \) are defined under the local coordinate system, which have variable \( \xi, \eta, \eta \) varying from -1 to 1 independently.

The eight-point isoparametric three-dimensional solid element is used for both domains. The displacement \( u,v,w \) in a three-dimensional solid element is interpolated in the same way as the geometry in equation (2).

\[
u = \sum_{i=1}^{m} N_i \nu_i \quad v = \sum_{i=1}^{m} N_i \nu_i \quad w = \sum_{i=1}^{m} N_i \nu_i
\]

(2)

In NASTRAN, there is a common linear multi-point constraints equation, named RBE3, which can be used to constrain mechanical degrees of freedom and relationship of displacement among different nodes. The function of the multi-point constraint equation is quite similar to the function of displacement interpolate function of finite element, see equation (3).

\[
u = \sum_{i=1}^{m} q_i \mu_i \quad v = \sum_{i=1}^{m} r_i \nu_i \quad w = \sum_{i=1}^{m} t_i \nu_i
\]

(3)
where $u_i, v_i, w_i$ are displacement components of element nodes; $q_i, r_i, t_i$ are the weight factors of displacement. Therefore, if the weight factors are equal to the interpolation function value, the multi-point constraints element has the same displacement results as comes from interpolation function between fiber nodes and global nodes during finite element analysis. The multi-point constraint RBE3 between a node in fiber domain and nodes in global domain is established as shown schematically in Fig. 2. The interpolation function of eight-node isoparametric element is as follows:

$$N_i = \frac{1}{8}(1 + \xi_0)(1 + \eta_0)(1 + \zeta_0)$$  \hspace{1cm} (4)

where $\xi_0 = \xi/\xi_0, \eta_0 = \eta/\eta_0, \zeta_0 = \zeta/\zeta_0, i = 1,2,...8$

Fig. 2. Constraint equation schematic between a node in fiber domain and nodes in global domain.

The interrelationship supposes that the node displacement in fiber domain only interacts with the node displacements of an element in global domain where the fiber node is located. To establish all the constraint equations of the interrelationship between the fiber domain and the global domain, it should identify each element in the global domain related to each node in the fiber domain. This can be easily addressed by establishing the global and fiber domains in different groups in MSC.Patran. From the geometry, the distances of the node and each plane of the element are calculated by equation (5).

$$d = \frac{Ax_0 + By_0 + Cz_0 + D}{\sqrt{A^2 + B^2 + C^2}}$$  \hspace{1cm} (5)

where $A, B, C, D$ are the constants of the plane equation consisting of three nodes that are not collinear. From the six distances of the node and the planes, we can identify if the node of the fiber domain is located in the element in the global domain. This function is carried out with the development language PCL of MSC.Patran to establish the finite element model of textile composite model.

2.2. Material Constitute Model

In a DST model, the RVE model is divided into a global domain $\Omega^G$ and a fiber domain $\Omega^F$, where the matrix domain $\Omega^M$ is the global domain $\Omega^G$ minus fabric domain $\Omega^F$. Since the material properties in global domain is the matrix properties, and the increased volume is the fiber volume, this additive effect can be decreased by reducing the mechanical property values of fiber in the fiber domain.

From DST, dealing with the material properties for the PWF element model follows two principles:

1) All the elements in the global domain are assumed to be isotropic, and the stiffness matrix is $D^M$, in other words, the properties are same as matrix.

2) The stiffness matrix of the fiber domain is:

$$D^F - D^M$$

3 Numerical Analysis of the Textile Composite

3.1. Finite Element Model

In this section, a type of plain weave fabric composite model is built using the domain superposition technique in MSC.Nastran. The warp and fill are the same material, density and bending shape, thus the RVE is symmetric in x and y coordinates. The dimension data of the RVE are given in table 1, where L is the length of RVE, H is the height of RVE, while a and b are the major and minor axis of the section of fiber respectively.

To validate the calculation results by domain superposition technique model, a traditional finite element model based on voxel mesh technique is proposed for comparison. The advantage of voxel mesh is that it can avoid the irregular elements on the resin rich area. However, the disadvantage of this technique is that it needs huge element number and cost much computation time. The statistical node number and element number of both DST model and voxel model are listed in table 2.

The RVE finite element models of textile composite based on DST are shown in Fig. 3 with both the fiber domain and the matrix domain. While such models based on voxel technique are presented in Fig.4 with different group separately.
Table 1. RVE dimensions of Woven Fabric Composite.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>L</th>
<th>H</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length(mm)</td>
<td>1.67</td>
<td>0.362</td>
<td>0.36</td>
<td>0.085</td>
</tr>
</tbody>
</table>

Table 2. Statistical properties of different FEM

<table>
<thead>
<tr>
<th>Node number</th>
<th>Element number</th>
</tr>
</thead>
<tbody>
<tr>
<td>DST model</td>
<td>4584</td>
</tr>
<tr>
<td>Voxel model</td>
<td>137781</td>
</tr>
</tbody>
</table>

Fig. 3. RVE of Textile Composite Based on DST.

(a) Meshes of Fiber Domain

(b) Meshes of Matrix Domain

(c) Meshes of total RVE model

Fig. 4. RVE of Textile Composite Based on Voxel Technique.

(a) Meshes of Fiber Group

(b) Meshes of Matrix Group

(c) Meshes of total RVE model

The material properties of fiber and matrix as well as PWF come from the data of HTA/977-2 with the fiber volume is 65%, the test standards follow the ASTM standard, ASTM D3039 for tension elastic modulus and Poisson ratio, and ASTM D5379 for shear modulus. The material properties are given in table 2.

Table 2. Mechanical properties of fiber, matrix and PWF(fiber volume = 65%).

<table>
<thead>
<tr>
<th>Properties</th>
<th>Fiber(HTA)</th>
<th>Matrix(977/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_x$ (Gpa)</td>
<td>230.974</td>
<td>3.5</td>
</tr>
<tr>
<td>$E_y$ (Gpa)</td>
<td>14.479</td>
<td>3.5</td>
</tr>
<tr>
<td>$G_{xy}$ (Gpa)</td>
<td>22.7527</td>
<td>1.3</td>
</tr>
<tr>
<td>$G_{yz}$ (Gpa)</td>
<td>5.1707</td>
<td>1.3</td>
</tr>
<tr>
<td>$v_{xy}$</td>
<td>0.27</td>
<td>0.35</td>
</tr>
<tr>
<td>$v_{yz}$</td>
<td>0.27</td>
<td>0.35</td>
</tr>
</tbody>
</table>

A PCL program is presented to define the different orientation for both weft and warp tows due to the curvature of the tow. The two perpendicular tows interlace each other and the principal direction of the tow is slightly tilted relatively to the plain weave. The transverse direction of the tow is just follow the $x$ or $y$ axis of the global coordinate which perpendicular to the principal direction and keeps constancy. Fig. 5 give the illustration of the material
coordinates of one tow.

Fig. 5. Material Coordinates of the tow.

To carry out the elastic properties of the RVE, displacement boundary conditions are imposed to the domain superposition model. Axial loading is modeled by a displacement acting on the plane yz at $x = L$. Due to Poisson’s ratio effects, nodes placed in $y = L$, $z = H$ undergo a displacement of $\delta_x$ and $\delta_z$.

The displacement constraints applied to the finite element model are:

\[
\begin{align*}
    u_x(0, y, z) &= 0 & u_x(L, y, z) &= kL \\
    u_y(x, 0, z) &= 0 & u_y(x, L, z) &= \delta_y \\
    u_z(x, y, 0) &= 0 & u_z(x, y, H) &= \delta_z
\end{align*}
\]

Where $k$ is the load factor for forced displacement and is usually taken to be 0.01.

Transverse loading is modeled by a displacement acting on the plane $xz$ in $y = L$ in order to evaluate $E_y$ and on the plane $xy$ in $z = H$ for $E_z$.

To determine $E_y$:

\[
\begin{align*}
    u_x(0, y, z) &= 0 & u_x(L, y, z) &= \delta_x \\
    u_y(x, 0, z) &= 0 & u_y(x, L, z) &= kL \\
    u_z(x, y, 0) &= 0 & u_z(x, y, H) &= \delta_z
\end{align*}
\]

To determine $E_z$:

\[
\begin{align*}
    u_x(0, y, z) &= 0 & u_x(L, y, z) &= \delta_x \\
    u_y(x, 0, z) &= 0 & u_y(x, L, z) &= \delta_y \\
    u_z(x, y, 0) &= 0 & u_z(x, y, H) &= kH
\end{align*}
\]

Where $\delta_x$ is the displacement (caused by Poisson’s ratio effects) of nodes placed in $x = L$. The Poisson ratios for the woven fabric composite can be derived from Equations. (6) to (8). Due to the fact that the RVE is symmetric in $x$ and $y$ coordinates, shear modulus $G_{xy}$ is equal to $G_{yx}$. To determine $G_{xy}$, the displacement constraints applied to the finite element model are:

\[
\begin{align*}
    u_x(x, 0, z) &= 0 & u_x(x, L, z) &= kL \\
    u_y(0, y, z) &= 0 & u_y(L, y, z) &= kL \\
    u_z(x, y, 0) &= 0 & u_z(x, y, H) &= \delta_z
\end{align*}
\]

To determine $G_{xy}$:

\[
\begin{align*}
    u_x(x, y, 0) = 0 & u_x(x, y, H) = kH \\
    u_y(x, 0, z) = 0 & u_y(x, L, z) = \delta_y \\
    u_z(0, y, z) = 0 & u_z(L, y, z) = kL
\end{align*}
\]

\[
\begin{align*}
    u_x(0, y, 0) = 0 & u_x(L, y, 0) = kL \\
    u_y(x, 0, 0) = 0 & u_y(x, L, 0) = \delta_y \\
    u_z(x, 0, z) = 0 & u_z(x, L, z) = \delta_z
\end{align*}
\]

To determine $G_{yz}$:

\[
\begin{align*}
    u_x(x, y, 0) = 0 & u_x(x, y, H) = kH \\
    u_y(x, 0, z) = 0 & u_y(x, L, z) = \delta_y \\
    u_z(0, y, z) = 0 & u_z(L, y, z) = \delta_z
\end{align*}
\]

\[
\begin{align*}
    u_x(0, y, 0) = 0 & u_x(L, y, 0) = kL \\
    u_y(x, 0, 0) = 0 & u_y(x, L, 0) = \delta_y \\
    u_z(x, 0, z) = 0 & u_z(x, L, z) = \delta_z
\end{align*}
\]

\[
\begin{align*}
    u_x(x, y, 0) = 0 & u_x(x, y, H) = kH \\
    u_y(x, 0, z) = 0 & u_y(x, L, z) = \delta_y \\
    u_z(0, y, z) = 0 & u_z(L, y, z) = \delta_z
\end{align*}
\]

\[
\begin{align*}
    u_x(0, y, 0) = 0 & u_x(L, y, 0) = kL \\
    u_y(x, 0, 0) = 0 & u_y(x, L, 0) = \delta_y \\
    u_z(x, 0, z) = 0 & u_z(x, L, z) = \delta_z
\end{align*}
\]

3.2. Numerical Results of Mechanical properties

The mechanical properties are determined by the constitutive equations by calculating the weight average value of all the stresses and strains in the center of each element in the model. The volume average stress and strain over the RVE are determined by:

\[
\bar{\varepsilon}_{ij} = \frac{1}{V} \int_V \varepsilon_{ij} dV = \frac{1}{V^{G} + V^{F}} \left[ \sum \varepsilon_{ij}^G V_i^G + \sum \varepsilon_{ij}^F V_i^F \right]
\]

\[
\bar{\sigma}_{ij} = \frac{1}{V} \int_V \sigma_{ij} dV = \frac{1}{V} \left[ \sum \sigma_{ij}^G V_i^G + \sum \sigma_{ij}^F V_i^F \right]
\]

where $V$ represents the volume of the RVE, $\sigma_{ij}^G$ and $\varepsilon_{ij}^G$ are the element stress and strain in global domain, $\sigma_{ij}^F$ and $\varepsilon_{ij}^F$ are the element stress and strain in fiber domain, respectively.

The predicted effective elastic constants are listed in Table 3, along with test results.

Table 3. Comparison of elastic constants of the textile composite.

<table>
<thead>
<tr>
<th>Properties</th>
<th>DST model</th>
<th>Voxel model</th>
<th>Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_x = E_y (Gpa)$</td>
<td>52.3</td>
<td>64.5</td>
<td>53.4</td>
</tr>
<tr>
<td>$E_z (Gpa)$</td>
<td>10.2</td>
<td>12.4</td>
<td>9.1</td>
</tr>
<tr>
<td>$G_{xy} (Gpa)$</td>
<td>12.2</td>
<td>12.2</td>
<td>6.2</td>
</tr>
<tr>
<td>$G_{yx} = G_{yz} (Gpa)$</td>
<td>9.0</td>
<td>9.1</td>
<td>4.5</td>
</tr>
<tr>
<td>$\nu_{xy} = \nu_{yz}$</td>
<td>0.047</td>
<td>0.044</td>
<td>0.035</td>
</tr>
<tr>
<td>$\nu_{xz}$</td>
<td>0.41</td>
<td>0.43</td>
<td>0.46</td>
</tr>
</tbody>
</table>

As shown in Table 3, results obtained from the domain superposition model are fairly close to experimental results. The longitudinal modulus $E_x$ and transverse modulus $E_y$ are well estimated with a
small error below 2.10%. The shear modulus, through generally higher, compared favorably with the experimental results. The predicted Poisson ratios are also in good agreement with test results. The displacement contours of four different load cases by DST model are presented in Fig.5 and are also carried out by voxel model in Fig.6 for comparison.

Fig. 5. Displacement contours of four different load cases by DST model.

(c) xy-shear  (b) xz-shear

Fig. 6 Displacement contours of four different load cases by voxel model.

(c) xy-shear  (b) xz-shear

From the contour results, the warps in x-tension are the main micro-structure undertaking the load. The highest value of the stress is located in the intersect areas due to stress concentration. Even though the stiffness values obtained from the two models are similar which dominated by the average volume stress, the stress values in the intersection area of voxel model are higher than that of the DST model. This is because the mesh of voxel model is stepwise and interaction by different tow meshes. The other reason is the stress concentration phenomena in the step corner area.

Fig.7 and Fig.8 illustrate the RVE stress contours under tension load of x direction by DST model and voxel model separately, which relates to the mechanical properties Ex. The stress in tows for unidirectional tension is the max principle stress while the stress showed in matrix is the Von Misses stress.

Fig. 7. Stress of weft (a), warp (b), Tows (c) and matrix (d) by DST model under x-tension load.

(c)                                (b)

(a)                                (b)

Fig. 8. Stress of weft (a), warp (b), Tows (c) and matrix (d) under x-tension load.

(c)                                (d)

From the contour results, the warps in x-tension are the main micro-structure undertaking the load. The highest value of the stress is located in the intersect areas due to stress concentration. Even though the stiffness values obtained from the two models are similar which dominated by the average volume stress, the stress values in the intersection area of voxel model are higher than that of the DST model. This is because the mesh of voxel model is stepwise and interaction by different tow meshes. The other reason is the stress concentration phenomena in the step corner area.

Fig.9 and Fig.10 illustrate the RVE stress contours under tension load of z direction by DST model and voxel model separately, which relates to the mechanical properties Ez. The stress in tows for unidirectional tension is the max principle stress while the stress showed in matrix is the Von Misses stress.

From the contour results, the stress of wefts and warps are the same level with that of matrix. The highest value of the stress is located in the intersect areas due to stress concentration. Because the stress is dominated by the matrix, so the stress value is closer of both the different model.
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Fig. 9. Stress of weft (a), warp (b), Tows (c) and matrix (d) by DST model under z-tension load.

Fig. 10. Stress of weft (a), warp (b), Tows (c) and matrix (d) by voxel model under z-tension load.

Fig. 11 and Fig. 12 illustrate the RVE stress contours under in plane shear load by DST model and voxel model separately, which relates to the mechanical properties $G_{xy}$.

Under xy-shear load, there occur rigid body displacements in the tows elements. The stress of matrix and the tows are in the same level. The highest value of the stress is also located in the intersect areas due to stress concentration.

Fig. 13 and Fig. 14 illustrate the RVE stress contours under out plane shear load by DST model and voxel model separately, which relates to the mechanical properties $G_{xz}$.

Fig. 11. Stress of weft (a), warp (b), Tows (c) and matrix (d) by DST model under xy-shear load.

Fig. 12. Stress of weft (a), warp (b), Tows (c) and matrix (d) by voxel model under xy-shear load.

Fig. 13. Stress of weft (a), warp (b), Tows (c) and matrix (d) by DST model under xz-shear load.

Fig. 14. Stress of weft (a), warp (b), Tows (c) and matrix (d) by voxel model under xz-shear load.
Under xz-shear load, the tows are the dominated micro-structure which undertaken the load. The highest value of the stress on both tows and matrix are located in the intersect areas due to stress concentration.

Compare to the voxel model, the DST model can obtain sufficient results from more coarse mesh model and less computing cost. The calculation time of the DST model for the four load cases is 20 minutes while the computing time of voxel model for the same load cases is over 5 hours. Moreover, to build a voxel model with huge element and node numbers is more complicate and cost much time on operating the model by pre & post processor software.

4 Conclusions

A new approach for modeling the textile composite finite element model is proposed. The domain superposition technique was used in establishing the individual finite element model (FEM) of tows and matrix while their interrelationships of displacement are established with multi-point constraints element implemented in MSC.Nastran. This approach is proved that it is an efficient way for numerical simulation of complex composites materials and structures by modeling the textile composite model quickly and easily while avoiding the matrix elements in resin rich regions in the cross area of fibers. The numerical results are also validated by the traditional voxel model and test results.

References