1 Introduction
The design of composite stiffened panels is of great importance in aeronautics, automotive or nautical fields since they are used in building Principal Structural Elements (PSE). Besides the presence of technological defects, the residual strength of such PSE can be of course affected by in-service damage due to stress concentrations at stiffener/skin ply drop-off, at joining regions, as well as due to accidental loading such as impact. The damage ends generally in a delamination crack which proceeds into the PSE due to in-service conditions, among them due to fatigue loading. A numerical method able to reproduce the fatigue delamination of such a composite part is therefore attractive to predict its residual strength.

A relationship between the applied stress intensity factor and the fatigue crack growth (FCG) rate of a defect is generally expressed as a power law. In the case of polymers, adhesives and composites the relationship is traditionally written as a function of the range of strain energy release rate ($\Delta G$) as

$$\frac{da}{dN} = B(\Delta G)^\delta$$  \hspace{1cm} (1)

where $B$ and $d$ are parameters depending on the material and load mixity ratio, and $a$ is the defect length. In general applications the strain energy release rate can only be computed numerically by using, for example, Finite Element (FE) simulations, where $G$ is obtained using the contour integral or the virtual crack closure technique (VCCT). This latter, initially proposed by [1] for four noded elements, was extended to higher order elements by [2] and to three-dimensional cracked bodies by [3], while in recent years it has been successfully implemented in commercial finite element codes, both in two and three dimensions. The prediction of crack growth can be then carried out by a stepwise analysis, each step corresponding to a user-defined crack growth increment, hence the number of cycles can be obtained by manually integrating the crack growth rate computed from the Paris law. To speed up the process, in some finite element softwares this procedure is integrated in special features.

An alternative way for dealing with fatigue crack growth problems is using a cohesive zone model (CZM). This model is largely used for the simulation of the quasi static fracture problems, especially in the case of interface cracks such as delamination in composites. The possibility to simulate the growth of a defect without any remeshing requirements and the relatively easy possibility to manipulate the constitutive law of the cohesive elements makes the cohesive zone model attractive also for the fatigue crack growth simulation. In fact, most of the works where the cohesive zone model is used for the simulation of the fatigue crack growth deal with interfaces [4-10].

In this work a CZM developed and validated by some of the authors in a previous work [11] starting from the concept presented in [7] and then extended to 3D geometries [12] is applied to simulate fatigue delamination. A comparison of the performances with the VCCT embedded in the software Abaqus is made on mode I, mode II and mixed-mode I/II loaded, three-dimensional specimen geometries.

2 Modelling
2.1 VCCT
The VCCT is a well-established (see for example [13]) method to calculate the strain energy release
rate, G. In two-dimensional finite element models, an advancing crack is considered with an initial crack front at a point l, the point l splits into two points l₁ and l₂ forming a new crack front at point i as seen in Figure 1. If u and u' are the displacements in local x-direction and v and v' are the displacements in the y-direction of the points l₁ and l₂ respectively then the strain energy release rate G, based on VCCT may be evaluated as:

\[ G_I = \frac{1}{2t \delta a} F_y (v - v') \]
\[ G_{II} = \frac{1}{2t \delta a} F_x (u - u') \]  

(2)

The total energy release rate is \( G = G_I + G_{II} \). The Abaqus software uses VCCT to evaluate the increment in the number of cycles, \( \Delta N \), related to an increment of crack length, \( \Delta a \), based on Paris-like crack growth rate relationship, Eq. 1, where B and d are taken to be independent of mixed-mode loading.

\[ \frac{dN}{dD} = \frac{1}{B} \]  

(3)

where, \( n \) represents the number of terms in the Fourier series, \( \omega \) is the angular frequency, and \( U_o, U'_o, U_k^s, U'_k^s \) are the coefficients of displacement corresponding to each degree of freedom. The residual vectors are of the same form as the displacement function and are represented by

\[ R(t) = R_o + \sum_{k=1}^{n} [R_k^s \sin k\omega t + R_k^c \cos k\omega t] \]  

(4)

Where \( R_o, R_k^s, \) and \( R_k^c \) are in correspondence with the displacement coefficients \( U_o, U'_o, U_k^s, \) and \( U'_k^s \) respectively, and this vector \( R(t) \) is tracked for each instance of time in the loading cycle. The integration of this function \( R(t) \) over the entire cycle yields the following Fourier coefficients

\[ R_0 = \frac{2}{T} \int_{0}^{T} R(t) dt \]
\[ R_k^s = \frac{2}{T} \int_{0}^{T} R(t) \sin k\omega t dt \]  

(5)
\[ R_k^c = \frac{2}{T} \int_{0}^{T} R(t) \cos k\omega t dt \]

These coefficients are then compared with the tolerances defined for convergence, with respect to the corresponding \( U_o, U'_o, U_k^s, \) and \( U'_k^s \). If the tolerance is met, convergence is achieved and the solution is obtained for that crack length increment.

### 2.2 CZM

The CZM developed in [11] for bonded interfaces can also readily be applied to composite delamination. A damage parameter, D, is applied to the initial stiffness \( K_0 \) of the cohesive elements to yield \( K = (1 - D)K_0 \) where K is the current (damaged) stiffness. The damage evolution with the number of cycles is given by the equation:

\[ \frac{dD}{dN} = -\frac{1}{A_{CZ}} B(\Delta G)^d \]

(3)
where $A_{CZ}$ is the process zone area evaluated by finite element analysis during the simulation, and does not need to be derived from a theoretical model. The model is implemented in the USDFLD subroutine of the software Abaqus following the scheme in Fig. 2.

![Fig. 2. Scheme of fatigue CZM application in Abaqus.](image)

where $\Delta D_i^j$ is taken as follows:

$$
\Delta D_i^j = \Delta D_{\text{max}} \quad \text{if } 1 - D_i^j > \Delta D_{\text{max}}
$$

$$
\Delta D_i^j = 1 - D_i^j \quad \text{if } 1 - D_i^j < \Delta D_{\text{max}}
$$

being $\Delta D_{\text{max}}$ a user-selected value. The computation of the strain energy release rate has been done using $J$-integral. Since $J$ is not available in output for cohesive elements in Abaqus, a procedure based on Abaqus URDFIL user subroutine has been developed to calculate it on-the-run. Neglecting geometrical nonlinearity, the $J$-integral is calculated in two-dimensions along path on the boundaries of the cohesive zone as:

$$
G = J = \int_{\Omega} \left( -\sigma_{12} \frac{\partial u_2}{\partial x_1} - \sigma_{22} \frac{\partial u_2}{\partial x_2} \right) d\Gamma
$$

where the opening/sliding and the stresses in the cohesive elements are taken at the beginning of the increment. For 3D simulations the basic framework of 2D is maintained, but in this case the $J$-integral is evaluated along parallel paths in the thickness direction [12], therefore the mesh of the cohesive zone has to be mapped accordingly. The scheme of Fig. 2 is then applied on each path, looking for the value of $\Delta N_{\text{min}}^i$ among all paths. Under mixed-mode loading, the fatigue crack propagation model of [15] is applied. The fatigue crack growth rate is always represented by Eq. 1, where this time $B$ and $d$ are functions of the mixed mode ratio $MM=G_y/G$:

$$
d = d_j + (d_{II} - d_j) \cdot (MM)^{n_d}
$$

$$
\ln B = \ln B_{II} + (\ln B_I - \ln B_{II}) \left(1 - MM \right)^{n_B}
$$

and $d_j$, $B_I$ and $d_{II}$, $B_{II}$ are, respectively, the parameters of the Paris law in mode I and mode II and $n_d$, $n_B$ are material parameters.

### 2.3 FE models

The fatigue delamination models are tested on various joint geometries characterised by varying mixed mode ratios, in order to verify accuracy, robustness and performance in terms of computational time. In particular, pure mode I loading is simulated with a Double Cantilever Beam (DCB) geometry, pure mode II loading with an End Loaded Split (ELS) geometry and mixed mode I/II loading with a Mixed Mode End Loaded Split (MMELS) geometry, as shown in Fig. 3a-c. Additionally, a Single Lap Joint (SLJ) has been modelled as a representative case of real joint geometry (see Figure (Fig. 3d)). The propagation of the crack in the SLJ was allowed only on one side to simplify the comparison of the models results.

![Fig. 3. Simulated geometries: a) DCB, b) ELS, c) MMELS and d) SLJ. B (width) not represented.](image)
The elastic constants are $E_{11} = 54000\text{MPa}$; $E_{22} = 8000\text{MPa}$; $G_{12} = 2750\text{MPa}$ and $\nu_{12} = 0.25$ corresponding to those of a woven ply composite tested in [16]. The quasi-static cohesive parameters and fatigue crack growth (FCG) properties are taken from [7]. The dimensions are reported in Tab. 1.

**Tab. 1. model dimensions and applied load per unit thickness.**

<table>
<thead>
<tr>
<th></th>
<th>DCB</th>
<th>ELS</th>
<th>MMELS</th>
<th>SLJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>P [N/mm]</td>
<td>10</td>
<td>20</td>
<td>15</td>
<td>200</td>
</tr>
<tr>
<td>$a_0$ [mm]</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>*</td>
</tr>
<tr>
<td>h [mm]</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>10.56</td>
</tr>
<tr>
<td>L [mm]</td>
<td>175</td>
<td>175</td>
<td>175</td>
<td>285.8</td>
</tr>
<tr>
<td>B [mm]</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Lo [mm]</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>110.8</td>
</tr>
</tbody>
</table>

* An initial crack length of 1 mm (1 element) has been specified for the SLJ when simulated using VCCT, while no initial crack length was needed in the case of CZM.

In all the simulation a load ratio $R = 0.05$ is assumed. The mesh is comprised of 8-nodes, 3D continuum shell elements with reduced integration and unit aspect ratio. The cohesive elements are kinematically tied to the two delaminating halves. The mesh sizes are reported in Tab. 2, they represent a good balance between convergence on strain release rate and computational cost.

**Tab. 2. Element sizes**

<table>
<thead>
<tr>
<th>Model</th>
<th>3D Continuum Shell Size (mm)</th>
<th>Cohesive element Size (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCB</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>ELS</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>MMELS</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>SLJ</td>
<td>1.0 (next to cohesive elements)</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Other parameters, specific of each model, are:
- a maximum damage increment, $\Delta D_{\text{max}} = 0.2$ has been used for CZ [17];
- a number of Fourier series terms 49 and a time increment 0.01 have been set for VCCT except for ELS where the time increment was set to 0.1 due to observation of little variation at higher time points. The choice of a very small time increment in the VCCT solution has followed from a convergence study.

Finally, VCCT at present does not allow to modify the coefficient (B) and exponent (d) of Eq. 1 according to the mixed-mode ratio MM, as CZM instead does.

### 3 Comparison of Cohesive zone and VCCT on fatigue delamination/debonding

The two methods are compared with respect to: i) agreement with each other; ii) calculation time. Concerning i), the average value of $G$ (or $G_I$, $G_{II}$) and of crack length along the crack front were considered. Regarding ii), it is the time the analyst has to wait for the crack to reach the knee of the a-N diagram, that is close to fracture. In the cases studied here this means a crack length of 30 mm for all the geometries except SLJ, for which the analyses have stopped at 10 mm of crack length even though still far from fracture. Only the outputs strictly necessary for each model were required, in order to minimize time spent in storing data. The PC used for calculations for CZM is an Intel ® Core™ I, -2630QM 2GHz CPU, with 6 Gb RAM and 579Gb HD (7200rpm, 6Mb cache) and for VCCT Intel® Xeon™ E5645 (Nehalem, 1 core) 2.4 GHz CPU, with 48Gb RAM and 900Gb HD (10k rpm, 12.3Mb cache). Due to crack front bowing, the results are reported in terms of average crack length. An average G has also been evaluated from the analysis output for the sake of comparison.

#### 3.1 Mode I loading (DCB)

Fig. 4 shows the values of $G_I$ obtained by CZM and VCCT. The two sets show quite a good correspondence with each other after an initial phase of about 2mm.

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**Fig. 4. Comparison of $G_I$ obtained by CZM and VCCT in the case of DCB.**
The reason of this initial difference is that CZM needs to develop a process zone ahead of the crack front before starting the propagation, which is not the case of VCCT. Moreover, the process zone develops from inside to outside due to constraint, which results in a bowed crack front in both cases. To evaluate which solution is closer to reality is beyond the scope of the paper.

Skipping the first two millimeters where the cohesive process zone develops, the crack length vs. number of cycles is shown in Fig. 5. Even though the trend of $G_I$ of the two models is the same and the values are similar, the difference in the elapsed number of cycles for a given crack growth is high as the crack growth rate (Eq. 1) is strongly dependent on $\Delta G$. It has to be underlined that the size of cohesive elements is a compromise between the computation time and a simulation of the process zone with sufficient detail, as testified also by the small oscillations of $G$ in Fig. 4. By just decreasing element size, a better agreement would be found.

Fig. 5. Comparison of $a$-$N$ values in the case of DCB obtained by CZM and VCCT.

3.2 Mode II loading (ELS)

Fig. 6 shows the values of $G_{II}$ obtained by CZM and VCCT. The two sets show quite a good correspondence with each other concerning the trend, while CZM results lower than VCCT of 15% in average. The difference in the first millimeters is not so marked as in the case of Mode I DCB, since damage under Mode II develops almost uniformly along the crack front (no bowing).

Skipping the first two millimeters where the cohesive process zone develops, the crack length vs. number of cycles is shown in Fig. 7. Even though the trend of $G_I$ of the two models is the same, the average 15% difference causes a sensible difference in the elapsed number of cycles for a given crack growth.

Fig. 6. Comparison of $G_{II}$ obtained by CZM and VCCT in the case of ELS.

As discussed concerning Mode I, the size of cohesive elements is a compromise between the computation time and a simulation of the process zone with sufficient detail. By just decreasing element size, a better agreement would be found.

Fig. 7. Comparison of $a$-$N$ values in the case of ELS obtained by CZM and VCCT.

3.3 Mixed-Mode I/II loading (MMELS)

Fig. 8 shows the values of $G_I$ and $G_{II}$ obtained by CZM and VCCT. The two sets show quite a good correspondence with each other concerning $G_I$, being CZM in this case higher than VCCT of 10% in average, after an initial phase of about 2mm. The reason of this initial difference was already discussed in 3.1.

The Mode II component instead, results lower than VCCT more markedly than in the case of ELS. A possible reason for this fact is the development of a
Mode III, as described later in the case of SLJ, which origin is now under investigation.

![Graph](image)

**Fig. 8.** Comparison of $G_I$ obtained by CZM and VCCT in the case of MMELS.

From the comparison of the results of MMELS with DCB and ELS, a general statement whether CZM under- or overpredict the strain energy release rate outcome from VCCT cannot be therefore drawn unless further investigations, especially concerning mesh size effects, are done. Skipping the first two millimeters where the cohesive process zone develops, the crack length vs. number of cycles is shown in Fig. 9. Given the differences shown in Fig. 8 in the values of $G_I$ and $G_{II}$ between the two models, a sensible difference in the elapsed number of cycles for a given crack growth obviously turns out.

![Graph](image)

**Fig. 9.** Comparison of $a$-N values in the case of MMELS obtained by CZM and VCCT.

### 3.4 Single-Lap Joint (SLJ)

![Graph](image)

**Fig. 10.** Example of $G_I$, $G_{II}$ and $G_{III}$ distribution along the crack front.

3.4 Single-Lap Joint (SLJ)

Fig. 10 shows the distribution of $G_I$, $G_{II}$ along crack front. It is clear that values are unevenly distributed probably due to the development of an asymmetrical crack front. It is also evident that where $G_{II}$ values drop, a comparable $G_{III}$ shows up. Since Mode III should be concentrated in correspondence of the surfaces, the occurrence of non-negligible $G_{III}$ at some points in the interior is peculiar. Keeping in mind that this results may affect the comparison with CZM, the values of $G_I$ and $G_{II}$ obtained by CZM and VCCT are reported in Fig. 10. The two sets show quite a good correspondence anyway with each other concerning $G_{II}$, being CZM in this case higher than VCCT after about 5mm of crack growth. The Mode I component instead, results lower than VCCT more markedly than in the case of DCB, and differently from the case of MMELS, which is probably related to the uneven distribution shown in Fig. 10.
initial crack and, therefore, the necessity of some millimeters of crack growth for the process zone to develop. Oscillations visible afterwards are instead related to the relatively coarse mesh used to keep calculation time within affordable limits.

As already said in the case of MMELS, a general statement whether CZM under- or overpredict the strain energy release rate outcoming from VCCT cannot be drawn, unless further investigations, especially concerning mesh size effects, are done.

Skipping the first five millimeters where the cohesive process zone develops, the crack length vs. number of cycles is shown in Fig. 12. Given the differences shown in Fig. 11 in the values of $G_I$ and $G_{II}$ between the two models, a sensible difference in the elapsed number of cycles for a given crack growth obviously turns out.

Conclusions

The comparison of the performances of the cohesive zone model presented in [12] and the Virtual Crack Closure Technique (VCCT) embedded in the software Abaqus on mode I, mode II and mixed-mode I/II loaded cracks in composite assemblies yielded the following results:

- the two models are in good agreement concerning Mode I and Mode II conditions and also under Mixed-Mode I/II loading concerning the total $G = G_I + G_{II}$. However, as the crack growth rate (Eq. 1) is strongly dependent on $\Delta G$, even a limited difference causes a sensible difference in the elapsed number of cycles for a given crack growth. Since the element size is a compromise between the computation time and a simulation with sufficient detail, by just decreasing it a better agreement would be found. Hence, a general statement whether CZM under- or overpredict the strain energy release rate outcoming from VCCT cannot be drawn, unless further investigations, especially concerning mesh size effects, are done.
- While the modeling effort is a bit higher (need of introducing a layer of cohesive elements), quicker than VCCT, with calculation times of the order of minutes instead of hours. In the case of SLJ, the increase in calculation time is related to the finer mesh, but the time required by VCCT is becoming so important that high performance computing may be needed if the model complexity would increase further.

The origin of this large difference in performance between the in-house CZ subroutine and the built-in VCCT, both run using the Abaqus solver, can be at least partly found in the Direct Cyclic procedure that is associated with VCCT in Abaqus. Indeed, this procedure requires quite a large number of iterations to satisfy convergence on $\Delta G$ value. On the other hand, relaxing the convergence on $\Delta G$ may affect the number of cycles to failure in a hardly predictable way.

<table>
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<tbody>
<tr>
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<td>186</td>
<td>54</td>
<td>90</td>
<td>57</td>
</tr>
<tr>
<td>VCCT</td>
<td>1012</td>
<td>86</td>
<td>229</td>
<td>1015</td>
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Fig. 11. Comparison of $G_I$ obtained by CZM and VCCT in the case of SLJ.

Fig. 11. Comparison of $a$-N values in the case of SLJ obtained by CZM and VCCT.

3.5 Calculation time

The calculation times are reported in Tab. 3 The CZM results on average two-order of magnitude quicker than VCCT, with calculation times of the order of minutes instead of hours. In the case of SLJ, the increase in calculation time is related to the finer mesh, but the time required by VCCT is becoming so important that high performance computing may be needed if the model complexity would increase further.

The origin of this large difference in performance between the in-house CZ subroutine and the built-in VCCT, both run using the Abaqus solver, can be at least partly found in the Direct Cyclic procedure that is associated with VCCT in Abaqus. Indeed, this procedure requires quite a large number of iterations to satisfy convergence on $\Delta G$ value. On the other hand, relaxing the convergence on $\Delta G$ may affect the number of cycles to failure in a hardly predictable way.

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</table>

Fig. 11. Comparison of $G_{II}$ obtained by CZM and VCCT in the case of SLJ.
CZM results of easier use (no need to identify the proper number of Fourier terms and time increment to represent cyclic loading). At the same time, it results more efficient as the computation is 1-2 order of magnitude faster, even though run on a less powerful PC. Anyway, the origin of this large difference in performance can be at least partly found in the Direct Cyclic procedure that is associated with VCCT in Abaqus.

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References