1 General Introduction
As a result of their superior specific strength and stiffness characteristics, excellent fatigue properties and impressive corrosion resistance, composite materials, such as carbon fibre reinforced plastic (CFRP) are currently finding widespread use in a wide range of high-performance engineering structures. An additional attractive feature of these lightweight materials is their ability to absorb significant energy under certain well-defined loading conditions. Extensive testing has shown that composites, when produced in a tubular form and loaded in compression, are capable of absorbing significant energy through a range of failure mechanisms including fibre fracture, matrix cracking, debonding and delamination [1]. Over the years, this impressive energy-absorbing capability has attracted the interest of many vehicle manufacturers, including Chrysler and Ford. Indeed, Jacob et al [2] calculated that only 600 grams of composite is required to absorb the energy of a medium-sized car travelling at 35 mph. Figure 1 highlights the extraordinary failure characteristics associated with composite materials, where a 10 mm diameter CFRP tube is being crushed at a low rate of strain. These failure modes are typical of those observed in larger diameter tubes, with extensive splaying, fibre fracture and matrix cracking being in evidence.

Figure 2 shows the progress failure of carbon and glass fibre rods subjected to compression. The failure of both rods is initiated on the top end. The carbon fibre rod under compression demonstrates more ductile failure than the glass fibre one. Clearly, the failure pattern of the rod influences its energy absorbing capacity. If buckling failure can be avoided the energy absorption will be maximized. Therefore, it is necessary to introduce constraints to composite tubes or rods. An effective way to apply such constraints is to embed them into PVC foam, so that a progressive crushing of composite tube or rod can be realised. Figure 3 exhibits failed PVC foam core with embedded carbon fibre and glass fibre pins. Both carbon and glass fibre rods turn into dust, which indicate tests with successful constraints offered by PVC foam.

Keywords: PVC foam, impact, finite element, CFRP pin, damage

Fig. 1. Images of a 10 mm tube during and following crushing.

Fig. 2. Progressive failure of carbon and glass fibre reinforced rods under compression.

The energy-absorbing capacity of a composite tube or rod is most frequently evaluated by determining its specific energy absorption (SEA) capability in J/kg. SEA values can vary greatly, for example, from 20 kJ/kg for a pultruded glass fibre/epoxy [2] to values well in excess of 100 kJ/kg for carbon...
Modelling structural behavior of PVC foam panels reinforced by CFRP pins

Fig. 3. Failure modes of sandwich cores with embedded CF (left) and GF (right) rods.

Fig. 4. (a) The variation of the specific energy absorption of circular CFRP tubes with diameter/thickness ratio [7] (b) Photograph of a partially-inserted tube in a polymer foam (note that the tube has not been fully inserted).

Fibre-based systems [3]. The precise value depends on a number of parameters, including the geometry of the tube, its fibre architecture, as well as the mechanical properties of the matrix phase. For example, Hamada and co-workers [3] showed that the energy-absorbing capacity of a 55 mm diameter CFRP tube decreased by fifteen percent in passing from a unidirectional 0° tube to one with its fibres oriented at +/-25°. A number of researchers have studied the influence of specimen geometry on the energy-absorbing capability of composite tubes. Thornton and Edwards [4,5] investigated geometrical effects in the energy-absorbing response of tubes based on circular, square and rectangular cross-sections and showed that the former outperformed both their square and rectangular tubular counterparts. Farley [6] conducted tests on carbon and Kevlar fibre reinforced tubes, with ply orientations typical of those used in sub-floor beam structures and showed that the tube diameter to thickness ratio played a significant role in determining its subsequent strain energy-absorbing capacity.

Similar trends have been observed by Ruzanna and Cantwell [7] following tests on circular composite tubes, Figure 4a, with values increasing by over fifty percent as the D/t ratio is reduced from approximately 42 to 6. This evidence suggests that the use of very low values of D/t can lead to greatly enhanced energy absorption in tubular structures. Following these initial tests on small diameter reinforcements, individual tubes were embedded in a polymer foam (as shown in Figure 4b) and crushed at quasi-static rates of strain [7].

Composite sandwich structures are increasingly finding use in a wide range of lightweight load-bearing engineering structures. Sandwich structures, such as those used in high-performance aerospace components, are typically based on thin composite skins bonded to a low density foam or honeycomb core. The skins are usually thin, often rendering these lightweight panels highly susceptible to damage by a hard projectile, such as that associated with runway debris or hail. A number of investigations have focused on the potential hazard resulting from an uncontained turbine engine failure on outer parts of an aircraft [8-10].

In such sandwich structures, the skin sheets carry bending loads, whilst the core resists transverse shear and through-thickness indentation forces. Therefore, to enhance the load carrying capacity it is desirable to maximize the through-thickness stiffness and strength of the core. One approach to achieve this goal is to add reinforcing pins to the core, with the ends of the pins embedded in woven carbon fibre skin sheets. Cartie and Fleck [11] undertook the theoretical analysis and revealed that the through-thickness stiffness and strength are relatively insensitive to the pin arrangements in pyramidal, tetrahedral and random patterns.

This paper presents numerical modeling of compressive structural behavior of PVC foam core panels reinforced by CFRP pins. Here, the foam was modeled as a crushable foam material with strain hardening, whilst CFRP as an orthotropic linear elastic material up to failure followed by damage initiation and evolution using Hashin criteria. Energy absorption of the sandwich panels made with the cores of different densities was also investigated. Modeling results were compared with the experimental results, in terms of load-displacement relationships, deformation and failure modes. Reasonably good correlation was obtained.
2 Finite element modeling

2.1 PVC foam

The core in the sandwich structures was modeled as a crushable foam using hardening curves obtained following compression tests on square samples. It was assumed that the Poisson’s ratio of all of the foams was 0.32. Deshpande and Fleck [12] proposed a phenomenological yield surface for a closed-cell foam material, given by:

\[
\varphi = \frac{1}{1+\left(\frac{\alpha}{3}\right)^2} \left[q^2 + \alpha^2 \sigma_m^2\right] - \sigma_y^2 \leq 0
\]

(1)

where \(\sigma_y\) is the uniaxial yield strength (in tension or compression) of the foam, \(q\) is the Von Mises stress, and \(\sigma_m\) is the mean stress. The term \(\alpha\) describes the shape of the yield surface, which is related to the ratio of the initial uniaxial yield stress, \(\sigma_y\), and the hydrostatic tensile yield stress, \(p_t\), to the hydrostatic compressive yield stress, \(p_c\), respectively. The yield stress in hydrostatic compression, \(p_c\), describes the development of the size of the yield surface and is given as:

\[
p_c(E_{pl}^{vol}) = \frac{\sigma_c(E_{pl}^{vol})}{\sigma_c^2 + \frac{1}{3} \left(\frac{\sigma_c}{\sigma_c^2 + \frac{1}{3}} \right)}
\]

(2)

where \(E_{pl}^{vol}\) is defined as the plastic volumetric strain in the volumetric hardening model, and is set equal to \(E_{pl}^{axis}\) the compressive plastic strain. The term, \(p_c\), can therefore be determined from a compression test on the foam. Mechanical properties of the foams investigated are shown in Table 1.

Damage development in the foams was modelled by adopting a ductile damage criterion along with a shear damage criterion [13,14]. In the current loading condition (compression dominant), the former criterion is the predominant one. There may be some shear damage introduced by possible buckling failure of CF pins when they are not fully restrained by the foam. The applied strains associated with the initiation of ductile and shear damage, as well as the strain-rate, need to be established. Here, a linear softening law was adopted, involving a linear relationship between the softening stress and the displacement after the onset of the damage for elasto-plastic materials. Damage development was controlled by the fracture energies in tension and in shear. Figures 5 and 6 show load-displacement traces obtained following Mode I (opening) and Mode II (shear tests) tests on the foams. The former was obtained using the single edge notch bending geometry as shown in Figure 1a and the latter using a recently-developed shear geometry as shown in Figure 6a. An examination of

<table>
<thead>
<tr>
<th></th>
<th>C40</th>
<th>C130</th>
<th>C200</th>
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<tbody>
<tr>
<td>Density (kg/m³)</td>
<td>50</td>
<td>130</td>
<td>200</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.32</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>Compressive modulus (MPa)</td>
<td>49</td>
<td>160</td>
<td>280</td>
</tr>
<tr>
<td>Compressive strength (MPa)</td>
<td>0.7</td>
<td>2.6</td>
<td>4.8</td>
</tr>
<tr>
<td>Compressive fracture strain</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>Tensile modulus (MPa)</td>
<td>34</td>
<td>110</td>
<td>175</td>
</tr>
<tr>
<td>Tensile strength (MPa)</td>
<td>1.0</td>
<td>3.8</td>
<td>6.0</td>
</tr>
<tr>
<td>Shear modulus (MPa)</td>
<td>15</td>
<td>47</td>
<td>75</td>
</tr>
<tr>
<td>Shear strength (MPa)</td>
<td>0.6</td>
<td>2.3</td>
<td>3.5</td>
</tr>
<tr>
<td>Shear fracture strain</td>
<td>0.16</td>
<td>0.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>
Fig. 5 indicates that the crosslinked and PET foams fail in an unstable manner, whereas the linear PVC foam fails in a more ductile fashion involving gross plastic deformation in the cell walls. Failure in shear generally occurred in a stable fashion although some load drops were apparent in the crosslinked foams. The toughness properties of the foams were characterised by determining the work of fracture from the energy under the load-displacement traces and the area of the fractured ligament. The resulting values are presented in Table 2. Here, it is evident that the Mode I fracture properties of the linear PVC foams are significantly higher than those associated with its crosslinked counterpart. It is also evident that the Mode II work of fracture properties of the foams are much higher than the Mode I values, with the difference being most pronounced in the crosslinked systems.

<table>
<thead>
<tr>
<th>Foam</th>
<th>Work of fracture in tension (kJ/m²)</th>
<th>Work of fracture in shear (kJ/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C40</td>
<td>0.22</td>
<td>6.31</td>
</tr>
<tr>
<td>C80</td>
<td>0.44</td>
<td>12.6</td>
</tr>
<tr>
<td>C100</td>
<td>0.62</td>
<td>18.4</td>
</tr>
<tr>
<td>C130</td>
<td>0.76</td>
<td>27.6</td>
</tr>
<tr>
<td>C200</td>
<td>1.33</td>
<td>44.2</td>
</tr>
<tr>
<td>L90</td>
<td>6.06</td>
<td>21.2</td>
</tr>
<tr>
<td>L140</td>
<td>12.1</td>
<td>27.3</td>
</tr>
<tr>
<td>PET105</td>
<td>2.3</td>
<td>7.38</td>
</tr>
<tr>
<td>PET135</td>
<td>2.5</td>
<td>18.2</td>
</tr>
</tbody>
</table>

2.2 CFRP pin

Prior to damage initiation, the CFRP pins were modeled as an orthotropic elastic material. The elastic modulus values of the plain weave skins were assumed to be equal in the longitudinal and transverse directions. Damage initiation was modelled using Hashin’s failure criteria [15] which assumes four damage initiation mechanisms, namely fibre tension, fibre compression, matrix tension and matrix compression. Using the longitudinal, transverse and shear effective stress tensor components within the plane of the CFRP, the damage initiation criteria can be determined [16] and are expressed as:

Fibre tension:

\[ F^f_T = \left( \frac{\varepsilon}{X_T} \right) + p \left( \frac{\varepsilon}{X_L} \right) \]

(5)

Fibre compression:

\[ F^c_T = \left( \frac{\varepsilon}{X_C} \right) \]

(6)

Matrix tension:

\[ F^t_m = \left( \frac{\varepsilon}{Y_T} \right) + p \left( \frac{\varepsilon}{S_L} \right) \]

(7)

Matrix compression:

\[ F^c_m = \left( \frac{\varepsilon}{2Y_T} \right) + p \left( \frac{\varepsilon}{2S_L} \right) \]

(8)
where $X_T$, $X_C$ are the tensile and compressive strengths in the longitudinal direction, $Y_T$, $Y_C$ are the tensile and compressive strengths in the transverse direction, $S_L$, $S_T$ are the longitudinal and transverse shear strengths, and $\beta$ is a coefficient that specifies the contribution of the shear stress to the fibre tensile initiation criterion. Here $\beta$ is set to zero, i.e. it is assumed that there is no shear stress contribution involved in the initiation of fibre tensile failure. Table 3 shows the related material parameters that were used.

<table>
<thead>
<tr>
<th>$X_T$ (MPa)</th>
<th>$X_C$ (MPa)</th>
<th>$Y_T$ (MPa)</th>
<th>$Y_C$ (MPa)</th>
<th>$S_L$ (MPa)</th>
<th>$S_T$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>850</td>
<td>500</td>
<td>850</td>
<td>500</td>
<td>90</td>
<td>90</td>
</tr>
</tbody>
</table>

The damage elastic matrix, which relates the stress and strain and controls degradation of the material stiffness, can be expressed as:

$$C_d = \frac{1}{D} \begin{bmatrix}
(1-d_f) \frac{1}{E_1} & (1-d_m) \frac{1}{E_1} & 0 \\
(1-d_f) \frac{1}{E_2} & (1-d_m) \frac{1}{E_2} & 0 \\
0 & 0 & (1-d_s) G D
\end{bmatrix}$$

(9)

In the above equation, $G$ is the shear modulus and $D$ is an overall damage variable, which is given by:

$$D = 1 - (1-d_f) (1-d_m) \nu_1 \nu_2$$

(10)

where, $d_f$, $d_m$, and $d_s$ represent the current state of fibre, matrix and shear damage, respectively.

Damage development is characterised by the negative slope of the equivalent stress-displacement relation following damage initiation. The fracture energies associated with tensile fibre failure $G^f$, fibre compression failure $G^c$, tensile matrix failure $G^m$, and matrix compressive failure $G^c$ are required in order to determine the energy dissipated during the development of damage within the composite.

### 2.3 Mesh generation and boundary/loading conditions

Fig. 7 shows the mesh generation of a PVC foam panel with embedded CFRP pins. Here, the PVC core and the CFRP pins are meshed by 8-noded solid elements with reduced integration. The core size is 50×50×20 (in mm) and the diameters of the pins modelled are 3 and 4 mm. The loading platens on both the top and bottom of the panel are meshed using rigid surface elements. The compressive load is applied to the top platen, with an only degree of freedom in the vertical direction. The bottom platen is fully fixed.

There are various interfaces in the model to be dealt, which include interactions between the foam core and platens, between the CFRP pins and platens, between the CFRP pins and the foam. It is allowed the platens to contact the PVC core cell, i.e. the interior surface of the foam can touch the platens once its external surface fails.

### 3 Results and discussion

Here, PVC foam panels with densities of 50 and 200 kg/m³ are embedded CFRP pins in two diameters, i.e. 3 and 4 mm. Figure 8 shows load-displacement traces obtained from numerical modeling and the corresponding experimental results of the former.
panel. The test results of a PVC foam panel without any CF pin are also shown in the figure to assist better comparison. Clearly, reasonably good correlation has been obtained between the measurements and the FE predictions in terms of the initial stiffness, the peak load and damage evolution, especially for the foam panel embedded with CF pins in diameter of 3 mm. 78% increase on CF pin volume induces 150% enhancement on the peak load.

The comparison of the load-displacement traces for the foam panel with a density of 200 kg/m$^3$ and embedded CF pins in diameters of 3 and 4 mm are exhibited in Figure 9. Again, agreements between the experimental results and the finite element simulations are very good, with well captured features in the initial stiffness, the peak load, the damage evolution and the densification. The enhancement on the peak load is not as much as that of the PVC foam panel with a much lower density as shown in Fig. 8. It is anticipated that with further increasing on PVC foam core density the enhancement of the peak load will also be further reduced.

Figure 10 shows the comparison of energy absorptions obtained from experimental tests and FE predictions for C40 and C200 PVC foam panels with embedded CF pins in diameters of 3 and 4 mm. In general, correlation is quite good. The FE predictions for the foam panels with a lower density are slightly higher than those of experimental measurements, whilst such the predictions for the higher density panels are slightly lower. The possible reason is that due to the weak constraint offered by the foam with the lower density buckling failure of the pin likely occurs, which was not captured by the FE modelling. In the higher density case, the strong constraint from the foam forces the CF pins failure crushing along their longitudinal axis. However, in the modeling such crushing causes element penetration with each other, which underestimates resistance of the pin to the compressive load.
More comprehensive comparisons of energy absorptions related to PVC foam panels with different densities, with and without CF pin embedment are also carried out based on both experimental and numerical work. Here, the diameters of the pin cover 2, 3 and 4 mm. Figure 11 displays such the comparison. The energy absorption is enhanced significantly with increasing of diameter of the CF pin and density of the foam, as expected. However, the rate of enhancement on energy absorption in percentage is decelerated with increasing of foam density. Therefore, there should be an effective foam density range within the energy absorption enhancement can be maximized.

Figure 12 shows the deformation and failure modes of a foam core panel with embedded CFRP pins obtained from test and FE modelling. The core structure was deformed by 75% from its original configuration. The basic features of the foam crushing failure and the pin failure were captured. The failed pin is displayed in Figure 13, which indicates a complete collapse of all pins embedded in the PVC foam.

Conclusions

Finite element models have been developed to simulate load-displacement traces of PVC foam panels with embedded carbon fibre pins, which are
compared with the corresponding test results. Reasonably good correlation has been obtained between the experimental results and FE predictions, in terms of the initial stiffness, the peak load and the damage evolution. Here, three densities of the foam and two sizes of the CFRP pin are investigated. In addition, energy absorption features of the sandwich core structures are captured. The results show that the embedment of CFRP pins inside PVC foam core is a very effective way to enhance energy absorption of this novel sandwich structure. However, there is an optimum range of the PVC foam density, within which the energy absorption can be maximized in percentage sense.

References

[7]. A. Ruzanna and W.J. Cantwell, unpublished work.