Abstract
In this paper, a new dynamic reanalysis method for composite structures is presented. First we construct an iterative format on the basis of the matrix perturbation theory. By introducing a pair of initial eigensolution, we can obtain the natural vibration mode of the modified composite structure, and then use the Rayleigh quotient to compute the corresponding natural frequency. In this way, the computation accuracy can be greatly improved. Finally, a numerical example of composite laminate is included to demonstrate the validity of the proposed method.

1. Introduction
With the development of science and technology, many large complicated structures need to be designed. In structural design or optimization, the procedures are generally iterative and require repeated analysis as the structures are modified, so the equations must be solved and the corresponding analyses usually involve much computational effort. This difficulty motivates extensive studies on reanalysis techniques. The object of reanalysis is to evaluate the structural response for successive modifications in the design without solving the set of the modified implicit equations so that the computational cost is significantly reduced [1].

Structural dynamic reanalysis play an important role in structural design and optimization. Many scholars have paid their attentions to this subject. The traditional method is the Taylor series expansion method, and later approximation method using reduced basis vectors was proposed. In references [2, 3], the basis vector is the Taylor series expansion of the eigenpairs with respect to the structural parameters, and in references [4, 5], the basis vector is the binomial expansion by solving a statics problem. However, the choice of the basis vector is still a research issue. Kirsch et al. [6] introduced the combined approximation algorithm to the vibration analysis process, and made the equivalent treatment for the modal equations and the static equilibrium equations, then constructed the reduced basis vector in the Krylov subspace, finally, gave the error evaluation [7] and solving process [8]. He et al. [9] made the modal reanalysis research for structural large topological modifications with added degrees of freedom. Chen et al. [10-13] proposed the extended combined approximation method, and improved the calculation accuracy using the Rayleigh quotient. Zhang et al. [14] presented the modified combined approximation method based on the inverse iteration and combined approximation algorithm, and improved the calculation accuracy for structural large modifications. Liu et al. [15] made the research on the eigenvalue repeated analysis problems. Chen et al. [16, 17] made the research on the structural dynamic responds repeated analysis problems by introducing the Epsilon algorithm and combining with the Newmann series expansion.

Though many algorithms above are involved, the matrix perturbation method is the classical approach. Yang et al. [18] proposed a method based on the Padé approximation to improve the perturbation theory. The reference [19] improved the calculation accuracy of the perturbation method by combining the matrix perturbation method with the Rayleigh quotient, but it isn’t usually applicable to the case of large parameter modifications [20]. So it is necessary to improve the perturbation method in the calculation accuracy and the range of application.

In this paper, we present a new improved matrix perturbation method. By combining the iteration algorithm with the Rayleigh quotient, we can obtain the eigensolution of the modified composite structure, and the computational accuracy can be greatly improved. A numerical example of
composite laminate is used to illustrate the validity of the proposed method.

2. Theoretical background

Consider the following structural vibration eigenvalue problem
\[ [K] \{u_i\} = \lambda_i [M] \{u_i\} \]  
where \([K]\) and \([M]\) are the structural stiffness and mass matrices, respectively, \( \lambda_i = w_i^2 \) is the ith eigenvalue, \( w_i \) is the ith natural frequency, \( \{u_i\} \) is the eigenvector corresponding to \( \lambda_i \), \( n \) is the total degrees of freedom, and \( \delta_{ij} \) is the Kronecker sign. It is assumed that the eigenvalue is distinct, and Equation (1) will be referred to as the original eigenvalue problem.

After the structural is modified, the stiffness matrix \([K]\) and the mass matrix \([M]\) are also changed, and the amounts of change are \( [K] \Delta \) and \( [M] \Delta \), respectively. The corresponding eigenvalue problem of the modified structure is:
\[ [K] \{u_i\} = \lambda_i [M] \{u_i\} \]  
where
\[ [K] \Delta = [K] + [\Delta K] \]  
\[ [M] \Delta = [M] + [\Delta M] \]

Then the ith eigenvalue \( \lambda_i \) and the corresponding eigenvector \( \{u_i\} \) can be expressed as
\[ \lambda_i = \lambda_i + \Delta \lambda_i \]  
\[ \{u_i\} = \{u_i\} + \{\Delta u_i\} \]

3. The proposed method

From the known methods, we know that firstly the eigenvalues of the structure are usually calculated and then the corresponding eigenvectors are calculated. The following we first construct an iteration format on the basis of the matrix perturbation theory to compute the natural vibration mode of the modified structure, and then use the Rayleigh quotient to compute the corresponding natural frequency.

The eigenvectors of modified structure are expressed as
\[ \{u_{m,i}^{k+1}\} = \{u_i\} + \{\Delta u_{m,i}^{k+1}\} \]  
\[ \{\Delta u_{m,i}^{k+1}\} = \sum_{j=1}^{n} c_{ij}^{n(k+1)} \{u_j\} \]

Substituting Equations (5)-(8) into Equation (3) and rearranging them yields
\[ -([K] - \lambda_i [M]) \{\Delta u_i\} = \]  
\[ ([\Delta K] - \lambda_i [\Delta M]) \{u_i\} + \{\Delta u_i\} \]  
\[ -\Delta \lambda_i ([M] + [\Delta M]) \{u_i\} + \{\Delta u_i\} \]

Let \( \Delta \lambda_i = \Delta \lambda_{m,i}^k \), \( \{\Delta u_{m,i}^k\} \) on equation (11) right-hand side, and we get
\[ -([K] - \lambda_i [M]) \{\Delta u_i\} = \]  
\[ ([\Delta K] - \lambda_i [\Delta M]) \{u_i\} + \{\Delta u_i\} \]  
\[ -\Delta \lambda_{m,i}^k [M] \{u_i\} \]

Premultiplying Equation (12) by \( \{u_j\}^T \) results in
\[ \{u_j\}^T ([K] - \lambda_i [M]) \{\Delta u_i\} = \]  
\[ \{u_j\}^T ([\Delta K] - \lambda_i [\Delta M]) \{u_i\} \]
\[ -\Delta \lambda_{m,i}^k \{u_j\}^T [M] \{u_i\} \]

In Equation (13), let \( \{\Delta u_i\} = \{\Delta u_{m,i}^k\} \). Substituting Equation (10) into Equation (13) and noting that \( \{u_i\} \) is M-normalized, we have the following iterative formula
\[ c_{ij}^{n(k+1)} = \frac{\{u_j\}^T ([\Delta K] - \lambda_i [\Delta M]) \{u_i\}}{\Delta \lambda_{m,i}^k \{u_j\}^T [M] \{u_i\}} \]

For the case of \( j = i \), the coefficient \( c_{ii}^{n(k+1)} \) can be obtained by the \( M \)-normalized condition
\[ \{u_i\} + \{\Delta u_i\}^T [M] \{u_i\} + \{\Delta u_i\} = 1 \]

Expanding Equation (15) and rearranging them yields
\[ -2 \{u_i\}^T [M] \{\Delta u_i\} = \{u_i\}^T [M] \{u_i\} + \{\Delta u_i\}^T [M] \{u_i\} \]

Similarly, noting the Equations (2) and (10), we can obtain
\[ c_{ii}^{n(k+1)} = \frac{-\{u_i\}^T [\Delta M] \{u_i\} + \{\Delta u_i^k\} [M] \{\Delta u_i^k\}}{2} \]
Based on the obtained $c_{ii}^{n(k+1)}$ and $c_{ij}^{n(k+1)}$, the eigenvectors $\{u_{ni}^{k+1}\}$ can be obtained from the Equations (9) and (10).

Then we use the Rayleigh quotient and get the eigenvalues
\[
\lambda_{ni}^{k+1} = \frac{\langle \{u_{ni}^{k+1}\}^T [\bar{K}] \{u_{ni}^{k+1}\} \rangle}{\langle \{u_{ni}^{k+1}\}^T [\bar{M}] \{u_{ni}^{k+1}\} \rangle}
\]
and further get
\[
\Delta \lambda_{ni}^{k+1} = \left\{ \{u_{ni}^{k+1}\}^T ( [\bar{K}] - \lambda [\bar{M}] ) \{u_{ni}^{k+1}\} \right\} / (\langle \{u_{ni}^{k+1}\}^T [\bar{M}] \{u_{ni}^{k+1}\} \rangle) \]

Throughout the iterative process, we first select the iteration-based initial values $\Delta \lambda_{ni}^{0}$, $\Delta u_{ni}^{0}$, and $\{u_{ni}^{0}\}$. After obtaining the $\Delta \lambda_{ni}^{k+1}$, $\Delta u_{ni}^{k+1}$, and $\{u_{ni}^{k+1}\}$, we regard them as the new iteration-based and substitute them into the Equations (14) and (17), and make the repeated iteration of the loop equation until the results converge to the required accuracy.

For the composite laminates, when the material properties or the lay-up angles of the layer are changed, the structural stiffness matrix will change, while the mass matrix remains the same, i.e. $[\Delta M] = 0$, the proposed algorithm is still applicable.

4. A numerical example

In this section, we use a numerical example of composite laminate to demonstrate the validity of the proposed method.

We consider a four-layers composite laminate $[0^\circ/30^\circ]_s$. The thickness of the composite lamina is 0.25mm, and the material properties are
\[
E_x = 1250MPa \quad E_y = E_z = 300MPa
\]
\[
\nu_{xy} = \nu_{xz} = 0.25 \quad \nu_{yz} = 0.01
\]
\[
G_{xy} = G_{xz} = 50MPa \quad G_{yz} = 20MPa
\]

We assume that the fiber orientation angles of the second layer and the third layer are increased by 15° and 30°, i.e. the modified composite laminates are $[0^\circ/45^\circ]_s$ and $[0^\circ/60^\circ]_s$. To compare the accuracies, we adopt the first-order perturbation method, the second-order perturbation method, the William.B.B method, the perturbation method combining with the Rayleigh quotient and the proposed method respectively to compute the eigenvalues and eigenvectors of the modified structure. In the proposed method, we choose the results of first-order perturbation method as the initial iteration base, and make iteration one time.

The error of eigenvalue is calculated by
\[
tr_i = \left| \lambda_{Ei} - \lambda_{Fi} \right| / \lambda_{Ei}
\]
$\lambda_{Ei}$ is the exact eigenvalue of the modified structure, $\lambda_{Fi}$ is the approximate eigenvalue computed by the other several methods. The errors are denoted by $tr_1$, $tr_2$, $tr_3$, $tr_4$, $trn_1$.

The error of eigenvector is calculated by
\[
ur_i = \left| \langle u_{Ei} \rangle^T * \{u_{Fi}\} / sqrt(\langle u_{Ei} \rangle^T * \{u_{Ei}\}) \right|
\]
\[
/ sqrt(\langle u_{Fi} \rangle^T * \{u_{Fi}\}))
\]
$\{u_{Ei}\}$ is the exact eigenvector of the modified structure, $\{u_{Fi}\}$ is the approximate eigenvector computed by the other several methods. The errors are denoted by $ur_1$, $ur_2$, $ur_3$, $ur_4$, $urn_1$.

The results of the first five eigensolutions are listed in table 1-4. From tables we know that the accuracy of the proposed method is the highest, and the validity of the proposed method is demonstrated.

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<th>i</th>
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<th>tr4</th>
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<table>
<thead>
<tr>
<th>i</th>
<th>ur1</th>
<th>ur2</th>
<th>ur3</th>
<th>ur4</th>
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<td>1.0000</td>
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<td>tr2</td>
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Table 4  The accuracy comparison of eigenvectors(fiber orientation angles change by 30°)

<table>
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<th>ur2</th>
<th>ur3</th>
<th>ur4</th>
<th>urn1</th>
</tr>
</thead>
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<td>0.9997</td>
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<td>1.0000</td>
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<td>0.9999</td>
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<td>1.0000</td>
</tr>
<tr>
<td>3</td>
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<td>0.9998</td>
<td>0.9999</td>
<td>1.0000</td>
</tr>
<tr>
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<td>0.9964</td>
<td>0.9763</td>
<td>0.9964</td>
<td>0.9950</td>
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<tr>
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<td>0.9958</td>
<td>0.8162</td>
<td>0.9958</td>
<td>0.9851</td>
</tr>
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</table>

5. Conclusions

A new improved matrix perturbation method for composite structures has been proposed. By constructing an iterative format and combining with the Rayleigh quotient, we can obtain the eigensolution of the modified composite structure, and the calculation accuracy can be greatly improved. A numerical example has demonstrated the validity of the proposed method.

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References


