Curved sandwich panels consisting of fiber-reinforced polymer skins and foam cores are used in naval structures. These hydro-dynamically shaped structures not only provide high specific stiffness and strength but are corrosion-free and low cost alternatives to their existing metal counterparts. An analytical model for the blast response of a double-curvature, shallow composite sandwich panel with polymeric foam core is presented in this paper. The model will elucidate not only the dynamic response of the sandwich panel but address its ultimate failure and the energy absorption of its core up to the point of failure. It follows from previous work by the authors on cylindrical, flat and single curvature sandwich panels [1-3].

2 Problem Formulation

Consider the double-curvature, composite sandwich shell with facesheet thickness \( h \) and core thickness \( H \), as shown in Fig. 1. The mid-surfaces of the composite facesheets are defined with radius \( R_{x_1}, R_{y_1}, R_{x_2} \) and \( R_{y_2} \). Curvilinear coordinates \( x_1, y_1, z_1 \) and \( x_2, y_2, z_2 \) are defined with respect to the outer and inner facesheets, respectively. The sandwich shell is fully clamped along all edges, and is considered to be shallow. The shell is subjected to uniformly distributed pressure pulse of amplitude \( p_o \) and duration \( \Delta T \).

3 Facesheet Kinematics

Donnell’s nonlinear shallow shell theory is used to obtain the strain-displacement relations in the outer facesheet \((i = 1)\) and inner facesheet \((i = 2)\):

\[
\varepsilon_{x_1} = \varepsilon_{x_1,m} + z_i \Gamma_{x_1} \\
\varepsilon_{y_1} = \varepsilon_{y_1,m} + z_i \Gamma_{y_1} \\
\gamma_{x_1,y_1} = \gamma_{x_1,y_1,m} + z_i \Gamma_{x_1,y_1}
\]

where the mid-surface strain and the change in curvature in the outer and inner facesheets are

\[
\varepsilon_{x_1,m} = \frac{\partial u_i}{\partial x_i} + \frac{w_i}{R_{x_1}} + \frac{1}{2} \left( \frac{\partial w_i}{\partial x_i} \right)^2 \\
\varepsilon_{y_1,m} = \frac{\partial v_i}{\partial y_i} + \frac{w_i}{R_{y_1}} + \frac{1}{2} \left( \frac{\partial w_i}{\partial y_i} \right)^2 \\
\gamma_{x_1,y_1,m} = \frac{\partial u_i}{\partial y_i} + \frac{\partial v_i}{\partial x_i} + \frac{\partial w_i}{\partial x_i} \frac{\partial w_i}{\partial y_i}
\]
\[ \kappa_i = -\frac{\partial^2 w_i}{\partial x_i^2} \]  

(7)

\[ \kappa_j = -\frac{\partial^2 w_j}{\partial y_j^2} \]  

(8)

\[ \kappa_{x,y} = -2\frac{\partial^2 w_i}{\partial x_i \partial y_j} \]  

(9)

### 4 Core Kinematics

As indicated in Fig. 1, the core mid-surface is located at mean radii:

\[ R_x = \frac{1}{2}(R_{x1} + R_{x2}) \]  

(10)

\[ R_y = \frac{1}{2}(R_{y1} + R_{y2}) \]  

(11)

and is defined with a set of curvilinear coordinates \( x, y, z \). Core deformations, \( w_0, u_0 \) and \( v_0 \), are assumed to be compatible with the facesheet deflections so that

\[ w_0 = \frac{w_1 + w_2}{2} \]  

(12)

\[ u_0 = \frac{u_1 + u_2}{2} \]  

(13)

\[ v_0 = \frac{v_1 + v_2}{2} \]  

(14)

While thin shell theory was used for the facesheet kinematics, a thick shell theory with first order shear deformation is used for the core. The in-plane strain-displacement relations for the core are

\[ \varepsilon_x = \varepsilon_{x,m} + z\kappa_x \]  

(15)

\[ \varepsilon_y = \varepsilon_{y,m} + z\kappa_y \]  

(16)

\[ \gamma_{xy} = \gamma_{xy,m} + z\kappa_{xy} \]  

(17)

where the mid-surface strain and the change in curvature are

\[ \varepsilon_{x,m} = \frac{\partial u_0}{\partial x} + \frac{w_0}{R_x} + \frac{1}{2} \left( \frac{\partial w_0}{\partial x} \right)^2 \]  

\[ \varepsilon_{y,m} = \frac{\partial v_0}{\partial y} + \frac{w_0}{R_y} + \frac{1}{2} \left( \frac{\partial w_0}{\partial y} \right)^2 \]  

\[ \gamma_{xy,m} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial y} \]  

(18)

(19)

(20)

\[ \kappa_x = \frac{\partial \phi_y}{\partial x} \]  

(21)

\[ \kappa_y = \frac{\partial \phi_x}{\partial y} \]  

(22)

\[ \kappa_{xy} = \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} + \frac{1}{2} \left( \frac{1}{R_x} - \frac{1}{R_y} \right) \left( \frac{\partial u_0}{\partial x} \frac{\partial v_0}{\partial y} - \frac{\partial v_0}{\partial x} \frac{\partial u_0}{\partial y} \right) \]  

(23)

and \( \phi_x \) and \( \phi_y \) are rotations of plane sections about the x- and y-axes, respectively. The nonlinear terms may be neglected for thick cores undergoing very small deflection.

Transverse shear strains are then given by

\[ \gamma_{xz} = \phi_y + \frac{\partial w_0}{\partial x} - \frac{u_0}{R_x} \]  

(24)

\[ \gamma_{yz} = \phi_x + \frac{\partial w_0}{\partial y} - \frac{v_0}{R_y} \]  

(25)

The following approximations are made for the rotation of plane sections and the radial compressive strains:

\[ \phi_x = \frac{u_1 - u_2}{H} \]  

(26)

\[ \phi_y = \frac{v_1 - v_2}{H} \]  

(27)
\[
\varepsilon_z = \frac{w_1 - w_2}{H} \tag{28}
\]

5 Dynamic Response of Shell

In order to be consistent with clamped boundary conditions, out-of-plane and in-plane deflections of the facesheets are expressed in Fourier series as follows:

\[
v_1 = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} a_{nm} \left( 1 - \cos \frac{2\pi x}{a_1} \right) \left( 1 - \cos \frac{2\pi y}{b_1} \right) \cos \frac{m \pi x}{a_1} \cos \frac{n \pi y}{b_1} \tag{29}
\]

\[
v_2 = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} a_{nm} \sin \frac{m \pi x}{a_1} \sin \frac{n \pi y}{b_1} \tag{30}
\]

\[
u_1 = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} b_{nm} \sin \frac{2 \pi x}{a_2} \sin \frac{2 \pi y}{b_2} \tag{31}
\]

\[
u_2 = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} c_{mn} \sin \frac{2 \pi x}{a_2} \sin \frac{2 \pi y}{b_2} \tag{32}
\]

\[
u_2 = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} d_{mn} \left( 1 - \cos \frac{2\pi x}{a_2} \right) \left( 1 - \cos \frac{2\pi y}{b_2} \right) \cos \frac{m \pi x}{a_2} \cos \frac{n \pi y}{b_2} \tag{33}
\]

\[
u_1 = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} e_{mn} \sin \frac{m \pi x}{a_2} \sin \frac{n \pi y}{b_2} \tag{34}
\]

The dynamic response of the sandwich shell is found by satisfying Lagrange’s equations of motion:

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial q_{mn}} \right) - \frac{\partial T}{\partial q_{mn}} + \frac{\partial U}{\partial q_{mn}} = Q_{mn} \tag{35}
\]

where \( T \) and \( U \) are the total kinetic and strain energy in the sandwich shell and \( q_{mn} \) are generalized coordinates, \( a_{mn}, b_{mn}, c_{mn}, d_{mn}, e_{mn} \) and \( f_{mn} \). Plastic work dissipated during core crushing is accounted for in the strain energy expression.

The kinetic energy of the two facesheets are

\[
T_f = \frac{1}{2} \rho_f h \int_{S_1} \left( \dot{w}_1^2 + \dot{v}_1^2 + \dot{w}_2^2 \right) dx dy_1 + \frac{1}{2} \rho_f h \int_{S_2} \left( \dot{w}_2^2 + \dot{v}_2^2 + \dot{w}_2^2 \right) dx dy_2 \tag{36}
\]

where \( S_1 \) and \( S_2 \) are outer and inner facesheet mid-surface areas. The kinetic energy of the core is

\[
T_c = \frac{1}{2} \rho_c H \int_{S_0} \left( \dot{w}_1^2 + \dot{v}_1^2 + \dot{w}_2^2 \right) dx dy_1 \tag{37}
\]

where \( S_0 \) is the core mid-surface area. Generalized forces \( Q_a \) are obtained from virtual work \( \delta W \):

\[
Q_{mn} = \frac{\partial (\delta W)}{\partial (\delta q_{mn})} \tag{38}
\]

For uniformly distributed external pressure pulse \( p(t) \),

\[
\delta W = \int_{S_1} p(t) \delta w_1 dx dy_1 = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} Q_{mn} \delta \tilde{q}_{mn} \tag{39}
\]

Hence,

\[
Q_{mn} = \frac{\partial p(t)}{\partial \delta q_{mn}} \int_{0}^{h_1} 1 - \cos \frac{2\pi x}{a_1} \cos \frac{m \pi x}{a_1} \sin \frac{2\pi y}{b_1} \sin \frac{n \pi y}{b_1} dx dy_1 \tag{40}
\]

The facesheets remain linear elastic even though the core may undergo inelastic deformation during crushing. The elastic facesheet strain energy is

\[
U_f = \frac{1}{2} \int_{S_1} \left( A_{11} \varepsilon_{x_1}^2 + A_{22} \varepsilon_{y_1}^2 + A_{12} \varepsilon_{x_1} \varepsilon_{y_1} + A_{66} \varepsilon_{x_1} \varepsilon_{y_2} + D_{11} \kappa_{x_1}^2 + D_{22} \kappa_{y_1}^2 + 2D_{12} \kappa_{x_1} \kappa_{y_1} + D_{16} \kappa_{x_1} \kappa_{y_2} \right) dx dy_1 \tag{41}
\]

\[
+ \frac{1}{2} \int_{S_2} \left( A_{11} \varepsilon_{x_2}^2 + A_{22} \varepsilon_{y_2}^2 + A_{12} \varepsilon_{x_2} \varepsilon_{y_2} + A_{66} \varepsilon_{x_2} \varepsilon_{y_2} + D_{11} \kappa_{x_2}^2 + D_{22} \kappa_{y_2}^2 + 2D_{12} \kappa_{x_2} \kappa_{y_2} + D_{16} \kappa_{x_2} \kappa_{y_2} \right) dx dy_2
\]
5.1 Core elastic response

During fully elastic response, the stress-strain relation for an orthotropic core is

\[
\begin{align*}
\sigma_x &= C_{11} \varepsilon_x + C_{12} \varepsilon_y + C_{13} \varepsilon_z + \tau_{xy} \\
\sigma_y &= C_{12} \varepsilon_x + C_{22} \varepsilon_y + C_{23} \varepsilon_z + \tau_{yz} \\
\tau_{yz} &= C_{13} \varepsilon_x + C_{23} \varepsilon_y + C_{33} \varepsilon_z + \tau_{xy} \\
\tau_{xy} &= C_{66} \varepsilon_x + C_{66} \varepsilon_y + C_{66} \varepsilon_z + \tau_{xy}
\end{align*}
\]

where \( C_{ij} \) are the elastic stiffness. The elastic strain energy is therefore given by

\[
U_e = \frac{1}{2} \int_0^V \left[ C_{11} \varepsilon_x^2 + C_{12} \varepsilon_y^2 + C_{13} \varepsilon_z^2 + 2C_{12} \varepsilon_x \varepsilon_y \right] dV
\]

where \( V_0 \) is the volume of the core enclosed between the two facesheets. If the foam is transversely isotropic with respect to the 1-2 plane, \( C_{22} = C_{11}, C_{13} = C_{23}, C_{44} = C_{44}, \) and \( C_{66} = (C_{11} - C_{12})/2. \)

If the foam is assumed to be isotropic, then \( C_{33} = C_{22} = C_{11}, C_{23} = C_{13} = C_{12} \)

and \( C_{66} = C_{55} = C_{44} = (C_{11} - C_{12})/2. \)

Although many foams are transversely isotropic, they are often assumed to behave in an isotropic manner. In a flat sandwich panel subjected to lateral pressure loading, the foam core resists primarily transverse shear and compression. Modeling the core as an isotropic material with transverse compression and shear properties is adequate for analysis of flat sandwich panels and often produces accurate solutions. However, in a curved sandwich panel or shell with lateral pressure loading, the foam core must resist in-plane compression in addition to transverse shear and compression. The transversely isotropic properties of the foam would therefore become important in curved sandwich panels with foam cores.

5.2 Core elastic-plastic response

Cellular foam exhibits elastic-plastic behavior under both compression and shear. One of the most commonly used criteria to describe plastic yielding is an isotropic foam yielding [4] of the form

\[
f = \sigma - \sigma_0 = 0
\]

where \( \sigma_0 \) is the flow stress and the effective stress \( \sigma \) is given in terms of the mean stress \( \sigma_m \) and von Mises equivalent stress \( \sigma_e \):

\[
\sigma_e^2 = \frac{1}{1 + (\alpha/3)} \left( \sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 + 2\sigma_{xy}^2 + 2\sigma_{yz}^2 + 2\sigma_{zx}^2 \right)
\]

where \( \alpha = 3\sqrt{2} \), assuming the foam plastic Poisson’s ratio is approximately zero. Once foam plasticity has initiated, the core strain energy may be expressed over elastic and plastic regions separately. The yield criterion described by Eqs. (44) and (45) separate the boundary between elastic and plastic regions.

The strain energy in the plastic core regions are approximated by nonlinear elastic strain energy density enclosed in Region OACD of Fig. 2. In Fig. 2, \( \varepsilon_{ij} \) and \( \sigma_{ij} \) are the strain and stress components at the initiation of plasticity. Following initial yield, stress components do not remain perfectly plastic so that the behavior shown in Fig. 2 is an idealization of the stress-strain behavior in the plastic region. The area under OACD is equivalent to the area under OBD minus the area of shaded triangle ABC. Hence the core strain energy can be expressed by
\[ U_c = \frac{1}{2} \int \int \int \left[ C_{11} \varepsilon_{xx}^2 + C_{22} \varepsilon_{yy}^2 + C_{33} \varepsilon_{zz}^2 + 2C_{12} \varepsilon_{x} \varepsilon_{y} + 2C_{23} \varepsilon_{y} \varepsilon_{z} + 2C_{31} \varepsilon_{z} \varepsilon_{x} + C_{44} \varepsilon_{xx} \varepsilon_{yy} + C_{55} \varepsilon_{yy} \varepsilon_{zz} + C_{66} \varepsilon_{zz} \varepsilon_{xx} \right] \, dx \, dy \, dz \]

\[ + \frac{1}{2} \int \int \int \left[ C_{11} (\varepsilon_x - \varepsilon_{x0})^2 + C_{22} (\varepsilon_y - \varepsilon_{y0})^2 + C_{33} (\varepsilon_z - \varepsilon_{z0})^2 \right] \, dx \, dy \, dz \]

\[ + 2C_{12} (\varepsilon_x - \varepsilon_{x0}) (\varepsilon_y - \varepsilon_{y0}) + 2C_{23} (\varepsilon_y - \varepsilon_{y0}) (\varepsilon_z - \varepsilon_{z0}) + 2C_{31} (\varepsilon_z - \varepsilon_{z0}) (\varepsilon_x - \varepsilon_{x0}) \]

\[ + C_{44} (\gamma_{xx} - \gamma_{x0}) (\gamma_{yy} - \gamma_{y0}) + C_{55} (\gamma_{yy} - \gamma_{y0}) (\gamma_{zz} - \gamma_{z0}) + C_{66} (\gamma_{zz} - \gamma_{z0}) (\gamma_{xx} - \gamma_{x0}) \, dx \, dy \, dz \]

where \( V_e \) and \( V_p \) are the volumes of foam core undergoing purely elastic and elastic-plastic behavior.

### 6 Examples

As an example consider a composite sandwich shell with \( h = 5 \) mm, \( H = 25 \) mm, \( R_{x1} = R_{y1} = 396.1 \) mm, \( R_{x2} = R_{y2} = 365.6 \) mm, and \( a_1 = b_1 = 138.4 \) mm. The facesheet is E-Glass/Vinyl Ester Woven Roving and the core is Divinycell PVC H250. This panel is subjected to uniformly distributed pressure pulse loading. The load duration is 1 ms, and peak pressure is 3.5 MPa.

Elastic and plastic properties of the Divinycell PVC H250 are taken from data provided in Gdoutos et al. [5]. Table 1 list the elastic properties and Fig. 3 shows the transverse compression stress-strain curve of the PVC H250 foam.

Three cases are considered for the foam behaviors: (a) isotropic linear elastic, (b) transversely isotropic linear elastic, and (c) isotropic elastic-plastic. In the isotropic cases, the transverse compression properties of the foam are assumed. Hence, for the isotropic linear elastic case, \( E = E_{33} = 403 \) MPa and \( \nu = \nu_{13} = 0.34 \).

### Table 1 Elastic properties of PVC H250 [5].

<table>
<thead>
<tr>
<th>( E_{11} )</th>
<th>( E_{22} )</th>
<th>( E_{33} )</th>
<th>( \nu_{12} )</th>
<th>( \nu_{13} )</th>
<th>( G_{12} )</th>
<th>( G_{13} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>236</td>
<td>403</td>
<td>0.2</td>
<td>0.34</td>
<td>85</td>
<td>117</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 2 Strain energy in plastic region.

Fig. 3 Transverse compressive stress-strain in Divinycell PVC H250 and other foams.

### 6.1 Elastic response

Equations of motion, assuming isotropic linear elastic core, were solved for the Fourier coefficients described in Eqs. (29)-(34) using MATLAB. The Fourier series was expanded to \( n = m = 6 \). This entailed a total of 242 terms in the solution. The transient deflections of the mid-surface of the core along the x-axis are shown in Fig. 4 for the isotropic linear elastic case. The distribution of stresses along the x-axis at the mid-surface of the core were calculated from Eq. (42), and are shown at \( t = 0.138 \) ms in Fig. 5 (a) and (b) for the in-plane and out-of-plane stress components, respectively. The normal
stresses, both in-plane and out-of-plane, are compressive and roughly the same order of magnitude. This indicates that transversely isotropic foam properties would be needed to accurately determine the foam stress. The maximum transverse shear stress is about twice the highest normal stress.

Solutions from finite element analysis, which will be discussed later, are also shown in these figures for comparison. While the core mid-surface deflections are in very good agreement with FEA, stresses at the core mid-surface are not captured as well, especially at the clamped boundaries. This is a consequence of the first-order shear and uniform transverse compression assumptions which were made to simplify the core kinematics. In addition to this, more terms in the double Fourier series would be needed to improve the accuracy of stresses.

Fig. 4 Transient deflections along x-axis at mid-surface of core, assuming isotropic linear elastic core.

To observe the effect of transversely isotropic properties in the foam, transversely isotropic elastic properties of the foam from Table 1 were assumed instead. The load was kept the same as in the isotropic case, i.e. peak pressure of 3.5 MPa and duration of 1 ms. A comparison of the center panel deflection at the mid-surface of the core is shown in Fig. 6 for both isotropic and transversely isotropic cases. It can be seen that the sandwich shell with transversely isotropic foam model is more compliant than the one with the isotropic foam model. This is

Fig. 5 Stress distribution along x-axis in core mid-surface at t=0.138 ms: (a) in-plane stress components and (b) out-of-plane stress components.

Fig. 6 Comparison of center panel deflections at core mid-surface assuming isotropic and transversely isotropic linear elastic core.
because the in-plane stiffness is much lower than the transverse stiffness. Modeling the foam as isotropic instead of transversely isotropic would result in deflections being under-predicted by 18%.

6.2 Elastic-plastic response

The elastic-plastic response of the composite sandwich shell is obtained in two parts: (1) complete linear elastic response up to the point of initial yielding in the core and (2) elastic-plastic response as the plastic zone spreads in the core. The yield criterion, Eq. (44), must be evaluated during elastic response until it is met. Yielding in the core mid-surface was found to initiate and spread into the four regions shown in Fig. 7. The shaded zones in Fig. 7 represent foam that is plastically deforming.

![Fig. 7 Elastic and plastic regions in core mid-surface.](image)

The yield criterion delineates between elastic and plastic regions. The equations of motion during elastic-plastic response incorporate the spread of the plastic zone because it is introduced in the integration limits of the core strain energy in Eq. (46). The transient deflection of the core mid-surface is shown in Fig. 8. The equivalent stress, as defined in Eq. (44) is also shown at various times in Fig. 9. At $t=0.125$ ms, plasticity would just initiate at $x=22.6$ mm, 110.3 mm. Note that there is relatively good agreement between predicted transverse deflection and effective stresses and FEA, which is described next.

![Fig. 8 Transient deflections along x-axis at mid-surface of core, assuming isotropic elastic-plastic core.](image)

![Fig. 9 Effective stress along x-axis at core mid-surface.](image)

7 Finite Element Analysis

Finite element analysis using ABAQUS Explicit was used to corroborate results from the analytical model. The FEA model for the sandwich shell is shown in Fig. 10. The dimensions of the sandwich shell are the same as in Section 6. Continuum eight node elements were chosen for both facesheet and core (C3D8). The facesheets were orthotropic, linear elastic with properties of E-Glass/Vinyl Ester Woven Roving.
As in Section 6, the core was modeled with PVC H250 properties assuming isotropic linear elastic, transversely isotropic linear elastic, and isotropic elastic-plastic. Crushable foam with isotropic hardening was used for elastic-plastic core response. Solutions for the mid-surface core deflections and stresses from the FEA are given in Figs. 4, 5, 6, 8 and 9.

8 Failure of Sandwich Shell

If the pressure pulse amplitude is high enough, the sandwich shell may fail by foam or facesheet fracture, and these are illustrated in Fig. 11. Simple failure criteria are established for each mode below.

8.1 Facesheet failure

A modified Hashin-Rotem criterion is used to examine lamina failure of the woven roving E-Glass/Vinyl Ester [6]. For the orthotropic shell, the relationship between the principal stresses and strains are given by

\[
\begin{pmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{pmatrix} = \begin{bmatrix}
\overline{Q}_{11} & \overline{Q}_{12} & 0 \\
\overline{Q}_{12} & \overline{Q}_{22} & 0 \\
0 & 0 & \overline{Q}_{66}
\end{bmatrix} \begin{pmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{pmatrix}
\]

(47)

where \( \overline{Q}_{11} = E_{11} / (1 - \nu_{12} \nu_{21}) \), \( \overline{Q}_{22} = E_{22} / (1 - \nu_{12} \nu_{21}) \), \( \overline{Q}_{12} = \nu_{12} E_{22} / (1 - \nu_{12} \nu_{21}) \), and \( \overline{Q}_{66} = G_{12} \).

According to the modified Hashin-Rotem failure criteria, the failure of the composite occurs when

\[
\frac{|\sigma_x|}{X_T} = 1 \text{ if } \sigma_x > 0 \text{ or } \frac{|\sigma_x|}{X_C} = 1 \text{ if } \sigma_x < 0
\]

(48)

\[
\frac{|\sigma_y|}{Y_T} = 1 \text{ if } \sigma_y > 0 \text{ or } \frac{|\sigma_y|}{Y_C} = 1 \text{ if } \sigma_y < 0
\]

(49)

and

\[
\frac{|\tau_{xy}|}{S_L} = 1
\]

(50)

where \( X_T \) and \( X_C \) are the tensile and compressive strength in the x-direction; \( Y_T \) and \( Y_C \) are the tensile and compressive strength in y-direction; and \( S_L \) is the in-plane shear strength.

8.2 Core failure

The stress distributions in the core shown Figs. 5 (a) and (b) indicate that transverse shear stresses dominate all other stress components, when and where yielding begins. The points of yielding are also where the foam would tear if the strains exceed the foam ductility. Although core fracture is of a mixed mode involving not just transverse shear, but also in-plane stress and out-of-plane compression, it
is driven mostly by transverse shear deformations.  
A simple criterion for the onset of core shear failure is to set the transverse shear strain in the core equal to the core transverse fracture strain, $\gamma_f$. Core shear failure occurs when

$$\gamma_{xz} = \phi_y + \frac{\partial \omega_0}{\partial x} - \frac{u_0}{R_x} = \gamma_f$$

(51)

and

$$\gamma_{yz} = \phi_x + \frac{\partial \omega_0}{\partial y} - \frac{v_0}{R_y} = \gamma_f$$

(52)

9 Effect of Foam Crushing

To examine the effect of foam crushing, a composite sandwich shell with $h = 5$ mm, $H = 25$ mm, $R_{x1} = R_{y1} = 396.1$ mm, $R_{x2} = R_{y2} = 365.6$ mm, and $a_1 = b_1 = 138.4$ mm, was considered with various foam cores. The compression stress-strain curves for these foams are shown in Fig. 3, and relevant mechanical properties are listed in Table 2. The facesheets were made of woven E-Glass/Vinyl Ester facesheets and the shell was subjected to a triangular pressure pulse with duration $\Delta T = 1$ ms.

Table 2 Various foam properties.

<table>
<thead>
<tr>
<th></th>
<th>H30</th>
<th>H100</th>
<th>H200</th>
<th>H250</th>
<th>HCP 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$ (kg/m$^3$)</td>
<td>30</td>
<td>100</td>
<td>200</td>
<td>250</td>
<td>400</td>
</tr>
<tr>
<td>E (MPa)</td>
<td>27</td>
<td>105</td>
<td>293</td>
<td>403</td>
<td>650</td>
</tr>
<tr>
<td>$\sigma_0$ (MPa)</td>
<td>0.3</td>
<td>1.66</td>
<td>4.35</td>
<td>6.3</td>
<td>10.3</td>
</tr>
<tr>
<td>$\gamma_f$</td>
<td>0.09</td>
<td>0.4</td>
<td>0.45</td>
<td>0.45</td>
<td>0.35</td>
</tr>
</tbody>
</table>

The peak pressure of the 1 ms-pulse that would just cause either core shear fracture of facesheet rupture is shown in Fig. 12 for the different core materials.

The corresponding plastic work of the core up to failure is given in Fig. 13. Panel failure was due to compressive failure of the distal facesheet in all cases except for the sandwich shell with PVC H30 foam, which failed by core shear fracture. The sandwich shell with the HCP 100 foam core had the highest blast resistance, while the panel with Divinycell H200 foam dissipated the most plastic work.

While the sandwich shell with the HCP 100 foam core was the most blast resistant, it is also the heaviest. The blast resistance of the sandwich shells was compared on a per unit areal weight density basis in Fig. 14. The areal weight density of the sandwich shell is defined by

$$\bar{W} = g(2\rho_f h + \rho_c H)$$

(53)

On a per unit areal weight density basis, the most blast resistant sandwich shell has the PVC H250 core.

Fig. 12 Failure pressure to just cause panel failure (blast resistance).
sandwich shells with various foam cores. The sandwich shell with the softest core, PVC H30 was the weakest, failing by core shear. The sandwich shell with the stiffest core, PVC HCP100, was the most blast resistant, failing by facesheet fracture and with negligible plastic core crushing. On a per unit areal weight density basis, however, sandwich shells with PVC H200 and H250 foam cores, were the most blast resistant. These foams also experienced core crushing and plastic work dissipation before facesheet rupture.

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