Analysis on Low-Velocity Impact Damage of Laminated Composites Using CDM and CZM Models

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**Abstract:** The hierarchical and heterogeneous structure characteristics of composite laminates give rise to the difficulty in the study of the composite laminates damage under low-velocity impact. A numerical method for the evaluation of impact damage is proposed on the basis of the continuum damage mechanics (CDM) and the cohesive zone model (CZM). The intra-ply damages including matrix crack and fiber fracture are represented by the constitutive model which takes into account the physical progressive failure behavior in the ply, using the damage variable to describe the intra-ply damage state. The delamination at the interface between two plies is characterized by the CZM which takes into account the normal crack and the tangent slip, using a specific correlation between traction and separation displacement to describe the initiation and development of delamination. The evolution of the impact damage is investigated, and the correlation between the mesoscale structure and the macroscopic response under impact is constructed effectively by finite element analyses. From the top to the bottom of the specimen, the delamination area increases, which is consistent with the conic distribution of the delamination under impact observed in the experiment. The dynamic analysis is helpful for a thorough understanding of the evolution of low-velocity impact damages in composite laminates.

**1 Introduction**

Composite materials are widely used in advanced structures, especially in the aerospace industry, due to their high specific strength and stiffness over conventional engineering materials [1,2]. Despite these desirable physical properties, composites are fragile and susceptible to transversal impact loading because they are laminar systems with weak interface [3]. Thus, composite structures should be designed to function safely despite the presence of flaws. So it is important to be able to assess the influence of such a damage event on the reduction of strength. The composite plays a crucial role in absorbing energy due to various interlaminar and intralaminar damage mechanisms such as fiber breakage, matrix cracking and delamination. Therefore, the prediction of damage and energy absorption is critical to the structural design of composites. The hierarchical and heterogeneous structure characteristics of composite laminates give rise to the difficulty in the study of the composite laminates damage under low-velocity impact [4-6].

Experimental testing of impact damage is expensive and time consuming [7]. The applicability of analytical solutions, which have a number of simplified assumptions, is limited to model the complex physical phenomena involved in an impact event [8-10]. So the numerical modelling, which can accurately capture the impact event and the damage ensuing, is essential to the analysis on the impact damage. The finite element method (FEM) can simulate impact on composite structures under realistic loads and boundary conditions. More important, both intralaminar and interlaminar failure can be taken into account and incorporated within FE code to accurately capture these damage mechanisms, making the prediction of the initiation and propagation of various damages available.

The numerical analysis of intralaminar damage can be divided into four areas: a failure criterion approach; a fracture mechanics approach; a plasticity or yield surface approach, and a continuum damage mechanics approach [11]. The failure criterion approach assumes that a lamina behaves in an ideally brittle manner and the dominant stiffness and stress components are reduced to a very low value or zero instantaneously after failure. The constraints that are imposed on the failed lamina by
the adjacent lamina and the undamaged elements in the neighborhood of the damage site are disregarded [12]. The fracture mechanics approach cannot be easily incorporated into a progressive failure methodology because its application requires an initial flaw [13]. The plasticity approach is suitable for composites that exhibit ductile behavior [14]. Continuum damage mechanics (CDM) has been increasingly used in the development of numerical damage models for composite materials. The internal variables are introduced for representing the density and/or distribution of the microscopic defects that characterize damage. After damage initiation, the material stiffness is degraded to simulate the propagation of damage until enough energy is absorbed for complete failure of the material. The stress or strain damage initiation criteria are used for the prediction of damage initiation, and failure progression can be modelled via a fracture mechanics approach by associating internal damage variables, representing each form of damages, with their respective fracture energies [4]. For the interlaminar damage, the most common approaches that have been proposed to simulate delamination are the virtual crack-closure technique (VCCT) and cohesive zone model (CZM). Unlike the VCCT approach, the range of application of CZM is not limited to structures with small fracture process zones confined to the crack tip.

In this paper, the intra-ply damages including matrix crack and fiber fracture are represented by the CDM, which takes into account the physical progressive failure behavior within the ply, using the damage variables to describe the intra-ply damage state. The delamination at the interface between plies is characterized by the CZM, which takes into account the normal crack and the tangent slip, using a relation between the stress and the displacement to describe the initiation and development of the delamination, based on the energy dissipation mechanisms.

2 Constitutive Model

2.1 CDM

In this paper, a CDM proposed by Matzenmillner et al. [15] is used for the intra-ply damage constitutive. This approach has been implemented in many research works, demonstrating promising results in predicting the impact response and damage extent [16,17]. The effective stress tensor is introduced as:

\[ \tilde{\sigma} = M\sigma \]  

where \( \tilde{\sigma} \) denotes effective stress tensor, \( M \) represents the rank-four damage operator, and \( \sigma \) denotes nominal stress tensor. For the plane stress condition, \( M \) can be expressed as:

\[
M = \begin{bmatrix}
\frac{1}{1-\omega_{11}} & 0 & 0 \\
0 & \frac{1}{1-\omega_{22}} & 0 \\
0 & 0 & \frac{1}{1-\omega_{12}} \\
\end{bmatrix} 
\]  

(2)

where \( \omega_{11} \) is the damage variable associated with longitudinal (fiber) failure, whereas \( \omega_{22} \) is the damage variable associated with transverse matrix cracking, and \( \omega_{12} \) is a damage variable influenced by longitudinal and transverse cracks. The damage parameters \( \omega_{11} \) and \( \omega_{22} \) are assumed to be different values for tension (\( \omega_{11} \) and \( \omega_{22} \)) and compression (\( \omega_{11c} \) and \( \omega_{22c} \)) in order to account for the phenomenon of one-sidedness. In contrast to \( \omega_{11} \) and \( \omega_{22} \), the damage parameter for shear \( \omega_{12} \) is independent of the sign of the shear stress \( \tau \). The compliance tensor for the damaged lamina is:

\[
H(\omega) = \begin{bmatrix}
\frac{1}{(1-\omega_{11})E_{11}} & -\frac{\nu_{11}}{E_{11}} & 0 \\
-\frac{\nu_{21}}{E_{22}} & \frac{1}{(1-\omega_{22})E_{22}} & 0 \\
0 & 0 & \frac{1}{(1-\omega_{12})G} \\
\end{bmatrix} 
\]  

(3)

where \( E_{11} \) denotes longitudinal Young’s modulus, \( E_{22} \) denotes transverse Young’s modulus, \( G \) denotes shear modulus, and \( \nu \) denotes Poisson’s ratio.

The shape of the loading surface is:

for fiber damage model,

\[
f_{\parallel} = \frac{\sigma_{11}^2}{(1-\omega_{11})^2} X_{c1}^2 - r_{\parallel} = 0 
\]  

(4)

for matrix damage model,

\[
f_{\perp} = \frac{\sigma_{22}^2}{(1-\omega_{22})^2} Z_{c1}^2 + \frac{r^2}{(1-\omega_{22})^2} S_{c}^2 - r_{\perp} = 0 
\]  

(5)

where \( X_{c1}, Z_{c1} \) and \( S_{c} \) denote longitudinal, transversal and shear strength, respectively. \( r \) denotes damage threshold, subscript ‘\( c \)’ denotes compression and ‘\( \perp \)’ denotes tension. Since the damage variables have no direct relation to the micromechanic crack and void growth, they are treated as the phenomenological internal variables for the thermodynamics of irreversible processes. Therefore, the thermodynamic conjugate force to the damage variables is:
where $W$ denotes the strain energy function. In CDM, the variable $Y$ has the meaning of energy, released per volume, due to the advancement of damage. In the space of the thermodynamic forces $Y$, a damage potential $Q(Y, \omega)$ is introduced and the gradient of $Q$ defines the vector-valued function for the damage rule:

$$\dot{\omega} = \sum_i \phi_i \frac{\partial Q_i}{\partial Y}$$

(7)

The scalar function $\phi_i(\sigma, \omega, \dot{\omega})$ controls the amount of damage growth. The models used to simulate ply damage are thoroughly described in references [15] and will not be elaborated here except for their main aspects mentioned above.

### 2.2 CZM

The CDM does not take the delamination between plies into account. In this paper, a cohesive zone model (CZM) [18-20], which is on the basis of energy dissipation mechanisms, is introduced to correlate traction to displacement jump at the interface between plies [21]. Both the normal crack and the tangent slip are taken into account. The damage initiation is related to the interfacial strength, i.e., the maximum traction in the traction-displacement jump relation. When the area under the traction-displacement jump relation curve is equal to the fracture toughness, the traction is reduced to zero and a new crack surface is formed, as shown in Fig. 1. For the normal delamination, the fracture toughness is: $G_{IC}=0.5T\delta^f_I$. But for the tangential delamination, the fracture toughness is $G_{IC}=0.5S\delta^f_{II}$. Here $T$ and $S$ denote the peak tractions in normal and tangential direction, respectively. $\delta^f_I$ and $\delta^f_{II}$ denote the ultimate displacements in normal and tangential direction, respectively.

In this cohesive material model, the total mixed-mode relative displacement $\delta_m$ is defined as:

$$\delta_m = \sqrt{\delta^2_I + \delta^2_{II}}$$

(8)

where $\delta_i=\delta_i$ means the separation in normal direction (mode I), and $\delta_{II} = \sqrt{\delta^2_I + \delta^2_{II}}$ means the separation in tangential direction (mode II). The mixed-mode damage initiation displacement $\delta^0$ is given by:

$$\delta^0 = \delta^0_I \delta^0_{II} \sqrt{\frac{1+\beta^2}{(\delta^0_{II})^2 + (\beta\delta^0_I)^2}}$$

(9)

where $\delta^0_I$ and $\delta^0_{II}$ are the single-mode damage initiation separations, and $\beta=\delta_{II}/\delta_I$ is the mixed mode. The ultimate mixed-mode displacement $\delta^f$ for the power law is:

$$\delta^f = \frac{2(1+\beta)^2}{\delta^0_I} \left[ \left( \frac{T}{\delta^0_I G_{IC}} \right)^\eta + \left( \frac{S\beta^2}{\delta^0_{II} G_{IC}} \right)^\eta \right]^{\frac{1}{\eta}}$$

(10)

### 3 Simulation of Low-Velocity Impact Test

#### 3.1 Geometric and Material Parameters

Two experimental results taken from Refs. [22] and [23] respectively, are used for the model verification. One specimen is made with UD HTS carbon fiber and MVR-444 epoxy, with 0.34 mm thick UD plies and with the total dimension of 150×100×5.44 mm$^3$. They are impacted at their center under 37 J impact energy using an impactor with a mass of 5 kg and a diameter of 15.875 mm. Over the perimeter of a rectangle, 76×127 mm$^2$, the specimen with the stacking sequence [±45/90/0]s is clamped by clamps of which the tip is a 3 mm diameter hemisphere. The independent ply material properties are listed in Table 1.

The other specimen is fabricated using a fiber placement machine and Hexply AS4/8552 carbon-epoxy taws. The test specimens of 150×100 mm$^2$ are fixed between a rigid support and a rigid pressure plate. Both of them have 125×75 mm$^2$ rectangular cuts in the center leaving the part of the specimens free for impact, and are fixed during the impact process. The impactor is also modelled as a rigid body which has a spherically-shaped impact surface with a diameter of 16 mm. The 4.368 mm thick specimen consists of 24 laminated AS4/8552 plies with the stacking sequence [±45/90/0/45/04/]s. The independent ply material properties needed for the definition of the CDM and CZM are measured by standard test methods and summarized in Table 2 which is taken from Ref. [23].

#### 3.2 Discretization

A four-node shell element is used to discretize the lamina with the CDM constitutive model. The stacking sequence is controlled by the material angle at the integration point. The laminates are treated as a stack of shell elements. Each ply is represented by one layer of shell elements. Two adjacent plies are tied together, using a ‘tiebreak’ contact with an interfacial traction-displacement law, which is described by the cohesive zone model. After a
delamination, this contact behaves as a surface-to-surface contact, and the interpenetration of plies can be avoided. Through this approach, the evolution of delamination can be simulated with large critical integration time step. Using the CZM approach, a much larger composite structure can be modelled efficiently, compared with the interfacial solid models which are computationally expensive. The mesh refinement of the impact zone should be taken into account for the correct prediction of the damage evolution, because the accurate computation of the energy release ($G_c$) is mesh-dependent. The impactor is discretized by eight-node solid elements, and has the same mesh refinement as the impact zone of laminates. The finite element model is shown in Fig. 2.

4 Results and Discussion

4.1 Verification of Load and Time

The load-time curves of the impactor for the specimen HTS Carbon/MVR-444 with the stacking sequence [+45/90/-45/0]$_s$ at 37 J impact energy are shown in Fig. 3, both for the test result and the predicted result. Before the time, 0.4 ms for test, 0.3 ms for FE, the impact loads have a steep rise, and then the increase of impact loads slows down. At the time, 1.8 ms for test and 1.9 ms for FE, the impact loads reach a maximum, 13800 N for test and 13200 N for FE, and begin to drop. The load-displacement relations of the impactor for the specimen HTS Carbon/MVR-444 are shown in Fig. 4. The ultimate impact strokes are 4.6 mm in the test and 4.8 mm in the simulation. The simulated results have a good agreement with the experimental results taken from Ref. [22]. The simulated reaction force history of the specimen Hexply AS4/8552 corresponding to 29.7J impact energy and the experimental result taken from Ref. [23] are plotted in Fig. 5. With the increase of the impact displacement, the reaction force on the impactor caused by the composite specimen rapidly increases. There is an obvious drop of the load at the time 0.8ms when the force is about 5500N, and it is observed both in the test and in the simulation. After a transient decline, the interaction force between the impactor and the laminates ascends again and reaches a maximum one, about 5800N in the test and 6000N in the simulation. Then the impactor velocity is eventually reduced to zero and the impact stroke is accomplished, when the ultimate displacement of the impactor is 6.2mm in the test and 6.8mm in the simulation. The kinetic energy is completely transferred to the elastic strain energy and the dissipated energy caused by all kinds of irreversible damage in the laminates. Subsequently, the accumulated elastic strain energy is transferred back to the kinetic energy of the impactor, which is impelled back by the specimen. The simulated result obtained by the numerical method has a good agreement with the experimental result except that there is a little response time delay for the simulation result compared with the test result, especially for the rebound process. However, the delay of the rebound does not influence the discussion of the damage evolvement which occurs mainly in the impact process.

4.2 Verification of Damage Morphology

By comparing the simulated result with the C-Scan result taken from Ref. [22], as delineated in Fig. 6, it can be concluded that the total matrix damage and delamination of specimen HTS Carbon/MVR-444 are well predicted.

The total predicted damage areas of specimen Hexply AS4/8552 are shown in Fig. 7, which also includes matrix damage in each ply and delamination at each interlayer. The simulated result (solid line) agrees well with the experimental data (dotted line) which is taken from Ref. [23].

4.3 Evolution of Energy

As shown in Figs. 8 and 9, for both of these specimens, the total energy of the system approximately does not vary during the impact process, which guarantees the energy conservation in the total system. The kinetic energy is transferred to the elastic strain energy and the dissipated energy caused by all kinds of irreversible damages in the laminates. Subsequently, the accumulated elastic strain energy is transferred back to the kinetic energy of the impactor, which is impelled back by the specimen. The specimen absorbs most of the kinetic energy with the appearance of the intra-ply damage, delamination and friction between different parts. The hourglass energy maintains a low level, which guarantees the accuracy of the numerical calculation.

4.4 Delamination Area

The predicted delamination area at each ply interface of specimen Hexply AS4/8552 is shown in Fig. 10. With the increase of the impact stroke, the delamination expands at the most of the interfaces, except the one between the 23rd and 24th plies, which is at the top of the specimen and close to the
impactor. The delamination first occurs at the interface between the 7th and 8th plies at 0.3ms, while the most of the delamination initiation occurs at 0.8ms when the impact load first declines, as shown in Fig. 5. At the time 1.5ms, the impact force increases again, whereas the delamination area does not increase obviously. At the end of the impact stroke, namely at the time 2.5ms, there is the biggest delamination at the interface between 2nd and 3rd plies which is at the bottom of the specimen and far away from the impactor. From the top to the bottom of the specimen, the delamination area increases, which is consistent with the conic distribution of the delamination under impact observed in experiments [7].

5 Conclusions

By combining the continuum damage mechanics and the cohesive zone model, an integrated numerical method for the evaluation of composite laminates damage under low-velocity impact is proposed. The progressive failure behaviors of fibers, matrix and interfaces between plies are taken into account. The predicted results by the numerical method have a good agreement with the experimental results, and the proposed method leads to stable simulations, overcoming the numerical instability for the evaluation of composite laminates damage under low-velocity impact.

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| Table 1 HTS Carbon/MVR-444 epoxy ply material parameters [23] |
| ----------------- | --- | --- | --- | --- | --- | --- | --- |
| $E_1$ | $E_2$ | $G_{12}$ | $\nu_{12}$ | $X_1$ | $X_c$ | $Z_1$ | $Z_c$ | $S$ | $G_{IC}$ | $G_{IIC}$ |
| GPa | GPa | GPa | GPa | GPa | GPa | MPa | MPa | N/mm | N/mm |
| 114 | 8.6 | 4.45 | 0.3 | 1.85 | 1.2 | 35 | 170 | 75 | 0.17 | 1.2 |

| Table 2 Hexply AS4/8552 ply material parameters [23] |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $E_1$ | $E_2$ | $G_{12}$ | $\nu_{12}$ | $X_1$ | $X_c$ | $Z_1$ | $Z_c$ | $S$ | $G_{IC}$ | $G_{IIC}$ | $\rho$ |
| GPa | GPa | GPa | GPa | GPa | GPa | MPa | MPa | MPa | MPa | N/mm | N/mm | kg/m$^3$ |
| 135 | 9.6 | 5.3 | 0.32 | 2207 | 1531 | 80.7 | 199.8 | 114.5 | 0.28 | 0.79 | 1590 |

Fig. 1. Cohesive zone model

Fig. 2. Finite element model

Fig. 3. Load-time curve of the impactor corresponding to specimen HTS Carbon/MVR-444
Fig. 4. Load-displacement curve of the impactor corresponding to specimen HTS Carbon/MVR-444

Fig. 5. Impactor reaction force and displacement histories corresponding to specimen Hexply AS4/8552

Fig. 6. Total predicted damage (solid line) and C-Scan damage (dotted line) of specimen HTS Carbon/MVR-444

Fig. 7. Total predicted damage (solid line) and C-Scan damage (dotted line) of specimen Hexply AS4/8552

Fig. 8. Energy histories of specimen HTS Carbon/MVR-444

Fig. 9. Energy histories of specimen Hexply AS4/8552

Fig. 10. Delamination area for the different interlaminar zones of specimen Hexply AS4/8552
References


