RESPONSE SURFACES OF MECHANICAL BEHAVIOR OF DRY WOVEN FABRICS UNDER COMBINED LOADINGS

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1 General Introduction

Woven fabric composites are among the most promising alternatives in today’s design of advanced composite structures. High specific strength, flexibility in design and most importantly their ease of manufacturing can be counted among the main reasons for this success. Woven fabrics in dry forms are draped around 3D surfaces (molds) to create complex geometries of parts. Simulation of such forming process is necessary in the application of advanced composites to: (1) predict the deformed shape of the draped fabric; this can in turn be important in estimating critical design characteristics of the final structure such as anisotropic permeability and local direction of reinforcement; (2) choose proper tooling parameters (mould shape, punch speed, force, etc.) with a goal of achieving a near net shape product and avoiding undesirable deformation mechanisms such as wrinkling [1].

This article presents some of the authors’ recent findings regarding meso-level simulation of woven fabrics under combined (axial and shear) loading conditions, which are often the case during aforementioned forming processes. The outcome of these meso-level simulations are used as virtual experiments on a commercial fabric under different combinations of in-plane tension and shear loadings to construct its response surfaces. Consequently, the obtained response surfaces have been implemented into a Finite Element (FE) shell model to simulate a hemi-sphere forming process of the fabric with and without tension-shear coupling effect.

2 Multi-Scale Nature of Woven Fabrics

Woven fabrics are heterogeneous fibrous materials with different structural levels. Macro-level is the scale of the final product and the effective material behavior at this level is affected by the interaction of yarns at a lower scale/level called meso-level. Meso-level is the level of yarns with a characteristic length normally in the order of mm. Geometrical configuration of yarns and their interlacing patterns are the main features of this level. Nonetheless, yarns themselves are made of bundles of thin and long fibers that are the characteristic elements of the next lower scale/level called micro-level with a length scale of µm.

Simulation of the mechanical behavior of woven fabrics at one scale (e.g., meso-scale) is often used to predict their effective properties at a higher scale (e.g., macro-scale) [2]. This can be an alternative solution to costly and time-consuming physical experiments. Moreover, occasionally for particular modes of deformations/ loading conditions, proper testing instruments may not be available, while they can be conveniently tested by user-define codes in numerical analysis tools. When analyzing yarns in meso-level, however, special material modeling techniques should be implemented in order to take into account the heterogeneity of yarns and presence of fibers at micro-level [3]. Similarly, arrangement and deformation of yarns during deformation of a fabric at meso-level must be considered in macro-level models.

2.1 Macro-level simulations

The most common method for simulating a fabric’s mechanical response at this level is the use of homogenous membrane/shell elements in FE solvers [3]. However, as mentioned above, due to the multi-scale nature of woven fabrics, modifications must be applied to macro-level models in order to include complex interactive effects of yarns. One of the most important modifications is due to the fact that there are two main structural directions along the warp
and weft yarns, which can have different relative (non-orthogonal) angles during deformation. Therefore, woven fabrics may not be modeled using standard orthotropic models, and non-orthogonal constitutive material models are required [4]. Recently the Abaqus FE software has added a new material type in its package which is able to track the warp and weft direction of yarns as well as trellising shear in a non-orthogonal coordinate system and take them into account during stress updates [5]. However, currently it needs a full characterization of the fabric mechanical response in order to provide input data for simulations.

The standard woven fabric material model used in Abaqus is called FABRIC where, as illustrated in Figure 1, \( N_i \) and \( n_i \) are the vectors along the directions of warp and weft (for \( i=1,2 \)) before and after deformation, respectively; \( \gamma \) is the trellising shear angle between the warp and weft yarns. The default material properties can be fed into the software by defining the normal stress-stress response of the fibers along warp and weft as well as the shear response due to yarns’ angle change (trellising mode). Coupling between the axial tension of yarns and shearing, or even between the axial components of the warp and weft (bi-axial tension), is not readily incorporated; however, there is the possibility of adding a customized material behavior to this model via a FORTRAN subroutine, named VFABRIC. The input to this subroutine can be deformation parameters such as strains and strain increments along the warp and weft directions as well as the change in the angle between yarns. The required outputs are the stress components along warp and weft and the shear stress. In Section 3 of this article, a combined macro-level experimental and meso-level numerical (virtual experiment) method is proposed in order to collect the required input data for the above mentioned subroutine (VFABRIC). Basically, what needs to be generated is the stress along warp and weft as well as shear stress as a function of axial elongation of yarns and the relative deformed angle between them.

2.2 Meso-level simulations

In order to make the fabric simulations viable, yarns at this level of analysis are assumed to be homogenous (not made of thousands of fibers in contact and with gaps among them). However, effects from micro-level fibrous nature of yarns should and can still be included in the analysis via proper modifications to the material constitutive behavior at the meso-models [6]. In particular, the fact is that the axial stiffness in yarns along the fiber directions is relatively high in tension only, whereas all other stiffness components are comparatively much lower and sometimes non-linearly dependent on the deformation. Therefore, special care must be taken to track the direction of fibers in yarns and apply an appropriate method of updating stress components [7].

3 Shell Model Parameters

3.1 Experimental

Extracting constitutive material parameters of an FE shell model can be achieved via experimentation on a given fabric in the macro level. This normally includes tests on the fabric under uniaxial and/or biaxial extensions and shear modes, which are most often done independently [4]. However, recent researches in the field have shown that the fabric behavior under these two basic deformation modes (tension and shear) can in fact be dependent \([8–10]\), and a considerably higher number of tests would be required for accurate characterization of this coupling phenomenon. Hence, FE simulations (virtual experiments) at this stage appear to be a more valuable tool than before to replace costly or complex experiments; especially that currently there are limited reports on standard instruments/test setups that can apply general in-plane deformations (tension + shear) on dry fabrics. Namely, most recent attempts in this area include facilitating shear testing under equi-biaxial tension of yarns [8], as well as modifications on the traditional picture frame test fixture [10]. The former does not completely impose a uniform trellising shear and the latter does not allow a fully controlled tension during deformation. The authors recently in collaboration with the City College of New York (CCNY) and the Naval Undersea Warfare Center (NUWC) have proposed the design and manufacture of a new test fixture for this purpose [11], which is demonstrated in Figure 2 and is at its final stage of production and validation.
3.2 Using virtual experiments

Numerical/virtual experiments on woven fabrics are normally performed on unit cell models at meso-level. However, this in turn needs the extraction of yarn mechanical properties which can be done using macro-level (fabric) test data and a multi-objective inverse identification. The multi-objectivity of the identification is a key issue to ensure that the resulting model parameters are valid for multiple deformation modes (not a single independent mode). In fact, if yarns are modeled to represent a fabric under general in-plane loading conditions, the inverse identification tests that are implemented to extract yarn properties must be under loadings close to these general conditions. For example, if material properties of yarns are characterized by testing a fabric under bi-axial tension and then running inverse identification, the identified parameters might not be as accurate for predicting the response of fabric under trellising shear (i.e., showing level-2 model errors). Apparently, with the current fabric constitutive models there is a tradeoff between number of test modes used during identification and the accuracy/generality of the identified yarn material properties. In the next section a method for this multi-objective characterization is demonstrated via a case study on a commercial fabric sample.

4 Sample Case Study and Numerical Results

A case study is conducted on samples of balanced plain weave commercially known as TWINTEX®. This material in dry form is made of E-glass fibers combed with Polypropylene (PP) fibers and is widely used in thermoplastic composite industries. First, two common uniaxial tension and shear frame tests were separately applied to the specimens to extract the corresponding response of the fabric (Figure 3). For the shear frame test, after clamping the fabric in the jig an initial tension is applied to the yarns before running the test. The Digital Image Correlation (DIC) technique was employed in the region of interest which is the center square region in the cross shape (region with overlaid contours in Figure 3) to measure the strain field due to initial tension. By averaging strain in this square region it was found that an average strain level of \( \varepsilon_{11} = 1.7 \times 10^{-3} \) exists in the middle region of the sample prior to the test.

Subsequently, the meso-level numerical model was run using a postulated yarn material model as follows.

\[
\{d\sigma\}_{6\times1} = [C]_{6\times6} \{d\epsilon\}_{6\times1} \quad (1)
\]

\[
C_{ij} = \begin{bmatrix}
E_{11} & 0 & 0 \\
0 & E_{22} & 0 \\
0 & 0 & 2G
\end{bmatrix} \quad (2)
\]

\[
E_{11} = \begin{cases} 
A_i & \text{if } \varepsilon_{11} \geq 0 \\
B_iA_i & \text{if } \varepsilon_{11} < 0 
\end{cases} \quad (3)
\]

\[
E_{22} = A_i \varepsilon_{11}^2 + B_2 \quad (4)
\]

\[
G = 25(1 + B_3 |\varepsilon_{11}|) \quad (5)
\]

where, the stiffness units are in MPa and \( A_i \) are the constants that need to be determined via inverse identification and \( B_i \) are constants providing stability to the numerical process or satisfying some physical aspects of yarns. For example in this case study, we considered \( B_1 = 0.05 \); the reason being that yarns in dry form cannot carry compression and because zero stiffness in numerical procedure may result in divergence. It was also considered \( B_3 = 5 \text{ MPa} \) in the yarn transverse crushing formulation in Eq. (4) to avoid zero stiffness in the beginning of the simulation, and \( B_3 = 1000 \) to increase shear stiffness of the yarns after they go under tension (i.e., coupling effect). If a pure trellising deformation occurs, \( \varepsilon_{11} \) will be vanished and \( G \) will be automatically decoupled. In the case of no coupling, the intra-yarn shear modulus (\( G \)) of fabrics in the dry form is usually set at a small value to mimic the small resistance of yarns’ filaments to shear. Ideally, this value is recommended to be zero, however, for stability of numerical methods a small non-zero
value must be used [12]; here a value of 25 MPa was selected.

4.1 Inverse identification

The main steps of inverse identification can be summarized as:
1- Postulating a yarn material model with a set of constants;
2- Running actual tests on a set of selected fabric deformation modes and obtaining the force-displacement responses;
3- Running the corresponding simulations with the FE model in step 1 and an initial (guessed) set of constants;
4- Comparing results of experiments in step 2 to the results of simulations in step 3 and defining a measure that shows the deviation in the results (error); and
5- Running a minimization (optimization) algorithm with a set of constants in step 1 as input and the error function from step 4 as the objective function.

This procedure is also summarized in Figure 4. For the objective/error function, square of the difference between experiments and numerical simulation at selected points is a widely used measure. Because running real experiments on yarns on meso-level while they are interlaced in the fabric is almost impossible, the above inverse identification using macro-level data can be effectively used to extract meso-level yarn material properties. Komeili and Milani [13] applied a similar multi-objective inverse identification from results of three different deformation modes (uni-axial and bi-axial tensions and shear frame test) and extracted a general set of yarn model constants for a glass fabric. It was argued in their work, however, that multi-objective optimization is associated with the concept of Pareto front where there may be more than one optimal variables set depending on the analysis preference/emphasis on one deformation mode over another [14]. In order to handle this challenge systematically, the set of optimization parameters and objectives should be partitioned into two parts. For the current case study, with two sets of experimental data (uniaxial and shear as shown in Figure 4), minimization of the following partitioned objective function is proposed.

\[ \text{Obj}_{\text{total}}(A) = w_{\text{uniaxial}} \cdot \text{Obj}_{\text{uniaxial}}(A) + w_{\text{shear}} \cdot \text{Obj}_{\text{shear}}(A); \quad i = 1, 2, 3 \]  

where \( \text{Obj}_{\text{total}} \), \( \text{Obj}_{\text{uniaxial}} \) and \( \text{Obj}_{\text{shear}} \) are the total objective to be minimized, and objectives for uniaxial tension and shear frame tests, respectively. \( w_{\text{uniaxial}} \), \( w_{\text{shear}} \) are the corresponding weights. \( A_i \) (\( i = 1, 2, 3 \)) are the three model constants that need to be identified (according to Eqs. (3) and (4)). All the parameters assumed to have contributions in the total objective function, and the priority of individual objective functions (deviation of simulation response from particular deformation mode) is taken into account by weights \( w_{\text{uniaxial}} \), \( w_{\text{shear}} \). The challenge of this approach, especially for an inexperienced analyst, is the correct assignment of these weights. An alternative and simpler approach, which is also used here, is that first the optimization is done only on \( \text{Obj}_{\text{uniaxial}}(A_1) \), which means the axial stiffness of yarns are determined from uniaxial tension data. In doing so, for the other two parameters \( (A_2 \) and \( A_3) \) some reasonable fixed values (initial guess) can be used. This is particularly a reasonable approach for woven fabrics given that under uniaxial mode the main dominant factor in the material response is \( A_1 \).

Then, after optimizing \( \text{Obj}_{\text{uniaxial}}(A_1) \) and finding the best fitting \( A_1 \), it can be fixed for the next step to minimize \( \text{Obj}_{\text{shear}}(A_2, A_3) \) and find the optimum values of \( A_2, A_3 \). In order to make results even more accurate, the above ad-hoc optimization procedure can be repeated one or more times by using the new \( A_2, A_3 \) in \( \text{Obj}_{\text{uniaxial}}(A_1) \) and a second iteration on \( \text{Obj}_{\text{shear}}(A_2, A_3) \). Table 1 shows the results of this minimization procedure in the current case study.

Figure 5 shows the comparison between the numerical model results and experiments using the optimized constants in Table 1. In order to ensure that the numerical data are reliable, an extra experiment was conducted in the picture frame mode with no pretension and compared with the simulation using the same model constants identified previously (Figure 6).
4.2 Extracting response surfaces

After extracting the material properties of the yarns and ensuring that the meso-model can fairly predict the response of the fabric under individual modes simultaneously, a 10-level full factorial design of experiment (DOE) was conducted on the unit cell model to extract response surfaces of the material behavior under combined loading conditions. More specifically, this process included running different combinations of macro-level $\varepsilon_1$ and $\varepsilon_2$ (which here are equal to average strain values over the unit cell) as well as the shear strain $\gamma$, and extracting the average stress values at the end of each run. It required $10^3$ virtual experiments which took about 20 hours on a standard workstation.

The resulting response surfaces are illustrated in Figure 7. As it can be seen, the state of axial strain and shear angle deformation has a significant influence on the resulting shear stress, whereas the axial stress is predominantly a function of axial strains in warp and weft direction and not much affected by shear.

Next, using data points on the response surfaces in Figure 7, the following polynomial functions were identified for the fabric in-plane stress components as a function of in-plane axial strain and shear components.

$$\sigma_{ii} = 1.33 \varepsilon_i + 0.422 \varepsilon_j + 12.43 \varepsilon_j^2 \text{ MPa}$$

$$\tau_{ij} = \gamma \left[ 433 - 56.4 \gamma + 2.97 \gamma^2 + 620 \times 10^3 (\varepsilon_{11} + \varepsilon_{22}) 
- 599 \times 10^6 (\varepsilon_{11} - \varepsilon_{22})^2 
- 3.72 \times 10^5 (\varepsilon_{11} + \varepsilon_{22})^2 
- 26.6 \times 10^3 \gamma (\varepsilon_{11} + \varepsilon_{22}) + 501 \gamma^2 (\varepsilon_{11} + \varepsilon_{22}) \right]$$

The resulting curve-fitted response surfaces in the above functions along with the corresponding $R^2$ values are depicted in Figure 8. The reason for choosing these particular forms of functions in Eqs. 7 and 8 rely on the ease of polynomial functions in interpolating between untested points of the actual response surface, and also on a performed sensitivity analysis for adding or removing different terms to polynomials (this subject requires a broader discussion and will be published separately). However, it must be pointed out that for axial tension stress component ($\sigma_{ii}$), although graphs in Figure 7 show some dependency on shear angle initially, however with the development of higher axial strains, that dependency is almost vanished. Accordingly, because the stress levels at this stage are very low, it can be considered that shear angle has no considerable effect on the whole axial tension simulations.

4.3 Simulation of fabric forming with the new material model

As addressed in Section 1, the final goal of most research programs in the field including this article has been to arrive at more accurate material models for better predicting fabrics behavior during composite forming processes. Figure 9 shows a case of forming a dry fabric into a hemisphere die using a punch and die set-up. Figure 10 reveals the shear stress distribution results of this (macro-level) simulation using a fabric material model that assumes the shear and axial stresses are decoupled (by setting the coefficients of $(\varepsilon_{11} + \varepsilon_{22})$ and $(\varepsilon_{11} - \varepsilon_{22})$ to 0 in Eq. 8), and compares it to the actual coupled material model. As it can be seen in Figures 10(a) and (b), the deviation between the two simulations is notable (can be up to 4 times higher in terms of stress level at some spots). It should be added that in this case there was not much force from blank holders (the applied force on the blanks were taken to be 40N), but in other cases there may be even higher tension induced to the fabric during draping. The latter can occur also due to more complex 3D tool geometries.

5 Conclusions

Uniaxial tension test and picture frame test with pretension along yarns in a dry fabric were used in conjunction with a multi-objective inverse identification to extract material properties of dry TWINTEX glass/PP woven yarns at meso-level. The model was verified against an independent picture frame test with no pretension. Then, virtual/numerical experiments on the meso-level unit cell of the fabric were employed to extract the material response surfaces at macro-level, which in
turn were used in shell modeling of the fabric in a FE package to simulate the fabric forming during manufacturing. A focus of the work was on highlighting the effect of combined axial tension-shear loading where the interaction between the shear and axial loading modes is present. The proposed response surface methodology may also be used in characterizing the coupling behavior of fabrics under other complex loading modes, especially when experimentation with customized fixtures is infeasible or costly. Finally, an interpolated surface response for axial tension along yarns and trellising shear stress was postulated and implemented into a hemisphere forming simulation. Results suggested the importance of accounting for the coupling behavior between tension and shear stress in the fabric.

As a future work, the research can include studying the coupling phenomenon on more complex shapes and more practical case studies, and validating them with experimental measurements from actual manufacturing set-ups.

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![Figure 2](image2.png)

Figure 2. (a) The new test fixture developed for characterizing the coupling of shear-tension in dry fabrics (© CCNY-UBC-NUWC)

![Figure 1](image1.png)

Figure 1. Illustration of the non-orthogonal material model used in Abaqus FABRIC model [5]

![Figure 3](image3.png)

Figure 3. Experiments used for multi-objective inverse identification of yarns’ model constants; (a) the uni-axial tension; (b) shear frame test with pre-tension
Figure 4. Flow diagram of the suggested multi-objective inverse identification

Figure 5. Comparison between results of axial experiments and simulations with the set of input data generated from inverse identification (the share frame test is done with pre-tension); along with side view of unit cell in tension and top view in shear

Figure 6. Validating the meso-level model by using the parameters from inverse identification in an independent shear frame test with no pretension
Figure 7. Stress response of the representative fabric unit cell under different combined bi-axial-shear loading conditions; (a) the axial stress response along the yarns; different layered surfaces are representing different levels of shear angle ($\gamma$) fixed from 0 to 32 degrees. (b) the shear response, different layered surfaces are representing different levels of axial strain ($\varepsilon_{11}$) fixed from 0 to 0.01.

Figure 8. Response surfaces of the stress field fitted from the data in Figure 7, along with the resulting $R^2$-values.

Figure 9. Geometry of a punch and die in forming simulation (dimensions are in mm)
Figure 10. Comparison of shear stress in the fabric with (a) coupling effect between tension and shear stress; (b) decoupled tension and shear stress.

Table 1. Optimum values of variables obtained after two steps of the minimization procedure

<table>
<thead>
<tr>
<th>Variable</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value (GPa)</td>
<td>9.39</td>
<td>4.58</td>
<td>14.15</td>
</tr>
</tbody>
</table>

References


