DELAMINATION DETECTION IN ROTORCRAFT FLEXBEAMS USING FRACTAL DIMENSIONS

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Keywords: Flex beam, Torsional mode shape, Fractal dimensions, Homogenized delaminated sectional properties, Finite element model.

1 Introduction

Rotorcrafts employ flexbeams in bearingless rotors inherently introducing virtual hinges and bearings respectively for the flap, lead-lag, and pitch motions of the blades [1]. Laminates, in general, and laminated flexbeams, in particular, are prone to defects such as matrix cracking, strength and stiffness degradation due to aging/corrosion, and edge delamination [2-4]. Delamination in a composite structure may occur either during the manufacturing process or due to fatigue load during the service period of the structure. Delaminations may not be visible or barely visible to external inspection when embedded within the composite structures. However, delamination reduces the stiffness of the structure and hence it affects the load-carrying ability and natural frequencies of the structure [3]. Structural health monitoring of a composite flexbeam becomes necessary to evaluate the integrity and operational limits of the structure in the presence of delamination, as failures of flexbeam often have tragic consequences [1,3].

There are two methodologies for modeling a rotating cantilever beam: one uses classical Cartesian approach; and the other uses mixed or hybrid bases, (a non-Cartesian arc length basis and two Cartesian bases along the cross-sectional dimensions of the beam) [4]. The mixed or hybrid basis introduced by Kane et al. [5] for modeling the general beam is prominently used by recent researchers [6-8]. The advantages of employing the mixed bases system are that the effect of centrifugal stiffening, Coriolis forces etc. are accounted for in the derivation itself rather than by adding just the terms of interest to equations derived for the beam with an inertially fixed support [5].

Many delamination models are mentioned in the literature [9]. Most of the delamination models for beams, mentioned in the literature, are for beams with through-width delamination; exceptions, which address the issue of beams with partial delamination in the width-wise direction, are restricted to cross-sectional analysis [10-12]. A novel partial delamination model for composite laminated beam is developed in the present work and implemented on a rotating flexbeam.

Modal parameters of the rotating cantilever beam are typically obtained either using assumed modes method / Rayleigh-Ritz method [4,5,7,8] or by finite element method [6,13]. The finite element method is more amenable for parametric study in the presence of delamination and for obtaining time response of the beam for transient analysis.

There is a plethora of methods available for damage detection, using vibration-based techniques. Most of these methods are reviewed in the article by Fan and Qiao [14]. The presence of delamination reduces the local stiffness of the structure, and hence there will be a change in the modal parameters of the structure. Therefore, the essence of model-based delamination detection is to capture and amplify the changes in modal parameters of the structure using suitable quantification methods. Katz fractal dimension [15,16] is one such method, which is used in the present article to characterize the changes in the mode shape of a composite rotating flexbeam in the presence of delamination.

Many models available in the literature for rotating composite cantilever beam often do not include the torsional degree of freedom. Exceptions include works of Hodges [17] and co-workers, who treat beams as 1-D structures with arbitrary loading, including torsion. The present article includes the following contributions: a) Rotating composite beam with flap-wise flexure, pitch-wise torsion and stretch degrees of freedom; b) Modeling of partial delamination in the width-wise direction of the beam; c) Effect of angular speed on the natural frequencies of delaminated beam; and d) Fractal dimension method for delamination detection.

2 Strain energy and kinetic energy

2.1 Assumptions

Following assumptions are made for the displacement-based formulation of the beam.

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2 Strain energy and kinetic energy

2.1 Assumptions

Following assumptions are made for the displacement-based formulation of the beam.
1. The plane section normal to the centerline of the beam remains plane after deformation [18].
2. Through-the-thickness deformation and stresses are negligible [19].
3. Beam bending is predominantly in the flapping plane as shown in Fig. 1. In other words, lead-lag bending is ignored.
4. The warping of the cross-section of the beam is ignored.

2.2 System description and system variables

Rotating flexbeam is as shown in Fig. 1. The system consists of a rigid rotating hub referred as A, on which a flexbeam is mounted. The motion of reference frame A is known with respect to another inertial reference frame N. A Cartesian coordinate system associated with reference frame A is centered at the rotating hub, having unit orthogonal triads $\vec{a}_1, \vec{a}_2, \vec{a}_3$. The angular velocity of the hub with respect to the inertial reference frame is denoted by $\vec{\omega}^H$. Deformed beam is depicted using solid lines and undeformed beam is depicted using dotted lines. The fixed end of the reference curve is denoted by point O. Displacement of a reference point $P^*$ in the undeformed state of the beam to P in the deformed state of the beam is denoted by vector $\vec{u}$, but a non-Cartesian variable $s$ is used to denote axial stretch of the beam. Cartesian variable $u_i$ is used to denote flap-wise bending deformation. According to H. H. Yoo et al. [20], the relation between stretch $s$ and the Cartesian variables is given by

$$s = u_i + 1/2 \left[ \frac{\partial u_x}{\partial \xi} \right]^2 + \frac{\partial u_y}{\partial \xi} \right] d\xi + (H.O.T.)$$

(1)

where H.O.T. refers to higher order terms,

$$h_v = \frac{1}{2} \left[ \int_0^\infty \frac{\partial u_x}{\partial \xi} \right]^2 d\xi, \quad h_w = \frac{1}{2} \left[ \int_0^\infty \frac{\partial u_y}{\partial \xi} \right]^2 d\xi$$

and $\xi$ is a dummy variable. Similarly, the velocity of the reference point P with respect to the inertial reference is given by

$$\vec{v}^P = \vec{v}^A + \vec{\omega}^A \times ((a + x)\vec{a}_1 + \vec{u}) + \vec{a}^A \vec{v}^P$$

(2)

where $\vec{v}^A$ is the inertial velocity of point O and $\vec{a}^A \vec{v}^P$ is the relative velocity of P in frame A obtained by taking the time derivative of $\vec{u}$ in frame A. Taking the time derivative of Eq. 1 and rearranging the terms,

$$\dot{u}_x = \frac{d}{d\xi} \left[ \frac{u_x}{\sqrt{1 + \left( \frac{\partial u_x}{\partial \xi} \right)^2 + \left( \frac{\partial u_y}{\partial \xi} \right)^2}} \right] d\xi + (H.O.T.)$$

(3)

2.3 Displacement fields

Adopting the displacement field assumed in [7] and accounting for the displacement of material points of the cross-section due to torsion as well, the following expressions are obtained:

$$s(X,Z,t) = \psi(X,t) \theta(X,t)$$

$$u_x(X,Z,t) = -Z \psi(X,t)$$

$$u_z(X,Y,t) = 0 \psi(X,t)$$

(4)

Here, $X, Y, Z$ are the coordinates along the unit vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$ respectively; the transverse displacement of any material point initially located on the undeformed mid-plane is denoted by $u_3$; $s$ is the axial stretch of the mid-plane; $\theta$ is the rotation of the cross-section about the $Y$-axis, and $\psi$ is the rotation of cross-section about the $X$-axis. Independent spatial variables are $X, Y, Z$, while $t$ is the temporal variable.

2.4 Displacement field in the delaminated region

$$\dot{u}_x = \ddot{s} - \ddot{h}_v - h_w$$

(3)}
The partially delaminated beam cross-section is shown in Fig. 2. The delaminated beam cross-section is partitioned into six regions as depicted in Fig. 2. This particular configuration of delamination and partitioning of the cross-section of the beam are ideal to simulate most of the delamination cases arising in beam-like structures, through straightforward specialization and/or extrapolation of the analytical model being developed herein. Consider the interface of the healthy beam segment and the delaminated beam segment in the longitudinal direction. The following conditions have to be met:

1. Generalized displacement continuity: displacement function \( u_s(X,t) \) and rotations \( \psi(x,t) \) and \( \Theta(X,t) \) will have to remain continuous, as \( u_1 \) or \( \Theta \) can only be discontinuous when there is a cut or kink, respectively, at the section and the two sections slide relative to each other in the \( Y - Z \) plane.

2. Displacement \( s(X,Z,t) \) will be continuous at the interface.

3. Vectorial sum of the generalized forces should be zero, to satisfy equilibrium at the interface.

The axial stiffness of a stack of laminae is the sum of the axial stiffnesses of the individual laminae, and remains same irrespective of laminae being bonded together or not. On the other hand, the bending stiffness of the stack of laminae depends on how well the laminae are bonded together. Hence, in the delaminated composite beam, reduction in the bending stiffness of the beam occurs due to delamination, but the axial stiffness remains unchanged. To sum up, the axial stiffness of the cross-section is the sum of the axial stiffnesses of all the six subsections shown in Fig. 2. The bending stiffness of the beam is the sum of the bending stiffnesses of the six subsections evaluated about their respective midplanes, plus the contribution of the axial stiffnesses of the sections along with a modification factor, the need for which is explained below. The contribution of the axial stiffnesses of the subsections towards the bending stiffness of the beam is due to the eccentricity of the midplanes, or more generally the neutral axes of the subsections of the beam with respect to the neutral axis of the beam. The modification factor associated with axial stiffness is to compensate for the reduction in the bending stiffness due to delamination. The axial displacement function, \( s \) of the delaminated cross-section in the region \( x_s > X < x_s + l_s \) of the beam is piecewise continuous, and is given by Eq. 5.

\[
s(X,t) = \begin{cases}
0 & \text{region 1 and 6} \\
0 + Z_0\theta[m_s \phi(Y)] + Z_0\theta & \text{region 2} \\
0 - Z_0\theta[m_s \phi(Y)] + Z_0\theta & \text{region 3} \\
0 + Z_0\theta[m_s \phi(Y)] + Z_0\theta & \text{region 4} \\
0 - Z_0\theta[m_s \phi(Y)] + Z_0\theta & \text{region 5}
\end{cases}
\]  

(5)

The factor \( m_s \phi(Y) \) is a function, which specifies the amplification of the displacement in the delaminated region (due to the delamination), or equivalently the reduction in stiffness contribution of the delaminated region (due to the delamination). The function is introduced based on the following reasoning: Consider the case of total delamination in the widthwise direction of the beam. The following points are known/observed, either from the mechanics or from the literature on delamination studies:

1. The reduction in the bending stiffness of the laminate is a function of delamination length, manifesting in experiments as a reduction in the bending natural frequencies, as the delamination length increases for a given length of the beam.

2. When we have two beams placed one above another without bonding (total delamination in the widthwise direction as well as lengthwise direction of the beam), the bending stiffness of the combined beam is the sum of the bending-stiffnesses of the individual beams about their respective neutral axes.

3. The reduction in the bending stiffness of the laminated beam is also a function of the delamination location in the thickness direction of the beam.

Based on the above points, the modification function for the present study is assumed to be composed of two functions: one denoted as \( m_s \phi \) is a modification factor, and the other \( \phi(Y) \), which defines how displacement, and hence stress, varies at the junction of the delaminated section with the healthy section in the beam cross-section. The modification factor \( m_s \phi \) accounts for finiteness of the delamination length and width in the beam. For the present studies, \( \phi(Y) \) is assumed to be the unit step function, and the generic form of delamination factor is \( m_s \psi = 1 - f(l_s) f(w_s) \), where \( f(l_s) \) is a function of longitudinal delamination ratio.
\( I_d = \frac{L_d}{L}, \) and \( f(w_d) \) is a function of width delamination ratio \( w_d = \frac{B_d}{B} \), where, in turn, \( B_d \) is the breadth of the edge or internal delamination. In the present study, the modification factor is assumed to be \( m_y = 1 - I_d * (1 - w_d)^2 \).

### 2.5 Strains and curvatures

The non-zero strains in the beam are the axial normal and transverse shear strains. The axial strain is

\[
\varepsilon_x = \frac{\partial s}{\partial X} = \left( \frac{\partial u_x}{\partial X} + \frac{1}{2} \frac{\partial u_y}{\partial X} \right)^2 + \frac{1}{2} \left( \frac{\partial u_z}{\partial X} \right)^2 + (HOT.).
\]

This shows that the axial differential of the approximate \( s \) is equivalent to the well-known Von Karman strain measure for beams, \( \frac{\partial s}{\partial X} \) represents the stretching strain of the beam, and Eq. (1) provides a simple way of determining \( u_1 \) when the numerical values of \( s, u_2, \) and \( u_3 \) are known.

Shear strains are

\[
\gamma_x = \frac{\partial u_x}{\partial X} + \theta = \frac{\partial u_x}{\partial X} + Y \frac{\partial y}{\partial X} + \theta
\]

and

\[
\gamma_y = -Z \frac{\partial y}{\partial X}.
\]

Mid-plane strains and curvatures are

\[
\varepsilon_{0x} = \frac{\partial s}{\partial X}, \quad \kappa_x = \frac{\partial \theta}{\partial X}, \quad \text{and} \quad \kappa_y = \frac{\partial \gamma}{\partial X}.
\]

### 2.6 Strain energy, potential energy and kinetic energy

Employing the assumptions of classical laminated plate theory (CLPT), the stresses in the lamina [19] are given by

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}
= \begin{bmatrix}
\tilde{Q}_{11} & \tilde{Q}_{12} & \tilde{Q}_{16} \\
\tilde{Q}_{12} & \tilde{Q}_{22} & \tilde{Q}_{26} \\
\tilde{Q}_{16} & \tilde{Q}_{26} & \tilde{Q}_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
\]

where \( \tilde{Q}_{ij} \)'s are transformed lamina stiffness coefficients [19].

For bending of beams predominantly in \( \tilde{a}_1 - \tilde{a}_3 \) plane, resulting from the applied force and moment in \( \tilde{a}_1 - \tilde{a}_3 \) plane, the normal stress \( \sigma_x \) is equal to zero, while the strain \( \varepsilon_y \) may not be zero due to Poisson’s effect [21].

Thus, stresses in a lamina of the beam are

\[
\begin{bmatrix}
\sigma_x \\
\tau_{xy}
\end{bmatrix}
= \begin{bmatrix}
\tilde{Q}_{11} & \tilde{Q}_{16} \\
\tilde{Q}_{16} & \tilde{Q}_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\gamma_{xy}
\end{bmatrix}
\]

where,

\[
\tilde{Q}_{11} = \tilde{Q}_{11} - (\tilde{Q}_{12})^2 / \tilde{Q}_{22}, \quad \tilde{Q}_{16} = \tilde{Q}_{16} - (\tilde{Q}_{12} * \tilde{Q}_{26}) / \tilde{Q}_{22},
\]

and \( \tilde{Q}_{66} = \tilde{Q}_{66} - (\tilde{Q}_{26})^2 / \tilde{Q}_{22} \). In CLPT, the laminae are assumed to be in a state of plane stress and hence the shear stress in \( \tilde{a}_1 - \tilde{a}_3 \) plane is not considered, but in the case of Timoshenko beam theory, we do have non-zero shear stress in the thickness direction. Fortunately, for orthotropic laminae, with the thickness direction being transverse to the fibers, the shear-extension coupling coefficients are not present and hence shear stress in the \( \tilde{a}_1 - \tilde{a}_3 \) plane is given by \( \tau_{xz} = \tilde{Q}_{55}\gamma_{xz} \).

Torsion induces shear stress \( \tau_{xy} \) and \( \tau_{xz} \) in the beam. The expressions for shear stresses \( \tau_{xy} \) and \( \tau_{xz} \) can be obtained using membrane analogy, as discussed by Swanson [22]. Contribution of torsion-induced shear stress \( \tau_{xz} \) towards the total potential of the beam is negligible, for rectangular strips and hence can be neglected [22]. Presence of delamination in the beam will shift the shear center of the beam cross-section in the delaminated length segment of the beam; the shift in the shear center of the delaminated beam segment can be obtained by following the formulation similar to that for thin-walled beam cross-sections as followed by other researchers [23-25]. The cross-sectional torsional stiffness of the delaminated beam segment can be determined following the procedures of [24, 25].

The strain energy in the beam is given by

\[
V = \frac{1}{2} \int \int \left\{ \sigma \right\}^T \left\{ \varepsilon \right\} dZdYdX
\]

where,

\[
\left\{ \sigma \right\} = \begin{bmatrix}
\sigma_x \\
\tau_{xy}
\end{bmatrix} \quad \left\{ \varepsilon \right\} = \begin{bmatrix}
\varepsilon_x \\
\gamma_{xy}
\end{bmatrix}
\]
where, \( B_{11} \) and \( B_{16} \) are sectional stiffness coefficients populating \([D]_s\) are given in the appendix.

The potential energy due to centrifugal force acting on the beam is given by [26],

\[ W = \int_0^L \int_{A \rightarrow 0} \int_{A \rightarrow 0} \rho \left( \frac{d^2}{dZ^2} \right) (ds - d\zeta) dY dX \]

where, \( \zeta \) is a dummy variable in place of \( X \), and

\[ ds - d\zeta = \left[ \frac{d^2}{dZ^2} + \frac{\partial}{\partial \zeta} \right] \frac{1}{2} \left( \frac{\partial u}{\partial \zeta} \right)^2 d\zeta, \]

which can be approximated to \( \frac{1}{2} \left( \frac{\partial u}{\partial \zeta} \right)^2 d\zeta \), using binomial expansion.

Kinetic energy of the laminated composite beam is

\[ T = \frac{1}{2} \int_0^L \int_{A \rightarrow 0} \int_{A \rightarrow 0} \rho \left( \frac{d}{dZ} \frac{d}{dZ} \right) dZ dY dX \]

where \( \rho \) is the density of the equivalent homogenized material constituting the local lamina. Assuming all the laminae to be made of the same composite, albeit with different orientations in general,

\[ T = \rho Bh \left\{ \int_0^L \left( \delta_0 \right)^2 + \left( \dot{\delta}_0 \right)^2 + \frac{1}{12} \int_0^L \left[ (B^2 + h^2)\psi^2 + h^2 \dot{\theta}^2 \right] \right\} dX \]

\[ + \frac{1}{12} \int_0^L \left[ (\dot{\delta}_0)^2 - 2\dot{\delta}_0 \dot{\psi} + \frac{\ddot{\psi}^2 h^2}{12} (\dot{\psi} - \theta) \right] dX \]

\[ + \frac{1}{12} \int_0^L \left[ \left( \frac{\ddot{\psi}^2 h^2}{12} (\psi^2 + \dot{\theta}^2) + (\ddot{\psi}^2) (a + x - h_w - s_0)^2 \right) \right] dX \].

### 3 Finite element model

The finite element model for the composite beam consists of two-noded elements with four degrees of freedom (DoFs) per node. The DoFs at each node are: stretch \( s \), transverse displacement \( u_3 \), bending rotation \( \theta \), and twisting rotation \( \psi \). C0 continuous shape functions are considered for the axial displacement and the twist. Shape functions for transverse displacement and bending rotation are obtained from the Timoshenko functions (C1-type). C1-type shape functions for Timoshenko beam element are prone to shear locking phenomenon, but by using Timoshenko functions, the phenomenon of shear locking can be avoided [27]. The nonlinear and small parameters contributing to the kinetic energy and work potential are discarded to linearize the equations. Coriolis effect on the beam is also neglected while solving for the eigenvalues.

![Finite element discretization](image)

**Fig. 3: Finite element discretization**

Finite element discretization of the beam is shown in Fig. 3. \( K^e \), mentioned in Fig. 3, denotes the healthy composite beam element stiffness matrix, \( M^e \) stands for the mass matrix of the composite beam element, and \( K^d \) denotes the delaminated beam element stiffness matrix. The mass matrix of the delaminated segment of the beam is considered to be the same as that of the healthy segment of the beam, as there will be hardly any loss of material or any significant changes in the cross-section due to delamination.
3.1 Shape functions

Shape functions for stretch, bending, and twist of the composite laminated beam are given below by Eqs. 6-8, where \( x \) is the local co-ordinate system and \( l_e \) is the element length. The shape functions for stretch and twisting rotation are

\[
N_i = 1 - \frac{x}{l_e} \quad \text{and} \quad N_j = \frac{x}{l_e}
\]

where \( l_e \) is the length of the element under consideration.

The shape function for the transverse displacements, based on the Timoshenko displacement functions [27] or equivalently from the interdependent interpolation element of Reddy [18], are given by

\[
N_i^b = 1 + \frac{2\mu}{l_e^3} x^3 - \frac{3\mu}{l_e^3} x^2 + \left( \frac{\mu - 1}{l_e^2} \right)x
\]

\[
N_j^b = \frac{\mu}{l_e} x^3 - \frac{(3\mu + 1)}{2l_e} x^2 + \left( \frac{\mu + 1}{2l_e} \right)x
\]

\[
N_k^b = -\frac{2\mu}{l_e^3} x^3 + \frac{3\mu}{l_e^3} x^2 + \left( 1 - \frac{\mu - 1}{l_e^2} \right)x
\]

and

\[
N_l^b = \frac{\mu}{l_e} x^3 + \left( 1 - \frac{3\mu}{l_e} \right)x^2 + \left( \frac{\mu - 1}{2l_e} \right)x
\]

The shape functions for the bending slope, based once again on the Timoshenko displacement functions [27] or equivalently from the interdependent interpolation element of Reddy [18], are

\[
N_i' = -\frac{6\mu}{l_e} \left( \frac{x^2}{l_e} - x \right)
\]

\[
N_j' = -\frac{3\mu}{l_e} x^2 + \left( \frac{3\mu + 1}{l_e^2} \right)x
\]

\[
N_k' = \frac{6\mu}{l_e^3} x^3 - \frac{6\mu}{l_e^3} x
\]

and

\[
N_l' = -\frac{3\mu}{l_e} x^2 - \left( 1 - \frac{3\mu}{l_e^2} \right)x
\]

Here, \( \mu_e = \frac{1}{1+12\lambda_e} \) and \( \lambda_e = \frac{D_{11}}{A_{35}L^2} \). The displacements for the composite beam element in the form of nodal displacements are now given by

\[
\begin{align*}
\{s(x)\} &= \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} s_1 \\ w_1 \\ \theta_1 \\ \psi_1 \\ s_2 \\ w_2 \\ \theta_2 \\ \psi_2 \end{bmatrix} \\
\{u_3(x)\} &= \begin{bmatrix} 0 & N_1^3 & N_2^3 \end{bmatrix} \begin{bmatrix} u_3 \end{bmatrix} \\
\{\theta(x)\} &= \begin{bmatrix} N_1^\theta & N_2^\theta \end{bmatrix} \begin{bmatrix} \theta_1 \\ \psi_1 \end{bmatrix} \\
\{\psi(x)\} &= \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} N_1 \theta \end{bmatrix} \begin{bmatrix} 0 & 0 & N_2 \theta \end{bmatrix}
\end{align*}
\]

The above equation can be represented in compact form as \( \{\tilde{q}\} = [N]\{q_e\} \), where \( [N] \) is the shape function matrix, and \( \{q_e\} \) is the nodal displacement vector.

3.2 Stiffness matrix and mass matrix

Stiffness matrix of the beam element is composed of cross-sectional stiffnesses in extension, torsion, transverse shear, and bending; includes the rotational motion-induced stiffnesses; and is given by

\[
K_e = \int_0^L \begin{bmatrix} [B_i^T][D][B_i] + \left( \tilde{\phi}^\lambda \tilde{\phi}^\lambda \right) \{N\}^T \{S\} \end{bmatrix} \{N\} dL
\]

\[
+ \int_0^L \left( \tilde{\phi}^\lambda \tilde{\phi}^\lambda \right) \{N\}^T [N] \{N\} dL
\]

where

\[
S_1 = \begin{bmatrix} -\frac{\rho Bh^3}{12} & 0 & 0 & 0 \\ 0 & -\frac{\rho Bh^3}{12} & 0 & 0 \\ 0 & 0 & 0 & -\frac{\rho Bh^3}{12} \\ 0 & 0 & 0 & 0 \\ S_{z(2,2)} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{z(4,4)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \end{bmatrix}
\]

\[
S_{z(2,2)} = \rho A (\tilde{\phi}^\lambda \tilde{\phi}^\lambda) \left[ \frac{1}{2} \left( L^2 - (x_e + x)^2 \right) + a(L - (x_e + x)) \right]
\]

\[
S_{z(4,4)} = \rho I_z (\tilde{\phi}^\lambda \tilde{\phi}^\lambda) \left[ \frac{1}{2} \left( L^2 - (x_e + x)^2 \right) + a(L - (x_e + x)) \right]
\]
forces are applied in terms of a parameter: the frequency on the rotating beam. Fictitious body forces (centrifugal and Coriolis forces) as body elements. Delaminations in the beam are simulated. The beam is meshed with 40 elements. The hub is considered to be 254mm. laminate with the stacking sequence of \([0/90]_2\).

Three dimensional finite element model

Beam geometry and material properties considered in the simulation are as follows: a 127mm long, 12.7mm broad, and 10.16mm thick graphite/epoxy laminate with the stacking sequence of \([0/90]_2\). The beam is mounted on a rotating hub, and the radius of the hub is considered to be 254mm.

A three-dimensional finite element model of the rotating flex beam is simulated in COMSOL 4.2a. The beam is meshed with 40,000 hexahedral elements. Delaminations in the beam are simulated using thin elastic layers [28]. Assigning fictitious forces (centrifugal and Coriolis forces) as body forces to the beam simulates the effect of angular frequency on the rotating beam. Fictitious body forces are applied in terms of a parameter: the angular speed. Hence, a parametric study of variations in the natural frequencies of the delaminated beam with varying angular speed can be performed. The healthy beam as well as the delaminated beams are simulated for different angular speeds. The range of angular speeds considered is 0-4,500 revolutions per minute (roughly 0-471.2 radians per second).

5 Full-width delamination results

Results are obtained for the first bending and the first torsional modes of a healthy beam at zero angular speed and listed in Table 1. For comparison, the table also lists the results of 3-D COMSOL FE simulation and the experimental results available in the literature [29].

Table 1: Healthy non-rotating beam natural frequencies

The torsional frequency obtained using the present formulation is lower compared to that obtained using the COMSOL simulation; this may be due to the neglect of warping in the formulation, which could be large for the non-circular cross-section under consideration.

Through-width delamination at the interface of the fourth and fifth lamina, and two different delamination lengths of 50.8mm and 76.2mm, symmetrically located across the center of the beam are considered for the simulation. Effects of the angular speed and the delamination on the natural frequencies of the rotating cantilever beam are determined. The change in the first bending mode frequency of a rotating beam, healthy as well as delaminated, with angular speed is shown in Fig. 4. Solid lines in Fig. 4 represent the results obtained using COMSOL simulation while the results obtained using the present beam model are plotted using markers. The variations in the second bending mode and first torsional mode natural frequencies, due to the presence of delamination and varying angular speed are plotted in Figs. 5 and 6, respectively. At higher angular speeds, the stiffening effect due to the centrifugal force negates the bending stiffness reduction due to delamination, as can be seen from Fig. 4. Present delamination model...
predicts the natural frequency of the first bending mode reasonably well; but, the predictions of second bending mode and first torsional mode natural frequencies are on the lower side. At certain angular speeds, the nominal second bending mode and the first torsional mode will not be pure bending or pure torsion; instead they exhibit a coupled mode behavior.

Fig. 4: First bending mode frequency variation with angular speed
This coupled mode behavior is seen in the region, where second bending mode natural frequency crosses over the first torsional mode natural frequency.

Fig. 5: Second bending mode frequency variation with angular speed
The effect of delamination on the second bending mode and first torsional mode, even in the presence of centrifugal stiffening at high angular speeds, is quite predominant.

5.1 Partial delamination results

First bending and first torsional mode natural frequencies of an edge-delaminated, non-rotating beam are shown in Table 2. Results tabulated are for an edge delamination of length 76.2mm in longitudinal direction, located symmetrically across the center of the beam; and in the width-wise direction, the edge delamination size is mentioned in terms of percentage with respect to the beam width. Delamination is at the interface of the fourth and the fifth laminae, counted from the bottom-most lamina, in the thickness direction.

<table>
<thead>
<tr>
<th>Delamination size along $a_2$ (% of width)</th>
<th>FEM (Hz)</th>
<th>COMSOL (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Bending mode</td>
<td>50</td>
<td>79.46</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>77.71</td>
</tr>
<tr>
<td>First Torsional mode</td>
<td>50</td>
<td>494.13</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>446.00</td>
</tr>
</tbody>
</table>

Table 2: Delaminated beam frequencies
The effect of angular speed in the presence of an edge delamination of 70 percent in the width-wise direction, at the interface of the fourth and fifth laminae, and 76.2mm in the longitudinal direction, located symmetrically across the center of the beam, is shown in the Fig. 7. The present delamination model predicts all three natural frequencies, at various angular speeds, reasonably well. The second bending and the first torsional modes are not pure bending and torsional modes; they are bending-torsion coupled modes, as explained previously.
DELAMINATION DETECTION IN ROTORCRAFT FLEXBEAMS USING FRACTAL DIMENSIONS

Fig. 7: 70% delaminated beam frequency variation with angular speed

6 Fractal dimension

Fractal as described by James Theiler [30] is follows: “A fractal is self-affine if it can be decomposed into subsets that can be linearly mapped into the full figure. If this linear map involves only rotation, translation, and (isotropic) dilation, then the figure is self-similar. For a self-affine map, the contraction in one direction may differ from the contraction in another direction.”

Fractal dimension of a curve as defined by Katz [15] is given by

\[ FD = \frac{\log_{10}(n)}{\log_{10}(d/f) + \log_{10}(n)} \]

Here, \( l \) is the total length of the curve, \( d \) is the planar extent of the curve, and \( n \) is the number of segments in the curve. Fractal dimension is calculated for a normalized torsional modeshape, by considering a window of 5 nodes or 4 elements and the calculated fractal dimension is assigned to the mid-node of the window [31]. This window is slid along the length of the beam and thus the fractal dimension is calculated for each node of the beam, except for the first and the last two nodes of the beam. The fractal dimension curve of an edge-delaminated non-rotating beam is shown in Fig. 8. Peaks in the curve indicate the edges of delamination.

Fig. 8: Fractal dimension plot of torsional mode shape

6 Conclusions

Formulation for a rotating composite cantilever beam with 4 DoFs per node (stretch, transverse deflection, bending rotation, and twisting rotation) is presented. A generic delamination model to handle edge and internal delaminations, which could be partial both along the width and length in the composite beam is developed. The effects of the angular speed and the delamination on the first few natural frequencies of the rotating beam are discussed. The results obtained using the present model are compared with a 3-D finite element model of commercially available COMSOL software. The Katz fractal dimension method applied on the torsional mode shape of composite beam is used to detect the delamination.

The natural frequencies obtained using the present 1D formulation are comparable with those obtained using the 3D simulation in COMSOL at lower rotational speeds of the flexbeam. However, differences are observed at higher speeds necessitating further probing, to check whether the beam formulation needs to be tweaked or the way the centrifugal force is applied in the 3-D finite element simulation using COMSOL is appropriate. Though delamination results for the first bending in the presence of full-width delamination and natural frequencies of the first three modes of the edge-delaminated beam are quite comparable with the corresponding results of COMSOL, the second bending mode and first torsional mode frequencies in the presence of full-width delamination do not match. Hence, the delamination model needs to be checked with different modification factor functions. It is demonstrated that the partial delamination in the
composite beam can be detected using the torsional modeshapes and the fractal dimension method.

References


[28] COMSOL 4.2a documentation.
DELAMINATION DETECTION IN ROTORCRAFT FLEXBEAMS USING FRACTAL DIMENSIONS


Appendix

Delaminated cross-sectional stiffness coefficients:

\[
A_{11}^d = \bar{A}_{11} + \bar{A}_{11}^1 + \bar{A}_{11}^2 + \bar{A}_{11}^3 + \bar{A}_{11}^4 + \bar{A}_{11}^5
\]

\[
A_{55}^d = \bar{A}_{55} + \bar{A}_{55}^1 + \bar{A}_{55}^2 + \bar{A}_{55}^3 + \bar{A}_{55}^4 + \bar{A}_{55}^5
\]

\[
B_{11}^d = \bar{B}_{11} + \bar{B}_{11}^1 + \bar{B}_{11}^2 + \bar{B}_{11}^3 + \bar{B}_{11}^4 + \bar{B}_{11}^5 + \bar{B}_{11}^6
\]

\[
\beta_l z_l^a \bar{A}_{11}^2 - \beta_l z_l^b \bar{A}_{11}^3 + \beta_r z_r^a \bar{A}_{11}^4 - \beta_r z_r^b \bar{A}_{11}^5
\]

\[
D_{16}^d = \bar{D}_{16} + \bar{D}_{16}^1 + \bar{D}_{16}^2 + \bar{D}_{16}^3 + \bar{D}_{16}^4 + \bar{D}_{16}^5 + \bar{D}_{16}^6
\]

\[
\beta_l z_l^a \bar{B}_{16}^2 - \beta_l z_l^b \bar{B}_{16}^3 + \beta_r z_r^a \bar{B}_{16}^4 - \beta_r z_r^b \bar{B}_{16}^5
\]

\[\text{where } \beta_l \text{ is the delamination modification factor for the subsections 2 and 3, and } \beta_r \text{ is the delamination modification factor for the subsections 4 and 5.}\]