COMPARISON OF CONSOLIDATED COMPOSITES USING MECHANICAL TESTING AND A MULTI-CRITERIA DECISION MAKING TECHNIQUE UNDER VARIABLE MATERIAL PROPERTIES

J. Leung, M. Heinrick, A.S. Milani*
Composites Research Network- Okanagan Laboratory
School of Engineering, University of British Columbia, Kelowna, Canada
*Corresponding author (abbas.milani@ubc.ca)

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1. General introduction

Selecting an optimum composite material in a modern engineering application can be a difficult task and requires examining all the available options under multiple criteria and uncertainty in material properties. The inherent flexibility in design of composite materials is associated with a need for robust decision-making techniques to test and optimize desired material properties in a given application. Among different types of composites, glass fiber reinforced polymer composites are becoming more and more promising in a wide range of applications. The performance of these composites is tied to several independent variables in the raw material (e.g., textile preform architecture) and the production technique used to make the final part [1].

Modeling techniques are commonly used to predict the material properties of composite materials with no cost of actual experimentation. The models are subsequently used for material selection purposes; however there is also evidence of some inconsistencies with what is seen in practice [2]. Particularly, woven composites are challenging to model because of their intricate structural composition and multi-scale nature of twisted yarns [3]. The complex fiber reinforcement used in these composites gives the material an inherent anisotropy that is only seen when the model is highly detailed (e.g., at micro-level) with small-scaled unit cells [2]. Hence, in higher scale models, (e.g., macro-level) assumptions often need to be made to create an efficient composite model. Mechanical testing, on the other hand, can produce global results that are more applicable to a range of practical applications without requiring intense numerical modeling [4].

This article follows, and is based on, the multi-criteria material selection approach that was proposed in [5], but instead of calculating the composite material properties using numerical analyses, here experimental data along with actual variation of material properties are used. Namely, two consolidated fiberglass composites are compared under a set of mechanical properties where each desired property is assigned a weighting factor based on its significance in a structural application. Proper Multi-Criteria Decision Making (MCDM) techniques have been chosen to determine which of the two material options is more suited to the engineering application. The selection methodology is general and may be applied to other critical composite selection scenarios, especially under sets of multiple criteria driven from design requirements. Also, the variability in material properties, which is often unavoidable for composites, has been embedded in the proposed methodology for the first time to account for the performance reliability of the final part.

2. Decision Making Process

Different applications of composite materials have different material property requirements. These requirements can be fulfilled with the use of specialised material types and manufacturing techniques. The multi-criteria decision making (MCDM) method is a useful tool to weigh benefits of using different material options and make a final selection based on the overall performance of each material in a desired application [6].
The initial step for MCDM is to collect data with respect to the performance or properties of each composite material under design criteria. The attributes of the criteria can be categorised into “the higher the better” (like stiffness, strength, etc.), or “the lower the better” (like the material density, cost, etc.), or they can be qualitative in which case their assessments can be converted into numerical numbers (e.g., 0 for a poor performance, 1 for neutral, 2 for an acceptable performance, etc.). Eventually all numerical data are employed to form a decision table (matrix) (see Table 2 for an illustrative example of such a matrix), where the rows represent the alternatives and the columns represent the criteria. Since in general the magnitude or units of the criteria are different, a normalised form of the decision matrix is required for subsequent calculations. The normalization can be achieved using:

$$p_{ij} = \frac{y_{ij}}{\sum_{j=1}^{n} y_{ij}} ; \text{for all } i, j$$  

(1)

where $y_{ij}$ are the non-normalised values of the decision matrix (i.e., the measure values of the $j$-th property for the $i$-th material option), $p_{ij}$ are the normalised values and $n$ is the number of alternatives (material options) in the matrix [7].

To choose the best material option, the alternatives should be ranked based on an overall performance score which can be defined from their property values and the criteria weights. A simple way for defining this score is summing the normalized properties as:

$$\gamma_i = \sum_{j=1}^{k} (-1)^{m_j} w_j p_{ij}$$  

(2)

where $\gamma_i$ is the performance score of the $i$-th material and $k$ is the number of criteria. However, the summed normalised properties via Eq. (2) neglects that the relative importance of criteria that may be of interest to the designer for a given application. For example more emphasis on the density of a material may be given in an application compared to its shear stiffness. Therefore, the sum of weighted properties is suggested as a better option as follows:

$$\gamma_i = \sum_{j=1}^{k} w_j p_{ij}$$  

(3)

where $w_j$ are the weighting factors, and $m_j$ is included to distinguish the positive (i.e., the higher the better) and negative/undesirable (the lower the better) characteristics of the attributes. If an attribute is positive, $m_j = 0$ and if it is negative, $m_j = 1$. The latter method in MCDM is referred to as the “Weighed Sum Method” or WSM and has been most widely used in various disciplines [7].

There are three general approaches for weighing the decision criteria (material properties in this case): either assigning ‘objective’ weights using statistical measures in the collected data, or assigning ‘subjective’ weights based on preferred material properties by an experienced analyst. Combining these two methods allows the decision maker to include some input based on his/her experience in the given particular application, while maintaining a degree of objectivity. Following a comparative approach as in [5], three weighting methods will be discussed and compared in the present article. Namely, the “entropy” method [8] will be used to reply purely on statistical measures of the data and assign objective weights. A “Modified Digital Logic” (MDL) [9] method will be used to allow the designer to rank each material property based on its perceived importance. And the third weighting method combines (averages) the above two methods to calculate the final weights for material properties [5].

2.1. Entropy method

In the entropy method [8], the weight for each material property is calculated based on the Shannon entropy, $E_j$, of the corresponding measured dataset as follows:

$$E_j = -\alpha \sum_{j=1}^{n} p_{ij} \ln p_{ij} ; \text{for all } j$$  

(4a)

$$\alpha = 1 / \ln (n)$$  

(4b)

From this, the following formula is used to find the objective weights:

$$w_j = \frac{1 - E_j}{\sum_{j=1}^{n} (1 - E_j)}$$  

(5)

2.2. Modified digital logic method

If the designer has a priori for assigning the criteria’s relative importance, the modified digital logic (MDL) method is a suitable option [9]. In this method, the criteria are compared in a pairwise manner and assigned a number based on their relative importance. More specifically, the designer grades each pair of material properties by granting a
value of 3 for strict preference, 2 for an equal preference, and 1 for loose preference (e.g., the designer may prefer high shear stiffness over low density for a given application, thus assigning 3 and 1 to these two criteria, respectively). After all pairwise comparisons are made, the assigned values are summed and normalised for each criterion to find its final relative weight. This method is completely based on the inference and experience of the designer, hence called objective weighting. It is worth adding that if, in a final design, more than one decision makers (DMs) have been involved, the average of weights from each DM can be used.

2.3. Combined weighting method

As noted before, it is also possible to combine the effects from weights calculated by the (objective) entropy method with the subjective weights from the MDL method. In this case, the following formula may be used [5]:

$$w_j = \frac{w_j^{subjective} \cdot w_j^{objective}}{\sum_{i=1}^{k} (w_i^{subjective} \cdot w_i^{objective})}$$

(6)

3. Illustrative example

A case study was conducted to demonstrate the use of the above MCDM framework in a composite material selection application via mechanical testing. The MCDM goal is to find the preferred composite among two different fibreglass thermosetting candidates (Fig. 1) that are used in a pin-joined structure that presumably undergoes uniaxial tension during service in an aerospace application, where low weight is highly desired. The two materials being compared are referred to hereafter as A and E. They differ in their type of reinforcement architecture, ply thickness, resin type and the lay-up. Material A uses the Aropol resin with 8 layers of glass plain weave, while material E uses the Envirez resin with 2 layers of glass plain weave and 2 alternating layers of fibreglass that are stitched as non-woven fabric. The use of different fabrics changes both how the composite is made and how it performs [10]. Material A has a nominal ply thickness of 0.5mm, while material E has a nominal ply thickness of 1mm. The fibre volume ratios of the two composites were measured by conducting resin burn-off tests [11]. Results are summarized in Table 1.

3.1. Mechanical testing of the composite samples

Three mechanical tests were conducted to collect the data required to make a selection: uniaxial tension (Figure 2), bearing strength (Fig 3), and short beam shear (Figure 4). The operating workflows for these tests were based on the ASTM standards [12], [13], and [14], respectively. For compliance with the ASTM standards, two thicknesses of each composite material were used. The specimens used in the pure unidirectional tensile test were nominally 2mm thick, while the specimens for the remaining two tests were nominally 4mm thick. The 2mm samples had half the layers of the 4mm samples lay-up. Thus, material A was 4-ply of plain weave and material E was 2-ply, one plain weave and one stitched non-woven fabric under the uniaxial test. The samples were manufactured via hand-layup in plates and cut into their testing dimensions using a water jet machine. Holes required for the bearing tensile test were formed using a drill press.

Each test was run three times to account for variability in the composite materials response, and subsequently used in the MCDM matrix as a measure of performance reliability (the better the repeatability of the material response, the higher the reliability of the part performance in service). All the tests were conducted over a span of two weeks using the same universal testing machine (Instron 5960 dual column tabletop universal testing system) and with the same operator (to avoid blocking effects in a statistical sense). Resulting material properties from the mechanical tests are summarized in Table 2.

3.2. MCDM implementation and results

The material properties from the above mechanical tests were used to establish the applicable MCDM attributes as listed below.

- Composite stiffness under uniaxial extension along warp ($E_{uni}$), to ensure the part’s rigidity under moderate loads;
- Composite ultimate strength under uniaxial extension along warp ($UTS$), to ensure the part can withstand high design loads;
- Composite strain at failure under uniaxial extension along warp ($\varepsilon_{max}$), to determine the maximum failure extension;
- Composite inter-laminar shear stiffness ($F_{sbs}$), to quantify the inter-laminar strength;
- Composite ultimate strength under uniaxial extension through bearing along warp...
(UTSb), to ensure the bearing capacity of the part at joints:

- The density of the composite (ρ), to minimize the weight of the part.

In addition to these six properties, the relative standard deviations (SD) of the $E_{uni}$, UTS, $F_{sbs}$, and UTSb were included in the decision making, which needed to be minimized to ensure the reliability of part performance. Using the results, the decision matrix of Table 2 was constructed. The matrix was normalised via Eq. (1) to produce Table 3. The matrix in Table 3 was then used to calculate the weighted values for each property, based on the three methods explained in Section 2. Results of these weighting methods are summarized in Table 5.

In every design case, the value of each material property (based on three tests) would be of primary importance to meet the design requirements, when compared to their STD values. Thus, during the implementation of the modified digital logic method, RSD for each property was assigned a 1 unit of importance when compared to the corresponding average property value, which received a 3. The full pairwise comparisons performed for the MDL method by two independent decision makers are shown in Figure 5. A sample weight calculation, e.g., for the Young’s modulus $E$ with respect to the decision maker 1, follows the sum of scores in the corresponding row divided by the total sum at the right-side column; i.e., $21/(21+25+18+22+27+11+13+9+16+18) = 0.12$.

The overall (group) MDL weights were estimated by averaging the two sets of weights from the two decision makers. For example, the MDL weight given to the density criterion by the first decision maker was calculated to be 0.10, while for the second decision maker it was 0.14. The average of these two weights was calculated (0.12) and assigned for the density criterion in the subsequent MCDM calculations.

All the averaged weights for the material properties were re-normalised so that the total of final weights remains 1.0. The weights from the entropy and MDL methods were also combined into the third type of weights (Eq. (6)) that account for both statistical disorder of the measured data and the designers’ input in the decision process. Finally, using the normalised data matrix and the three weighting factors, a total score for each material was obtained via Eq. (3) and subsequently the ranking was made as shown in Table 5. All the three methods favoured material A, although the overall scores varied between the methods. The selection was well expected because material A performed almost universally better in the mechanical tests under each criteria of interest, while demonstrating comparable performance robustness as noted by RSD values in Table 2. The high values of RSD for the bearing strength of both materials can be of concern for a particular manufacturer, e.g., in aerospace application where there is very limited tolerance for inconsistency in material performance at joints [15]. In that case the material options may be revisited and the proposed mechanical testing-MCDM methodology be re-run without loss of generality.

4. Summary and conclusions

The use of mechanical testing for making a material choice using multi-criteria decision making (MCDM) was shown to be practical and effective. Next to measuring mean property values, repeated mechanical tests allow an estimation of the reliability of each material property using standard deviation. The entropy method was discussed as a way to give weights to each property with no direct input from the decision maker. The modified digital logic method was also discussed to allow a decision maker to favour the importance of one material property over another. A combination of the two methods can then be used to give a more complete weighting of the selection criteria.

Mechanical testing for the presented case study allowed the MCDM matrix to include the standard deviations of the material properties. Estimating these deviations in the material properties would not be possible using computational modeling. The mechanical tests compared two fibreglass composite materials under 1-dimensional loading, with an application as a joining part with a stress concentration at the bearing. The decision making process concluded that the 8-ply fibreglass plain weave Aropol resin composite would be a better choice for this application. This mimics the expectation of the case study because it was seen from Table 2 that Material A performs better in all but two criteria: short beam strength standard deviation and density. It should also be noted that in many cases, material A had both a better average performance and lower standard deviation. For example, material A had a higher tensile strength while maintaining a lower standard deviation of that
strength. The material ranking in this case study was independent of the employed weighting methods. Some other factors that can affect the selection results are the product price, styrene production, and cure conditions.

The benefit of the proposed MCDM technique is the ability to consolidate measured data effectively into a matrix and numerically rank the material choices based on requirements that the designer chooses to include, including the repeatability of materials performance during their service life cycles.

Acknowledgments

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Figure 1. Images of the two tested materials

Figure 2. (a) Instron tensile test machine fixture grips; (b) Sample of material E after the test

Figure 3. (a) Laminate bearing test fixture (manufactured by Wyoming Test Fixtures, Inc.); (b) sample of material A after the test
Figure 4. (a) Short beam shear test fixture (manufactured by Wyoming Test Fixtures, Inc.); (b) sample of material E after the test.

Figure 5. Chart of pairwise comparisons made in the modified digital logic method by two decision makers.

Table 1. Fibre volume ratios of the two materials measured from resin burn-off tests

<table>
<thead>
<tr>
<th>Material</th>
<th>Fibre Volume Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>A [Aropol resin, 0.5mm ply thickness fibreglass plain weave]</td>
<td>0.640</td>
</tr>
<tr>
<td>E [Envirez resin, 1mm ply thickness alternating fibreglass plain weave &amp; stitched non-woven]</td>
<td>0.599</td>
</tr>
</tbody>
</table>
Table 2. The decision matrix based on the performed tests on candidate materials (for definitions of symbols, refer to Section 3.2)

<table>
<thead>
<tr>
<th>Material</th>
<th>$E_{\text{uni}}$ (MPa)</th>
<th>$UTS$ (MPa)</th>
<th>$\varepsilon_{\text{max}}$</th>
<th>$F_{\text{sbs}}$</th>
<th>$UTS_b$</th>
<th>$\rho$ (Kg/m$^3$)</th>
<th>RSD</th>
<th>RSD</th>
<th>RSD</th>
<th>RSD</th>
<th>RSD</th>
<th>RSD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material A</td>
<td>8294</td>
<td>279.45</td>
<td>0.0349</td>
<td>63.49</td>
<td>314.9</td>
<td>10.09%</td>
<td>1</td>
<td>2.49%</td>
<td>13.43%</td>
<td>32.14%</td>
<td>1857</td>
<td></td>
</tr>
<tr>
<td>Material E</td>
<td>7860</td>
<td>249.02</td>
<td>0.0328</td>
<td>35.57</td>
<td>278.2</td>
<td>10.91%</td>
<td>1</td>
<td>3.93%</td>
<td>9.80%</td>
<td>32.70%</td>
<td>1701</td>
<td></td>
</tr>
</tbody>
</table>

Table 3. The normalized decision matrix based on the values of Table 4 and Eq. (1)

<table>
<thead>
<tr>
<th>Material</th>
<th>$E_{\text{uni}}$</th>
<th>$UTS$</th>
<th>$\varepsilon_{\text{max}}$</th>
<th>$F_{\text{sbs}}$</th>
<th>$UTS_b$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material A</td>
<td>0.513</td>
<td>0.529</td>
<td>0.515</td>
<td>0.641</td>
<td>0.531</td>
<td>0.480</td>
</tr>
<tr>
<td>Material E</td>
<td>0.487</td>
<td>0.471</td>
<td>0.485</td>
<td>0.359</td>
<td>0.469</td>
<td>0.520</td>
</tr>
</tbody>
</table>

Table 4. The criteria relative importance obtained from different weighting methods (note that for each method the weights sum to the unity).

<table>
<thead>
<tr>
<th>Weights</th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$w_3$</th>
<th>$w_4$</th>
<th>$w_5$</th>
<th>$w_6$</th>
<th>$w_7$</th>
<th>$w_8$</th>
<th>$w_9$</th>
<th>$w_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entropy</td>
<td>.006</td>
<td>.028</td>
<td>.008</td>
<td>.668</td>
<td>.032</td>
<td>.013</td>
<td>.025</td>
<td>.204</td>
<td>.001</td>
<td>.016</td>
</tr>
<tr>
<td>Modified Digital Logic</td>
<td>.114</td>
<td>.136</td>
<td>.094</td>
<td>.117</td>
<td>.147</td>
<td>.058</td>
<td>.072</td>
<td>.053</td>
<td>.089</td>
<td>.119</td>
</tr>
<tr>
<td>Combined</td>
<td>.007</td>
<td>.036</td>
<td>.007</td>
<td>.756</td>
<td>.046</td>
<td>.007</td>
<td>.017</td>
<td>.105</td>
<td>.001</td>
<td>.019</td>
</tr>
</tbody>
</table>

Table 5: The total scores and ranking of the composite candidates using different weighting techniques

<table>
<thead>
<tr>
<th>Material</th>
<th>A</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entropy</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Modified Digital Logic</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Combined</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

References


