CONSTITUTIVE THEORY OF YEOH TYPE ELASTIC DIELECTRICS POLYMER

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1 General Introduction

Dielectric elastomer(DE) is a kind of promising material bearing excellent activate properties including large deformations, high energy densities, high efficiency, high responsive speed, good reliability and durability [1-3]. Thus the DE actuator, sensors and energy harvester is widely used in the field of aeronautics and smart bionics.

The couple of mechanical field and electric field exerted on DE results in the electromechanical coupled system unstable. In recent years, the DE’s stability analysis is one of the most popular issues, especially after Suo et al proposed the electromechanical stability theory of dielectric elastomers [2]. To further research the electromechanical stability performance of undergoing-large-deformation dielectric elastomer, the electric energy density function is constituted by applying the linear function of the permittivity and the nonlinear expression proposed respectively by Suo and our group. Then the coupled formulation of the elastic strain function with two material constants and the electric energy density function is used to analyze the stability performance of dielectric elastomer electromechanical coupled system, and the results shows that the larger the material ratio is, the smaller the electrostrictive factor $a$ is and the higher the stability is. Furthermore, stability is analyzed by applying the couple neo-Hookean elastic strain energy and electric field energy density with nonlinear permittivity. Clearly, it is helpful in the design of dielectric elastomer actuators. Finally, such an analysis method is extended to analyze the performance of the dielectric elastomer coupling system with more material constants.

2 The constitutive theory of elastic dielectrics polymer

The electromechanical coupled theory is introduced when the coupled effect of the mechanical field and the electric field is alerted on elastic dielectrics. Clearly, the dielectric elastomer electromechanical coupled theory mentioned above is the foundation and the particular case of the large deformation theory of elastic dielectrics polymer (EDP), although the large deformation theory for EDP is the generalization of the dielectric elastomer electromechanical coupled theory. In its working condition, elastic dielectrics polymer is subject to an electric field applied by the compliant electrodes spreaded on the elastic dielectrics polymer film’s surfaces. It will contract along the thickness direction due to the electro-static force. Meanwhile an in-plane elongation occurs, developing large strains perpendicular to the voltage direction, then the film becomes thinner. It is the process of the coupling effect of the mechanical field and the electric field. The energy function of the dielectric elastomer electromechanical coupling system can be written as follows:

$$\Omega = \frac{\varepsilon}{2}\epsilon_{0}\lambda_{1}(\lambda_{2},\lambda_{3},D^{*}) - P_{1}\lambda_{1} - P_{2}\lambda_{2} - P_{3}\lambda_{3} - UQ \hspace{1cm} (1)$$

where $\lambda(\lambda_{1},\lambda_{2},\lambda_{3},D^{*})$ is the free energy function, $\lambda_{1}, \lambda_{2}, \lambda_{3}$ are the original dimensions of EDP, $U$ is the electric voltage, $Q$ is the charge on each surface, $P_{1}, P_{2}, P_{3}$ are the pre-stretch forces, $\lambda_{1}, \lambda_{2}, \lambda_{3}$ are the stretch ratios in the three main directions. The nominal stress and the nominal electric field are defined as follows:

$$s_{i} = \frac{\partial \lambda(\lambda_{1},\lambda_{2},\lambda_{3},D^{*})}{\partial \lambda_{i}} \hspace{1cm} (2)$$
\[
\sigma_1 = \frac{\partial \Lambda(\lambda_1, \lambda_2, \lambda_3, D^4)}{\partial \lambda_1} \\
\sigma_2 = \frac{\partial \Lambda(\lambda_1, \lambda_2, \lambda_3, D^4)}{\partial \lambda_2} \\
\sigma_3 = \frac{\partial \Lambda(\lambda_1, \lambda_2, \lambda_3, D^4)}{\partial \lambda_3} \\
E = \frac{\partial \Lambda(\lambda_1, \lambda_2, \lambda_3, D^4)}{\partial D^4}
\]

Where \( \sigma_1, \sigma_2, \sigma_3 \) are the nominal stresses, and can be derived from the un-deformed state by dividing the pre-stretch force in the area before deformation, i.e., \( \sigma_1 = \frac{P}{L_i^3}, \sigma_2 = \frac{P}{L_i^2}, \sigma_3 = \frac{P}{L_i} \). The stretch ratios are defined as \( \lambda_1 = \frac{L_1}{l_1}, \lambda_2 = \frac{L_2}{l_2}, \lambda_3 = \frac{L_3}{l_3} \) and the real stress as \( \sigma_1 = \frac{P}{L_i L_3}, \sigma_2 = \frac{P}{L_i L_2}, \sigma_3 = \frac{P}{L_i L_1} \), then the relation between the real stress and the nominal stress can be expressed as \( \sigma_1 = \frac{s_1}{\lambda_2 \lambda_3}, \sigma_2 = \frac{s_2}{\lambda_1 \lambda_3}, \sigma_3 = \frac{s_3}{\lambda_1 \lambda_2} \). Similarly, the nominal electric field can be defined as \( E = \frac{U}{l_1} \) and the nominal electrical displacement as \( D = \frac{Q}{l_2} \), while the real electric field defined as \( E = \frac{U}{\lambda_2 L_3} = \frac{E}{\lambda_3} \) and the real electrical displacement as \( D = \frac{Q}{\lambda_1 \lambda_2} = \frac{D}{\lambda_1 \lambda_2} \).

The free energy function of the elastic dielectrics polymer electromechanical coupling system is:
\[
\Lambda(\lambda_1, \lambda_2, \lambda_3, D^4) = \Pi(\lambda_1, \lambda_2, \lambda_3) + \Sigma(\lambda_1, \lambda_2, \lambda_3, D^4)
\]

where \( \Pi(\lambda_1, \lambda_2, \lambda_3) \) is the elastic strain energy density function and \( \Sigma(\lambda_1, \lambda_2, \lambda_3, D^4) \) is the electric energy density function. Furthermore,
\[
\Sigma(\lambda_1, \lambda_2, \lambda_3, D^4) = \frac{D^4}{2\epsilon(\lambda_1, \lambda_2, \lambda_3)} \lambda_1^{-1} \lambda_2^{-1} \lambda_3
\]

Because elastic dielectrics polymer undergoing large deformation under the coupling effect of the mechanical field and the electric field, it is necessary to apply the electric energy density function within changeable permittivity to analyze the mechanical performance and the electromechanical stability performance of the electromechanical coupling system. The formulation of the permittivity \( \epsilon(\lambda_1, \lambda_2, \lambda_3) \) is introduced as follows,
\[
\epsilon(\lambda_1, \lambda_2, \lambda_3) = [1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)]\epsilon^-
\]

Where \( \epsilon^- \) is the permittivity before any deformation of elastic dielectrics polymer, \( a \) and \( b \) are the electrostrictive factors, \( \lambda_1, \lambda_2, \lambda_3 \) are the stretch ratios in the three main directions, respectively.

Substituting Equation (8) into (7) and (6), the free energy of elastic dielectrics polymer with the linear-changing permittivity is expressed as follows,
\[
\Lambda(\lambda_1, \lambda_2, \lambda_3, D^4) = \Pi(\lambda_1, \lambda_2, \lambda_3) + \frac{D^4}{2[1 + a(\lambda_3 - 1) + b(\lambda_1 + \lambda_2 + \lambda_3 - 3)]\epsilon^[-]} \lambda_1^{-1} \lambda_2^{-1} \lambda_3
\]

In the following analysis, the electromechanical coupling theory of elastic dielectric polymer is extended to dielectric elastome. The structural symmetry of the macromolecular, the crosslinking degree, along with the tensile deformation can affect the dielectric permittivity enormously. For dielectric elastome with higher crosslinking degree, or higher degree of molecular structural symmetry, its permittivity is relatively low. In addition, stretching can guide the macromolecule to be arranged in order, this can increase the intermolecular forces and reduce the activities of polar group, as a results, the dielectric constant will decrease.

**References**

