MODELLING CRACK PROPAGATION IN PARTICLE-REINFORCED COMPOSITES USING THE ELEMENT-FREE GALERKIN METHOD

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1 ABSTRACT

In this work, a novel approach to predict the path of crack propagation through particle-reinforced composites is presented. Simulations are based on the Element-Free Galerkin (EFG) method and linear elastic fracture mechanics (LEFM). A modified interaction integral, suitable for heterogeneous materials, was used to obtain the stress intensity factors (SIFs). The maximum tangential principal stress (MTPS) criterion was used to determine the instantaneous direction of crack propagation. Crack extension paths and variation of energy release rates (ERRs) during the propagation are presented. Thereby issues such as the interaction of the crack with inclusions during its extension, crack tip shielding and stress amplification are resolved. While the crack meanders near the particle, the ERRs are observed to be affected even if the crack tip is located at a larger distance from the particle.

2 INTRODUCTION

Reinforced composites are of importance for aerospace and automobile applications where the strength-to-weight ratio and stiffness-to-weight ratio are crucial factors. There has been significant research in the failure mechanisms of these materials in order to optimize their design and utilization. Although initially more attention was paid to experimental and analytical investigations [1], the focus has shifted towards computational methods facilitated by the emergence of sophisticated modelling techniques and decreasing cost of computational hardware [2,3].

The failure process in composites is often initiated at micro-sized flaws, which may grow to macro-sized cracks leading to catastrophic failure. The damage propagation depends on multiple parameters such as the volume fraction of particles, particle strength, interface bonding strength, pre-existing cracks, shape of the particle and their locations within the matrix. In general, fracture in particulate composites consists of a combination of matrix failure, particle fracture and interface decohesion. It is reported that the dominant crack may stay completely within the matrix [4]. Although the basic mechanical properties of particle-reinforced metal matrix composites (MMC) are superior to alloys [5], it is often found that their fracture toughness is lower than that of the monolithic metal alloys [6,7].

Previous analytical and numerical studies were carried out to explain the mechanics behind the toughening process of particle-reinforced composites. Toughening mechanisms for symmetrically located cracks with respect to the particle were analyzed by Atkinson [8] who showed the existence of $1/\sqrt{r}$ singular stress fields for the crack tip very close to the inclusion but not in direct contact. Increase in fracture toughness due to crack path deviation around a secondary phase (inclusion) is explained by Faber et al. [9] using a numerical model.

There are numerous computational techniques, reported in the literature, for modelling crack propagation in the presence of particle reinforcements. The finite element method (FEM) [10,11] and the boundary element method (BEM) [12,13] have been widely employed to study fracture problems. Li et al. [10] observed that there is crack tip shielding when a rigid inclusion is approached. Furthermore, they showed that there is amplification of stresses as soon as the crack crosses the inclusion.


An accurate method of modelling crack propagation through particle-reinforced composites, is key to understand and prevent failure of such materials. Although the FEM and the BEM are the most established methods for numerical modeling of structures, they present problems in handling crack extension. These methods require remeshing as the crack propagates, which is time consuming. Moreover, the density of FE mesh has to be increased significantly near the crack tip and matrix-particle interface to accurately capture the stress state. The eXtended Finite Element Method (XFEM) and Meshless Methods (MMs) have been developed to address some of these shortcomings. Recently, XFEM was used to model crack propagation in particle-reinforced composites [3].

There are currently many MM formulations in the literature and the Element-Free Galerkin (EFG) method is one among the popular techniques available today to model crack propagation problems [14]. The eXtended EFG (XEFG) method [15,16] is a relatively recent development, based on the XFEM, which incorporates special functions to handle discontinuities in the EFG method using a partition-of-unity approach. In this work, a modified XEFG method is used to model the crack propagation through the particulate composites.

The ERRs were calculated using the modified M-integral/interaction integral method. The traditional M-integral cannot be applied when material interfaces or discontinuities exist inside the integration domain. Rigorous proof of this point can be found in [17] and its application to composite materials is shown in [3]. The direction of crack extension was computed using the MTPS criterion [18].

Several case studies were solved using the proposed XEFG method in conjunction with the MTPS and MTS criteria to demonstrate the advantages of the method used. Furthermore, the dependence of the ERR on the elastic modulus and distribution of the particles are presented.

3 MODIFIED XEFG METHOD

3.1 XEFG Method

The displacement, \( \mathbf{u}(x) \), approximation in XEFG method is given by

\[
\mathbf{u}(x) = \sum_{I \in W(x)} \Phi_I(x) \mathbf{u}_I + \sum_{I \in W(x)} \Phi_I(x) \{a_I H(f(x))\} + \sum_{I \in W(x)} \Phi_I(x) \sum_{k=1}^{d} b_{ik} B_{ik}^{\text{enr}}(\gamma, \theta) + \sum_{I \in W(x)} \Phi_I(x) \mathbf{c}_I \Psi_I(x) \tag{1}
\]

where \( \Phi_I(x) \) is the shape function of the node \( I \), \( f(x) \) is the signed distance function from the crack. The Heaviside or jump-enriched function \( H(f(x)) \) and branch (William’s expansion) enriched function \( B_{ik}^{\text{enr}}(\gamma, \theta) \) are given by

\[
H(f(x)) = \begin{cases} 1 & \text{if } f(x) \geq 0 \\ -1 & \text{if } f(x) < 0 \end{cases}
\]

\[
B_{ik}^{\text{enr}}(\gamma, \theta) = [\sqrt{\gamma \sin^2 - \gamma \cos^2} + \sqrt{\gamma \sin^2 - \gamma \cos^2}] 
\]

\[
\sqrt{\gamma \sin^2 - \gamma \cos^2} \sin \theta, \sqrt{\gamma \sin^2 - \gamma \cos^2} \cos \theta
\]

\[
\gamma \text{ and } \theta \text{ are polar coordinates of the sampling point } x \text{ from the crack tip. The function } \Psi_I(x) \text{ is used for displacement continuity across the material interface of the particle. } \Psi_I(x) = F^I(x) - F^I(x_I) \text{ where the function } F^I(x) = \sum_{I \in W(x)} |\phi_I| \Phi_I(x) - \sum_{I \in W(x)} \phi_I \Phi_I(x) \text{ .} \]

\( \phi_I \) is the scalar level set function (signed distance) from the particle boundary.

When the crack passes through the neighbourhood of a matrix-particle interface, it becomes necessary to refine the nodes around the crack tip. This helps to provide sufficient branch enrichment within the matrix while ensuring the domain of influence of such nodes do not overlap with the secondary phase.
or particle. This approach is computationally cumbersome and costly when the tip is very close to the interface.

### 3.2 Modified XEFG Method

Fig. 1. Particle reinforced domain with a crack.

To overcome the above difficulties, the branch enrichment through the partition-of-unity approach was discarded for a geometry with a crack and an inclusion as shown in Fig. 1. Instead, the visibility method, where the weight functions of the nodes that influence the crack tip are modified, was used to model the crack tip as shown in Fig. 2. Since global refinement is computationally cumbersome, the region around the crack tip was moderately refined, in a regular manner, to capture the stress field accurately.

Fig. 2. Visibility method to model the crack tip.

In this work, the visibility method was used in conjunction with the Heaviside enrichment to model the crack. Such a combination will also be useful for modeling the crack propagation in multiphase materials.

The thickness of the interface was assumed to be negligible in this work. Therefore, enrichment based on level set functions [19] was used to model this weak discontinuity/material boundary. Consequently, the displacement approximation, in the presence of a crack (strong discontinuity) and inclusion boundaries (weak discontinuity) takes the form

\[
\mathbf{u}(x) = \sum_{I \in \mathcal{W}(x)} \Phi_I(x) \mathbf{a}_I + \sum_{I \in \mathcal{W}_I(x)} \Phi_I(x) \left\{ \mathbf{a}_I H(f(x)) \right\} + \sum_{I \in \mathcal{W}_I(x)} \Phi_I(x) \mathbf{c}_I \Psi_I(x)
\]  

The proposed modified XEFG method, with its nodal enrichment as shown in Fig. 3, was used for all the subsequent numerical analysis.

Fig. 3. Nodal discretization for a domain with a crack and an inclusion.

### 4 ENERGY RELEASE RATE AND CRACK PROPAGATION DIRECTION

The energy release rate was calculated after obtaining SIFs using the M-integral/Interaction integral. For linear elastic materials, the ERR(G) is given by

\[
G = J = \frac{(K_I^2 + K_{II}^2)}{E_{ip}}
\]  

where \( E_{ip} \) = \( E_{ip} \) for plane stress and \( E_{ip} = E_{ip} / (1 - \nu_{ip}^2) \) for plane strain problems. \( E_{ip} \) and \( \nu_{ip} \) are Young’s modulus and Poisson’s ratio at the crack tip respectively.
4.1 Modified Interaction Integral

The interaction integral, when the integration domain consists of material interfaces or discontinuities, can be written as

\[ I = I_h + I_{nonh} \]

\[ I_h = \int_A \left( \sigma_{ij}^t \delta_{ij} - \sigma_{ik}^t \epsilon_{ik}^t \right) q_i dA \]

\[ I_{nonh} = \int_A \left( \sigma_{ij}^t \delta_{ij} - \sigma_{ik}^t \epsilon_{ik}^t \right) q_i dA \]

where \( I_h \) is the standard interaction integral for homogenous materials and \( I_{nonh} \) arises due to the heterogeneous nature of the material considered.

There are many forms of expressions of \( I_{nonh} \) [17,20,21]. Most of the expressions are suitable for application in functionally graded materials where mechanical properties vary smoothly. Yu et al. [3,17] gave an expression for \( I_{nonh} \) which is devoid of derivatives of material properties. This is particularly suitable for particle-reinforced composites where there is a sudden change in material properties. It is given by

\[ I_{nonh} = \int_A \left( \sigma_{ij}^t \delta_{ij} - \sigma_{ik}^t \epsilon_{ik}^t \right) q_i dA \]

\[ \theta_e = 2 \tan^{-1} \left( \frac{-2K_h/K_I}{1+\sqrt{1+8(K_h/K_I)^2}} \right) \]

Fig. 4. MTPS and MTS criteria.

This direction of crack extension is not a principal direction when more than one term of the eigenfunction expansion is used to calculate the crack tip stresses. In this case, the direction given by MTS \( \partial \sigma_{\theta \theta}/\partial \theta = 0 \), i.e., \( (\sigma_{\theta \theta})_{max} \) is not principal direction.

Maiti et al. [18] showed that the crack extends in a direction, \( (\theta)_{MP} \), corresponding to \( \tau_{\theta \theta} = 0 \) which is close to the direction given by \( \partial \sigma_{\theta \theta}/\partial \theta = 0 \), but not the same. The corresponding tangential stress is a principal stress. They termed this criterion as the maximum tangential principal stress (MTPS) criterion. The difference between MTS and MTPS criteria is illustrated in Fig. 4. Other popular criteria including MTS are compared with MTPS in [18].

The shear stress \( \tau_{\theta \phi} \) was computed around a contour of finite radius around the crack tip. The angle of extension was given by the radial direction corresponding to \( \tau_{\theta \theta} = 0 \). For a crack tip close to the matrix-particle interface, care was taken to ensure that the contour for plotting \( \tau_{\theta \phi} \) does not pass through more than one phase.
4 RESULTS

In the following case studies, a pre-existing crack was assumed in the domain of interest. The length to width (L/w) ratio of the domain geometry was set to unity with L = 2. The radius (2r/L) of the particle was fixed to 0.15. The domain was subjected to uniform normal stress of 1MPa under plane strain conditions.

The normalized energy release rates (G/G₀) is plotted during extension of crack in the presence of particles. G is the actual ERR for a propagating crack in the particle-reinforced composite. G₀ is the ERR for the same crack length (mode I) in the corresponding non-heterogeneous medium. Furthermore, in some examples, crack paths are shown to highlight the difference between the predictions by MTPS and MTS criteria.

4.1 Crack Propagation in the Presence of a Single Particle

![Crack geometry with a single particle embedded in it.](image)

**Fig. 5.** Crack geometry with a single particle embedded in it.

Fig. 5 shows a geometry with a single embedded particle. The ratio of d/r, where r is particle radius, was set to unity. The Poisson's ratio of both the phases was kept constant (ν = 0.33). The crack propagation is studied for two different ratios of Young's moduli (E_p/E_m = 0.5 and E_p/E_m = 20) using both MTPS and MTS criterion.

![Crack path for different Young's modulus ratio using MTPS and MTS criterion.](image)

**Fig. 6.** Nodal discretization.

The geometry was discretized with 40×40 regular nodes. The region around the crack tip was discretized with 13×13 nodes for capturing the asymptotic stress fields as shown in Fig. 6.

![Crack path for different Young's modulus ratio using MTPS and MTS criterion.](image)

**Fig. 7.** Crack path for different Young's modulus ratio using MTPS and MTS criterion.

Fig. 7 shows the crack propagation paths for two different ratios of E_p/E_m as per MTS and MTPS criteria. As expected, the crack gets attracted towards the softer inclusion and it gets repelled by a hard inclusion. Whilst both criteria predict almost the same crack path for a soft inclusion, the crack is...
repelled more as per the MTS criterion for a hard inclusion.

Fig. 8 shows the normalized ERR ($G/G_0$) for different $E_p/E_m$ ratios. A modified interaction integral was used to obtain the energy release rate. The plot increases steadily for the softer inclusion. However, in the presence of a hard inclusion, the phenomenon of crack tip shielding and amplification is observed. As the crack tip approaches the hard inclusion, the $G/G_0$ ratio reduces. When it crosses the inclusion, $G/G_0$ is amplified (Fig. 8).

Fig. 8. Crack tip shielding and amplification due to hard inclusion.

In the subsequent cases, the ratio of $E_p/E_m$ has been set to 6.43, $\nu_p = 0.17$ and $\nu_m = 0.33$. This set of data corresponds to silicon carbide (SiC) reinforcement of aluminum (Al) matrix. Such materials find applications in aerospace, automobile and general engineering purposes.

Fig. 9 shows the computed crack paths, as per MTPS criteria, for various $d/r$ ratios. The deviation of the crack increases significantly as it approaches the particle. As it crosses the particle, it experiences an increase in the $G/G_0$ (amplification effect). As the $d/r$ ratio decreases, the effect due to the particle increases.

Fig. 9. Crack path as per MTPS criteria for various $d/r$ ratios.

Fig. 10 shows the variation of non-dimensional ERRs with crack extension of various $d/r$ ratios. It can be concluded that the crack experiences a reduction in the $G/G_0$ (shielding effect) as it approaches the particle. As it crosses the particle, it experiences an increase in the $G/G_0$ (amplification effect). As the $d/r$ ratio decreases, the effect due to the particle increases.

Fig. 10. The ERRs for various $d/r$ ratios.
4.2 Crack Interaction with a Pair of Particles Positioned Symmetrically about $x$-axis

Fig. 11. Pair of particles located symmetrically about $x$-axis.

Fig. 11 shows the geometry with two particles and an edge crack located at the centre. The crack was allowed to propagate under uniform remote tension. It is a pure mode I problem. Both MTPS and MTS yield the same crack extension direction.

Fig. 12. Non-dimensional energy release rate for a propagating crack in the presence of inclusions.

Fig. 12 shows the predicted variation of the normalized ERR ($G/G_0$) as the crack propagates in the presence of the inclusions. It is observed that as the crack approaches the particle inclusions, it experiences the shielding effect and the $G/G_0$ ratio reduces. This trend reverses as the crack moves past the centre line of the inclusions. The extent of shielding or amplification is dependent on the modulus ratio of the two constituents; the effect of particle increases with increase in the ratio of $E_p/E_m$.

This effect also increases with a decrease in the inter-particle distance. In other words, this effect gets magnified with the reduction in the proximity of the crack to the inclusion.

4.3 Crack Interaction with a Pair of Particles Positioned Arbitrarily

Fig. 13 shows the geometry with a pair of particles located at an arbitrary angle $\theta$. Both MTPS and MTS criterion were used to examine the paths of crack propagation.

Fig. 13. Pair of particles at an arbitrary angle in the domain.

Two case studies were considered: $\theta=60^\circ$ and $\theta=30^\circ$. The ratio of $E_p/E_m$ was set to 6.43, $v_p=0.17$ and $v_m=0.33$.

Fig. 14 shows the predicted crack path by MTPS and MTS criteria for $\theta=60^\circ$. Although crack deflection as per the MTS criterion is more pronounced, the difference is not significant.
Fig. 14. Crack propagation path in presence of inclusions oriented at 60°.

However, the difference is more pronounced in the θ=30° (Fig. 15) case where the crack paths predicted by the two criteria are very different. The MTS criterion predicted that the crack path would meet the particle, while the MTPS criterion predicted that the crack would propagate around the particles.

Fig. 15. Crack propagation path for inclusions oriented at 60°.

Fig. 16 shows the variation of normalized ERR (\(G/G_0\)) as the crack propagates for θ=60° and θ=30° using both MTPS and MTS criteria.

(a) θ=60°: The crack experiences the maximum shielding at \(x/r=-1\) i.e., when the crack tip is at a distance equal to the radius of the inclusion. The amplification occurs as the crack crosses the second inclusion.

(b) θ=30°: The crack experiences the maximum shielding when its tip is at \(x/r=-1\). This is more pronounced than the previous case due to its proximity to the inclusion. As per the MTS criteria, as soon as the crack crosses the left inclusion, there is a sudden dip in the ratio \(G/G_0\); the crack propagation is halted as soon as the tip reaches the second inclusion. In contrast, as per the MTPS criterion, the crack propagates through the matrix. This agrees more closely with experimental observations [23]. Further, the ratio \(G/G_0\) increases significantly as soon as it crosses the second inclusion (Fig. 16).

5 CONCLUSIONS

A modified EFG method has been proposed to handle the crack propagation problem in particle-reinforced composites. It shows the utility of the visibility method with nodal refinement around the crack tip to capture the singularity in linear elastic fracture mechanics. A modified interaction integral has been used to calculate the energy release rates. Furthermore, this paper also highlighted the difference in crack propagation paths arising from
the use of the MTPS and MTS criteria. The key conclusions are:

a) The reduction and amplification of the energy release rate occur in the presence of hard inclusions.

b) This reduction in the ERR enhances with the increase in the $E_p/E_m$ ratio and reduction in distance of the crack tip from the inclusion. Generally, the amplification effect, observed after the crack crosses the inclusion, is lower than the shielding effect before the crack approaches the inclusion.

c) The MTS criterion predicts a larger extent of crack deviation than the MTPS criterion. In the case of closely spaced inclusions, the MTPS criterion predicts crack movements always through the matrix for all particle-matrix stiffness ratios.

d) The proposed XEFG method can handle the study of crack propagation through particulate composites, if necessary. Further it can simulate singularity in stresses other than $r^{-0.5}$.

References


