1. Introduction

Directed Carbon Fibre Preforming (DCFP) is an automated process for producing complex 3D discontinuous fibre preforms for liquid moulding. Fibre deposition is robotically controlled and net-shape, offering excellent repeatability and low material wastage. DCFP offers greater design freedom compared with conventional laminated composites, as the fibre orientation distribution, fibre volume fraction ($V_f$), and fibre length can all be locally varied in the component according to structural requirements.

The current lack of robust design tools makes it difficult to exploit the versatility of the DCFP process, limiting these materials to a ‘black metal’ design approach. This results in homogeneous isotropic fibre architectures and similar geometries to metallic counterparts. The most widely used structural optimisation methods for composite materials are considered to be unsuitable for DCFP because they adopt a metaheuristic approach [1]. These are only practical for handling discrete problems and are more widely used for optimising laminated composites where fibres are arranged in plies. Other methods, such as non-linear programming [2, 3], require constant re-evaluation of the design objectives and constraints, and are therefore very computationally expensive, particularly for large structures. In comparison, optimality criterion approaches [4] use simple local rules to update design variables, which are much more efficient and suitable for complex or larger problems.

This paper builds upon a finite element based topology optimisation method previously proposed by the authors in [5]. An optimality criterion approach was developed to concurrently optimise local thickness and material stiffness for each finite element, over the entire surface of a DCFP component. The optimisation algorithm solved Lagrangian multipliers for each optimisation constraint, which included material volume and material cost.

Optimised fibre architectures produced using the algorithm in [5] were not always suitable for manufacture however [6], as the local section thickness and material stiffness were both continuously varied over the surface of the component. In practice it is impossible to vary the stiffness on an element-by-element basis, as the precision of the fibre deposition is dependent on the resolution of the robotically controlled chopper gun. Neighbouring elements with similar material properties therefore need to be merged into larger zones using a common set of material parameters (fibre length and orientation, tow size etc.), controlling the local stiffness of the zone.

This paper extends the topology optimisation model in [5], by introducing a segmentation algorithm. The global stiffness and thickness distributions that result from the initial stiffness optimisation are combined to form one single distribution of areal mass, which is the basis for performing the model segmentation. The size and the shape of each zone are tailored to suit the fibre deposition process, so that small areas or patches with small dimensions are avoided. It is also rational that a critical zone size exists in order to achieve a representative fibre architecture. The size of the representative volume element for achieving a homogeneous distribution of discontinuous fibres is known to be a function of fibre length and volume fraction [7, 8].

The model has been demonstrated by optimising the bending performance of a flat plate with three arbitrary holes. The deflection and specific stiffness
of the optimised DCFP panel are compared against uniformly thick DCFP and mild steel benchmarks. Results highlight the quality of the segmentation criteria and demonstrate potential weight saves for the DCFP panel process over the benchmarks.

The modelling procedure has been split over three key areas: stiffness optimisation, material assignment and model segmentation.

2. Stiffness optimisation criteria

The stiffness optimisation algorithm aims to minimise the overall deflection of a component, whilst satisfying all of the design constraints. Strain energy is used as a global measure of the displacement and the problem is solved using a minimisation approach \[5\]. The model is constrained by global weight and material cost conditions, which remain constant throughout the analysis. Local section thickness and material stiffness are used as the design variables, which are updated iteratively for each element within the model. The problem can be constructed as:

\[
\begin{align*}
\text{min } & \quad U(E, t) \\
\text{subject to } & \quad V(t) = V_0, \quad C(E, t) = C_0 \\
\text{and } & \quad E \geq E_{\text{min}}, \quad t \geq t_{\text{min}}
\end{align*}
\]

where \(E\) and \(t\) denote the modulus and thickness design variables respectively. \(U\) denotes the total strain energy in the structure. \(V\) and \(C\) denote the overall volume and material cost of the structure, and \(V_0\) and \(C_0\) are the target volume and cost respectively. \(E_{\text{min}}\) is the lower bound for the tensile modulus and \(t_{\text{min}}\) is the lower bound for the thickness to prevent local buckling of the structure, which are both determined experimentally. The minimum thickness is influenced by \(E_{\text{min}}\) since the stiffness and strength of the component change with thickness due to the homogeneity effects \[9\] (see Fig. 2).

A simple linear relationship is assumed between the material cost and the tensile modulus of the composite material:

\[ C_i = \alpha E_i A_i t_i \]  

where \(A_i\) is the area of element \(i\), and \(\alpha\) is a constant which can be estimated by calculating the material cost for a range of DCFP laminates with known properties. The material cost is influenced by fibre and resin choice, but is largely dominated by the fibre cost, which is a function of tow size and fibre grade.

The optimisation process is performed iteratively, based on an initial strain energy density value from an isotropic model of uniform section thickness. For each subsequent iteration, the overall strain energy, component volume and material cost can be expressed as a summation from each element. The optimality criterion is derived by solving the Karush–Kuhn–Tucker (KKT) conditions of the Lagrangian expression from equation (1):

\[ L = U + \lambda_1(V - V_0) + \lambda_2(C - C_0) + \lambda_3(E_{\text{min}} - E_i) + \lambda_4(t_{\text{min}} - t_i) \]

where \(\lambda_1, \lambda_2, \lambda_3\) and \(\lambda_4\) are the Lagrange multipliers corresponding to each constraint. The stationary of the Lagrangian leads to the following KKT conditions:

\[
\begin{align*}
\frac{\partial L}{\partial E} & = \sum_i \frac{\partial U_i}{\partial E_i} + \lambda_2 \sum_i \frac{\partial C_i}{\partial C_i} + \lambda_3 = 0 \\
\frac{\partial L}{\partial t} & = \sum_i \frac{\partial U_i}{\partial t_i} + \lambda_1 \sum_i \frac{\partial V_i}{\partial t_i} + \lambda_2 \sum_i \frac{\partial C_i}{\partial t_i} + \lambda_4 = 0
\end{align*}
\]

The local thickness and modulus values are updated concurrently in an iterative loop using the optimality criteria derived in equations (2), (4) and (5). (A more detailed derivation is provided in \[5\]). The iteration stops when the overall strain energy value has converged.

3. Material assignment

This part of the model assigns the appropriate fibre architecture (selected based on material properties) to the optimised component. It takes the output from the initial stiffness optimisation (in the form of a local thickness and modulus distribution) and converts it into a fibre areal mass distribution of constant fibre tow size and fibre length. The optimisation routine is linked to a material database containing experimental Young’s moduli for a
range of DCFP plaques with various fibre architectures.

The database has been analysed to form simple relationships between each fibre architecture parameter and the resultant Young’s modulus. The model utilises these continuous functions to express the thickness and modulus distributions in terms of fibre volume fraction for any predetermined fibre length and tow size. The volume fraction result is then combined with the thickness result to calculate the areal mass distribution, which is used directly to produce the robot deposition profile.

A rule of mixtures (ROM) approach is employed to summarise the relationship between Young’s modulus and fibre volume fraction for DCFP. Figure 2 shows typical data for plaques with 6K tows, chopped to a length of 57.5mm. Experimental data is presented as points, with lines representing predictions from ROM. The Young’s modulus increases with increasing fibre volume fraction and the experimental points are in good agreement with the linear ROM function. Furthermore, Figure 2 suggests that there is a tendency for the modulus to reduce for thinner specimens, which is common for discontinuous fibre architectures. This thickness size effect has been considered in the current model by locally increasing the fibre volume fraction to compensate.

Figure 1 shows the relationship between Young’s modulus and specimen thickness in more detail. It is evident that the data follows a bi-linear relationship, with a transition point occurring at 3.5mm for this particular fibre architecture. The Young’s modulus increases linearly for increasing thickness, after which a plateau is reached. This transition point depends on factors such as fibre length, volume fraction and tow size, as they all influence the level of homogeneity and departure from isotropy [8]. However, due to limited experimental data, a 3.5mm transition point has been assumed for all DCFP materials in the current work.

It is worth noting that the current material model is only valid within certain thickness and volume fraction ranges: Very thin panels can cause buckling problems when used in real structures, and it is difficult to achieve uniform fibre distributions for very low fibre volume fractions or large tow sizes. Feasible thickness and fibre volume fraction ranges are also limited by the liquid moulding process: Preform permeability may be too low for volume fractions approaching 50%, whereas preform washing may occur if the volume fraction is too low (<15%). Whilst the thickness can be directly constrained within the stiffness optimality criterion method, the fibre volume fraction can be effectively constrained by restricting the Young’s modulus limits in equation (1).

4. Model segmentation strategies

This section of the model aims to generate more realistic fibre architectures by grouping elements of similar fibre areal mass into larger zones of uniform areal mass. The adopted approach shares many similarities with image segmentation techniques, in particular, those based on the similarity of the pixel intensity value. The resultant zones are tailored with length scale and size control in a secondary operation.

4.1 Segmentation with multi-level thresholding

Otsu’s method [10] is one of the most common thresholding methods for segmenting bi-level images. It states that the optimum threshold value should divide an image into two classes of pixels, so that the intra-class variance is minimised. A modified version [11, 12] has been used in the current paper to conduct multi-levelled thresholding.

The model searches within the areal mass range of 1000 to 4000gsm, using a fixed increment size of 10gsm, to determine optimum thresholds. Let \( \rho_A \) denote element areal mass, where \( \rho_{A_{\text{max}}} \) and \( \rho_{A_{\text{min}}} \) are the maximum and the minimum values. The range in areal mass is divided into N intervals \( K_1, K_2, ..., K_N \) by the end points \( [1000, 1010, ..., 4000] \), such that \{\( \rho_{A_{\text{min}}} \leq \rho_A < 1000 \)\} for \( K_1 \), \{1000 \leq \rho_A < 1010 \} for \( K_2, ..., \{4000 \leq \rho_A < \rho_{A_{\text{max}}} \} \) for \( K_N \).

Assuming \( K_1, K_2, ..., K_N \) are divided into M classes \( C_1, C_2, ..., C_M \) by \( M-1 \) thresholds, the intra-class variance can be written as:

\[
(\sigma_B)^2 = H_{C_1} + H_{C_2} + \cdots + H_{C_M} \tag{6}
\]

in which:
\[ H_{C_i} = \frac{S_{C_i}^2}{P_{C_i}} \] (7)

\[ P_{C_i} = P_{(K_a, K_b)} = \sum_{n=a}^{b} p_{K_n} \] (8)

\[ S_{C_i} = S_{(K_a, K_b)} = \sum_{n=a}^{b} (p_{K_n} \cdot \mu_{K_n}) \] (9)

where \( p_{K_n} \) is the probability that a \( \rho_{A} \) value belongs to interval \( K_n \), and \( \mu_{K_n} \) is the average areal mass value of elements enclosed by \( K_n \).

The adopted algorithm searches the optimum thresholds by calculating the intra-class variance \( H(K_{a_i}, K_{b_i}) \) for any possible class containing interval \( K_a \) to \( K_b \) (1 \( \leq a \leq b \leq N \)), using equations (7)-(9), and stores them in the following array:

\[
\begin{bmatrix}
H(K_{1a},K_{1b}) & H(K_{1a},K_{2b}) & \ldots & H(K_{1a},K_{N-b}) & H(K_{1a},K_{b}) \\
0 & H(K_{2a},K_{2b}) & \ldots & H(K_{2a},K_{N-b}) & H(K_{2a},K_{b}) \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \ldots & H(K_{N-a},K_{N-b}) & H(K_{N-a},K_{b}) \\
0 & 0 & \ldots & 0 & H(K_{N-a},K_{b}) \\
\end{bmatrix}
\]

The optimum thresholds \( t_{1}, t_{2}, \ldots, t_{M-1} \) can be determined by calculating \( \sigma_{b_i}^2 \) for any possible combination of \( H(K_{i+1},K_{i}), H(K_{i+2},K_{i}), \ldots, H(K_{M},K_{i}) \) using the H array, and identifies the combination that yields a minimum value for \( \sigma_{b_i}^2 \).

Once all elements have been classified according to their areal mass, the model is refined into zones. A zone is defined as a group of elements that all belong to the same class and are bounded with one continuous boundary without self-intersection. The zoning process starts by randomly choosing an element as the seed. The zone expands by continuously searching and merging surrounding elements if they belong to the same class. When no further merging can be performed, a new zone will be started with a random new seed. The process continues until all the elements have been allocated a zone.

### 4.2 Length scale and size control

The multi-level thresholding method uses the element areal mass value as the only criteria during segmentation, and does not take into account the connectivity between elements. Therefore, model zones can result in random shapes and sizes, especially for cases where the areal mass distribution is highly scattered. Length scale and size constrains have been enforced to remove small zones and repair any zone that contain regions narrower than the robotic fibre deposition head.

The minimum length scale of the zone is measured by constructing an equidistant curve from the zone boundary. The curve is formed by generating points that are perpendicular to the boundary, at a constant distance on the inside of the zone (see Figure 3).

If \( 2r \) denotes the minimum length scale of the zones, then \( r \) denotes the offset distance. Therefore the equidistant curve does not self-intersect unless the minimal length scale is smaller than \( 2r \). Figure 3 illustrates this approach, showing the approximate bounding polygon of the boundary nodes. The original zone (Figure 3a) contains a narrow region, therefore the equidistant boundary (dotted line) produces a singularity, due to self-intersection. The narrow region is removed by defining a new zone boundary (solid line, Figure 3b), which encompasses elements at a constant distance \( r \) in the outward direction to form a new equidistant boundary.

Elements removed from narrow regions are re-allocated into neighbouring zones, but care is taken to prevent narrow regions reforming. This is demonstrated in Figure 4, where elements removed during the previous iteration (white elements) are surrounded by four other zones (coloured and numbered). The white elements are reallocated to one of the other four regions using a ‘region growth process’. Only one layer of the white elements are merged at a time and the zone is selected based on the longest boundary (in this case the row of elements bordering zone 2). Size control is performed as the last stage and is to reduce manufacturing complexity by minimising the number of smaller zones. It can be easily achieved by adopting a threshold value of the minimum zone size, where areas smaller than the critical size will be discarded. A subsequent merging process is
5. Case study

A 400\times 600\text{mm} rectangular plate with three large arbitrary circular holes has been chosen to demonstrate the optimisation process. Comparisons of the deflection of the plate under a point load are made for three materials: 2\text{mm} thick mild steel, 4\text{mm} thick DCFP with uniform fibre architecture and DCFP with optimised fibre architecture. Both DCFP plates consist of 30\text{mm} long 3\text{K} tows, but the optimised version has variable fibre volume fraction and section thickness locally within the part. The plate is simply supported along the two 400\text{mm} sides and the point load is applied at the geometric centre.

5.1 Finite element modelling

The CAD geometry of the plate is defined as a 3D conventional shell and exported into ABAQUS/CAE for mesh generation and analysis. The geometry of the conventional shell represents the mid-surface of the plate, where the material properties and section thickness can be defined using the shell section definition. Quadratic elements (STRI65) are adopted for this study, but the optimisation algorithm is capable of handling linear or non-linear conventional shell elements of any shape.

The model is subjected to a static load case using appropriate boundary conditions and is analysed in ABAQUS/Standard. The point load is applied as an equivalent pressure of 0.764\text{MPa} over a circular region of Ø10\text{mm}, to reduce the stress singularity caused by applying a concentrated force and to match experimental methods. Isotropic material properties are adopted for all material options, where composites properties are defined by the effective linear-elastic stress-strain relationship. For the mild steel and un-optimised benchmark models, the corresponding shell section definition is assigned globally to the entire model, whereas for the optimised DCFP model, a *DISTRIBUTION TABLE is used to specify the shell thickness and the effective composites properties for each element. The material properties for the benchmark models have been obtained experimentally from tensile testing of straight sided samples, where the Young’s modulus and Poisson’s ratio values have been measured using digital image correlation.

Table 1 summarises the material properties for the benchmark models and the optimised DCFP model. $E_0$, $t_0$ denote the Young’s moduli and section thicknesses of the benchmark models. A 3\text{K} tow size and 30\text{mm} fibre length have been selected for the DCFP fibre architecture, with a 30\% Vf and 4\text{mm} section thickness for the un-optimised case.

5.2 Results

A summary of the simulation results is presented in Table 2, including the mild steel and un-optimised DCFP benchmarks. Results for the optimised model are presented in two stages. The first set of results are following the stiffness optimisation, but before segmentation (w/o zoning), which represent the fully optimised case. The second set of results demonstrates the compromise in performance due to the segmentation of the areal mass distribution. The performance is compared in terms of the total strain energy, maximum deflection, total mass and specific stiffness.

Figure 7 displays the deflection plot for the steel models and the three DCFP models. Convergence of the strain energy density was achieved after just seven iterations for the initial optimised DCFP model. The maximum deflection was reduced by \sim 50\% compared with the un-optimised case. The deflection increases by 10\% following segmentation (Optimised DCFP – w/o zones), but this still represents a 45\% reduction compared to the un-optimised DCFP and 52\% compared to the steel.

The thickness of the un-optimised DCFP panel was chosen to yield a comparable bending stiffness to
the steel benchmark. According to the strain energy density values in Table 2, the stiffness of the un-optimised DCFP solution was 7.3% lower than the steel and therefore the maximum deflection was 5mm compare with the target of 4.65mm. However, the overall mass of the DCFP plate was 70% lower, reflecting the high specific stiffness of this material compared with steel (3.6 times the overall specific stiffness of mild steel for the current case).

The thickness and modulus distributions from the initial stiffness optimisation stage are presented in Figure 8. These represent the idealised solution and are considered to be unrealistic from a manufacturing perspective. Both distributions are similar for the thickness and modulus profiles, as they are dominated by the stress concentrations at the holes and at the central load point. A high modulus and thickness region is often an indication of the localised stress concentration, and usually the affected area is relatively small due to the large stress gradient. Figure 9 presents the segmentation scheme based on a 5-level threshold, where the effects of applying length-scale and size controls have also been demonstrated. The number of levels is reduced to 4 due to subsequent merging following the length-scale control. Figure 10 shows a histogram of areal mass distribution before segmentation and the range subsequently covered by each thresholding level after segmentation. The overlapped regions between levels are a result of the length scale and size control, where elements have been merged into a zone at a different thresholding level. It is worth noting that the spatial distribution of elements with higher areal masses (5500gsm-6000gsm) tends to be highly scattered and localised, therefore these elements are often removed during the length-scale and size control. High areal mass elements are shifted into lower areal mass regions during the subsequent merging process.

Although the areal mass becomes a lot more uniform due to the segmentation, it has very little impact on the structural performance of the plate according to Figure 7 and Table 2. The expected deflection of the final model is very close to that of the idealised solution, where the maximum deflection increases to 2.84mm, which still shows a significant gain in overall stiffness compare with the un-optimised DCFP. The specific stiffness for the optimised DCFP structure is 5.5 times that of the mild steel, and 1.8 times that for the un-optimised DCFP. This is achieved by redistributing the DCFP material more effectively within the structure without incurring any additional material cost or weight compare with the un-optimised solution. This demonstrates the flexibility of the DCFP process and highlights the importance of understanding the characteristics of the material and process.

6. Conclusions

A stiffness optimisation method has been developed to locally vary the thickness and areal mass distribution for a DCFP component. The model is based on an optimality criterion approach for concurrently optimising multiple design variables. A segmentation algorithm has been implemented to divide the resulting fibre architecture into several zones of similar areal mass, where the size and shape of each zone is optimised to suit the precision of the fibre deposition process.

The optimisation algorithm has been demonstrated using a flat rectangular plate with three holes. Comparisons are made between mild steel, an un-optimised DCFP plate and two optimised DCFP versions. Results suggest that the current method can effectively improve the stiffness of the component without sacrificing weight-saving potential or adding additional cost over the original DCFP benchmark. The subsequent segmentation model ensures that the optimised structure is fit for manufacture, with very little impact on the overall structural performance compared with the idealised solution.

References


Figure 1: Young’s modulus vs. specimen thickness for 6K 57.5mm DCFP. Experimental data are normalised to 30% Vf using rule of mixture. Error bars indicate the standard deviation of each specimen type.

Figure 2: Young’s modulus vs. fibre volume fraction for 6K, 57.5mm DCFP. Experimental data include tensile specimens of different thickness ranges as indicated with marker shape and colour intensity. Error bars indicate the standard deviation of the Young’s modulus value.

Figure 3: The equidistant boundary approach to control the minimum length scale of a zone. All solid lines indicate reference boundaries and all dotted lines indicate corresponding equidistant boundaries.

Figure 4: Schematic diagram of the region growth process. The white region in the centre indicates the un-zoned elements and the coloured, numbered regions indicate the existing zones.
Figure 5: Flow chart of the structural optimisation algorithm
Figure 6: Deflection of the test plates. Left to right: mild steel, un-optimised DCFP, optimised DCFP (w/o zones), optimised DCFP (w/ zones). All values in mm. Negative signs indicate downward displacement.

Figure 7: Thickness (mm, left) and modulus (MPa, right) distributions from the initial optimisation results.

Figure 8: Areal mass distribution before and after segmentation: (a) before segmentation; (b) with 5-level thresholding; (c) with length-scale control; (d) with zone size control. Values in legend are expressed in g/m².
Table 1: Material properties for the structural optimisation model. Benchmark values for DCFP are taken from 3K 30mm fibre length at 30%Vf.

<table>
<thead>
<tr>
<th>Material</th>
<th>$E_0$ (MPa)</th>
<th>$E_{\text{min}}$ (MPa)</th>
<th>$E_{\text{max}}$ (MPa)</th>
<th>$\nu$</th>
<th>$t_0$ (mm)</th>
<th>$t_{\text{min}}$ (mm)</th>
<th>$t_{\text{max}}$ (mm)</th>
<th>Density (Kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mild steel</td>
<td>200000</td>
<td>-</td>
<td>-</td>
<td>0.3</td>
<td>2.1</td>
<td>-</td>
<td>-</td>
<td>7850</td>
</tr>
<tr>
<td>DCFP</td>
<td>27060</td>
<td>15000</td>
<td>45000</td>
<td>0.3</td>
<td>4</td>
<td>2</td>
<td>6</td>
<td>1230</td>
</tr>
</tbody>
</table>

Table 2: FEA results for the benchmark model and the optimised DCFP.

<table>
<thead>
<tr>
<th></th>
<th>Total strain energy (kN.mm)</th>
<th>Max. deflection (mm)</th>
<th>Total mass (kg)</th>
<th>Specific stiffness* (unity)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mild steel</td>
<td>138.33</td>
<td>4.65</td>
<td>3.47</td>
<td>1</td>
</tr>
<tr>
<td>Un-optimised DCFP</td>
<td>148.47</td>
<td>5.00</td>
<td>1.04</td>
<td>3.1</td>
</tr>
<tr>
<td>Optimised DCFP (w/o zones)</td>
<td>75.55</td>
<td>2.52</td>
<td>1.04</td>
<td>6.1</td>
</tr>
<tr>
<td>Optimised DCFP (w/ zones)**</td>
<td>84.61</td>
<td>2.84</td>
<td>1.04</td>
<td>5.5</td>
</tr>
</tbody>
</table>

*Values normalised to the benchmark case of mild steel.

**Zones created with 5-level thresholding, 50mm minimum length-scale and $10^4$mm$^2$ minimum zone size.