A MIXED MODE COHESIVE LAW INCLUDING INTERFACE DILATATION UNDER NEAR MODE II FRACTURE

B. F. Sørensen*, S. Goutianos
Department of Wind Energy, Section of Composites and Materials Mechanics, Technical University of Denmark, DK-4000 Roskilde, Denmark
* Corresponding author (bsqr@dtu.dk)

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Abstract
A mixed mode cohesive law is presented for the representation of interface dilation of a fracture process zone undergoing dominated tangential (Mode II) crack opening displacement. The model introduces two new material length parameters for the interface dilatation. The phenomenological model can represent a variety of micromechanical fracture process mechanisms such as the development of multiple shear cracks, surface roughness of crack faces as well as bridging fibers undergoing buckling. The mixed mode cohesive law is implemented as a user element in a commercial finite element program. The new cohesive element is used to model mixed mode delamination experiments reported in the literature. It is found that the model is capable of reproducing the detailed behavior of the macroscopic delamination experiments well.

1 Introduction

1.1 Background and motivation
Cohesive zone modelling, in which the fracture process zone is modelled in terms of traction-separation laws called cohesive laws, is widely used to simulate crack growth of materials [1,2] and structures [3,4]. One of the advantages of cohesive zone modelling is that a cohesive law comprises both a strength (the peak traction value) and a fracture energy (the area under the traction-separation curve), such that it embodies both crack initiation and crack growth [5]. A large number of cohesive laws have been developed [6-8]. Most of these cohesive laws are idealized cohesive laws and usually the traction-separation relationships are assumed to be linear and the tractions decay to zero at a critical separation.

However, it is becoming more apparent that the idealized mixed mode cohesive laws can be too simple to represent real fracture processes. A number of recent experimental investigations have shown that the behavior of a real fracture process zone can be significantly different from the idealized cohesive laws [4, 9-10]. In these studies, it was found that the Mode II fracture process generates a small separation in the direction normal to the crack plane (Fig. 1). This phenomenon, that an imposed pure sliding displacement induces a normal opening is what it is called here interface dilatation. Moreover, the cohesive laws for the normal and shear tractions are coupled. In contrast, all idealized mixed mode cohesive laws treat the fracture process zone as a smooth plane, so that a pure tangential displacement induces only shear traction (and vice versa).

1.2 Aim of the study
The aim of the present work is to formulate a cohesive law that incorporates interface dilatation in a phenomenological sense. The developed cohesive law is thus not based on a specific microscale fracture process mechanisms. Additional material length parameters are introduced. The values of these length parameters can be determined experimentally or from results of micromechanical models of the specific fracture process. The proposed phenomenological model formulation is expected to be capable of modeling the mechanisms inducing interface dilatation shown in Fig. 1. In addition, it is assumed that the cohesive tractions are coupled but in a way so that they can be derived from a potential function. As a result the mixed cohesive law is path independent.

The developed cohesive law is implemented as a user-defined element in the commercial finite element (FE) code Abaqus. In the present paper, the
model is used for the modeling of the behavior of mixed mode fracture experiments of polymer fiber composites of Sørensen and Jacobsen [4].

2 Summary of Experimental Results

2.1 Experimental procedure

Before the model is presented, it is useful to review the results of some mixed mode experiments that have documented interface dilatation [4]. In that study, mixed mode delamination of a unidirectional glass fiber/polyester composites was using double cantilever beam specimens loaded with uneven bending moments (DCB-UBM), see Fig. 2. The DCB-UBM test methods apply the same moment ratio throughout each experiment. All the test specimens experienced significant toughening due to large-scale crack bridging by cross-over bridging of fiber ligaments. The fracture resistance was characterized in terms of the J integral (an analysis method valid for both small and large-scale bridging problems [6]). In addition, the normal and tangential displacement at the original crack tip were measured.

2.2 Experimental results

A few representative results are shown in Fig. 3, where \( M_1 \) and \( M_2 \) are moments (both defined positive in the same direction, see Fig. 2) applied to the two beam of the cracked end of the DCB specimen. For the DCB-UBM specimen, \( M_1/M_2 = -1 \) is pure Mode I and \( M_1/M_2 = 1 \) corresponds to pure Mode II. The fracture resistance \( J_R \) is shown as a function of the magnitude of the end-opening \( \Delta' \) in Fig. 3a. With increasing moment ratio (increasing amount of Mode II) the \( J \) value at the onset of cracking increases. For all three moment ratios, the fracture resistance increases with increasing end-opening. This increase is attributed to the development of a crack bridging zone. For \( M_1/M_2 = -0.52 \) (near Mode I) and \( M_1/M_2 = 0.50 \), a steady-state fracture resistance, significantly higher than the initiation value, is attained. For \( M_1/M_2 = 0.97 \) (close to pure Mode II) the fracture resistance rises significantly, but does not approach a steady-state value. Fig. 3b shows the corresponding relationships between the end-opening, \( \Delta''_n \), and end-sliding, \( \Delta'_t \). The Mode I dominated experiment \( (M_1/M_2 = -0.52) \) results in a large normal opening and a relative small tangential end-opening. For both the mixed mode experiments with \( M_1/M_2 = -0.52 \) and \( M_1/M_2 = 0.50 \), the relationship between \( \Delta''_n \) and \( \Delta'_t \) is almost linear. In contrast, for the near full Mode II experiment \( (M_1/M_2 = 0.97) \), both an end-sliding and a normal end-opening are found. More specific, for small end-sliding (up to \( \Delta'_t \approx 0.5 \) mm), the end-opening increases near-proportionally, but for larger end-sliding ( \( \Delta'_t > 0.5 \) mm) the normal opening remains at a near-constant value of about \( \Delta''_n \approx 0.2 \) mm. It is this behaviour for the near full Mode II experiment \( (M_1/M_2 = 0.97) \) that cannot be recreated by traditional cohesive laws; they would not give any normal opening under pure Mode II. In Section 5 we will attempt to reproduce this behaviour using our new cohesive law with interface dilatation.

3 Formulation of Mixed Mode Cohesive Law

3.1 Description of interface dilatation

As mentioned above, the model developed is based on the assumption that the cohesive tractions can be derived from a potential function. Then, the values of the normal traction, \( T_n \), and the shear traction, \( T_s \), at a given position within the cohesive zone depend only on the current values of the local normal opening displacement, \( \Delta''_n \), and tangential displacement, \( \Delta'_t \); they do not depend on the opening history.

Fig. 4 illustrates the idea behind the model. An imposed tangential displacement (while keeping zero normal traction), \( \Delta'_t \), should induce a displacement in the direction normal to the cracking plane, \( \Delta''_n \). For small tangential displacements, the resulting normal displacement, \( \Delta''_n \), increases with increasing \( \Delta'_t \). This increase is assumed to be linear. However, for tangential displacements larger than a characteristic value, \( \Delta'_t \), the dilatation mechanism is assumed to be in full operation. Thus, the resulting normal displacement takes a constant value \( \Delta''_n \). Both \( \Delta'_t \) and \( \Delta''_n \) are considered material properties (i.e. properties of a given interface) and can be determined experimentally (see Sections 4 and 5). An outline of
the mathematical formulation is given below; full details are given in a journal paper [11].

### 3.2 The normal traction
For pure Mode I (\(\Delta = 0\)), we take a bi-linear cohesive law. For \(\Delta_n < 0\), \(T_n\) should take a negative value, representing compressive normal traction during contact (opposing interpenetration). For \(\Delta_n > 0\), the normal traction increases linearly from zero (at zero opening) to a peak traction value, \(\bar{T}_n\), at a separation denoted \(\Delta'_n\), and decreases linearly to zero at a critical separation called \(\Delta^0_n\). The mathematical description of the pure Mode I cohesive law is:

\[
T_n(\Delta_n) = \begin{cases} 
\bar{T}_n \frac{\Delta_n}{\Delta_n^0} & \text{for } \Delta_n < \Delta'_n \\
\bar{T}_n \frac{\Delta'_n - \Delta_n}{\Delta'_n - \Delta^0_n} & \text{for } \Delta'_n \leq \Delta_n < \Delta^0_n \\
0 & \text{for } \Delta^0_n \leq \Delta_n 
\end{cases}
\] (1)

For Mixed Mode, \(\Delta'_n\) and \(\Delta^0_n\) (denoted \(\bar{\Delta}_n\) and \(\bar{\Delta}^0_n\)) are translated to follow the interface dilatation:

\[
\bar{\Delta}'_n = \frac{\bar{\Delta}_n}{\bar{\Delta}} \Delta'_n, \quad \bar{\Delta}^0_n = \frac{\bar{\Delta}_n}{\bar{\Delta}} \Delta^0_n.
\] (2)

Then, for \(0 < \Delta_n \leq \bar{\Delta}_n\), before the interface dilatation is fully in play, the normal traction becomes:

\[
T_n(\bar{\Delta}_n, \Delta_n) = \begin{cases} 
\bar{T}_n \frac{\bar{\Delta}_n \Delta'_n - \bar{\Delta}_n \Delta_n}{\bar{\Delta}_n \Delta'_n - \bar{\Delta}_n \Delta^0_n} & \text{for } \Delta_n < \bar{\Delta}_n \\
\bar{T}_n \frac{\bar{\Delta}_n \Delta'_n - \bar{\Delta}_n \Delta_n}{\bar{\Delta}_n (\Delta'_n - \Delta_n)} & \text{for } \bar{\Delta}_n \leq \Delta_n < \bar{\Delta}^0_n \\
0 & \text{for } \bar{\Delta}^0_n \leq \Delta_n 
\end{cases}
\] (3)

Now \(T_n\) is negative for \(\Delta_n < \bar{\Delta}_n\). Note also that \(T_n\) now depends on both \(\Delta_n\) and \(\bar{\Delta}_n\).

For \(\Delta_n > \bar{\Delta}_n\), the dilatation effect is in full effect and should be constant at the value \(\bar{\Delta}_n\). Consequently, the values corresponding to \(\Delta'_n\) and \(\Delta^0_n\) of Mode I (now denoted \(\bar{\Delta}_n\) and \(\bar{\Delta}^0_n\)) should be translated as:

\[
\bar{\Delta}_n = \bar{\Delta}_n + \Delta'_n \quad \text{and} \quad \bar{\Delta}^0_n = \bar{\Delta}_n + \Delta^0_n.
\] (4)

The equations for \(T_n\) then become:

\[
T_n(\Delta_n) = \begin{cases} 
\bar{T}_n \frac{\Delta_n - \bar{\Delta}_n}{\bar{\Delta}'_n - \bar{\Delta}^0_n} & \text{for } \Delta_n < \bar{\Delta}_n \\
\bar{T}_n \frac{\Delta'_n - \Delta_n}{\Delta'_n - \Delta^0_n} & \text{for } \bar{\Delta}_n \leq \Delta_n < \bar{\Delta}^0_n \\
0 & \text{for } \bar{\Delta}^0_n \leq \Delta_n 
\end{cases}
\] (5)

The normal traction-separation relations, Eq. (1), (3), and (5), are shown in Fig. 5.

### 3.3 The shear traction
Next, we address the shear traction, \(T_s\). In order to have cohesive tractions that are derivable from a potential function \(\Phi(\Delta_n, \Delta_s)\), the following two requirements must be met [13]: (A) The \(\Delta_n - \Delta_s\) domain in which the cohesive tractions are defined must be simply connected, and (B) the functions for the tractions must fulfill:

\[
\frac{\partial T_s}{\partial \Delta_n} = \frac{\partial T_s}{\partial \Delta_s}.
\] (6)

The first requirement is fulfilled for a 2D domain if it is without holes. The second requirement sets some restrictions to \(T_s\). Equations for \(T_s\) that fulfill (6) are slightly more complicated and less straightforward to understand than the equations for \(T_n\) given above. Therefore, the equations for \(T_s\) will not be given here; they can be found elsewhere [11].

### 4 Implementation in Finite Element Model

#### 4.1 Development and testing

The developed mixed mode cohesive laws were implemented in a 4-node user-defined element for the commercial finite element program Abaqus. The cohesive element was first tested with simple opening histories [11]. Next, the cohesive element with interface dilatation was used in a large finite element model for the simulation of fracture of mixed mode specimens.
4.2 Model of DCB-UBM tests
The finite element model of DCB-UBM specimens was made using 4-node isotropic plane strain (quadrilaterals) and 3-node reduced integration elements. The elastic properties were specified in terms of a Young's modulus of 37 GPa and a Poisson's ratio of 0.3. The moments were applied using prescribed rotations of the crack beam-ends in order to facilitate a stable solution.
The fracture plane was modeled using the cohesive zone element of the interface dilatation law. The two length parameters that describes the interface dilatation, $\tilde{\Delta}_n$ and $\tilde{\Delta}_r$, were determined by an iterative guessing procedure. That is, several sets of values of the cohesive zone parameters were used in simulation of the mixed mode DCB-UBM experiments and cohesive zone parameters were fine-tuned to obtain a good agreement between the experimental results and the model simulation. Although such a procedure may not guarantee a unique solution for the set of interface parameters, it seems to be reasonably efficient in terms of speed for parameter determination.

5 Model Results
Overall, model simulations using the new cohesive element were found to run fairly stable for models that involve many elements.

5.1 Fracture resistance
Fig. 6a shows some simulated results for the fracture resistance $J_r$ as a function of the magnitude of the end-opening $\Delta^*$. The values of the cohesive law parameters are given in Table 1. For the Mode I dominated experiment ($M_1/M_2 = -0.52$), the fracture resistance increases to a steady-state fracture resistance value of nearly 1 kJ/m². For the mixed Mode experiment ($M_1/M_2 = 0.50$), the fracture resistance also rises, and reaches a value of 2.7 kJ/m² at an opening of nearly 3 mm at which the simulation was ended. For the Mode II dominated experiment ($M_1/M_2 = 0.97$), reaches 1.2 kJ/m² at a small opening and thereafter increases near-linearly with increasing $\Delta^*$.

5.2 End-opening and end-sliding
Fig. 6b shows the corresponding relationships between the end-opening, $\Delta_n^*$, and end-sliding, $\Delta_r^*$. For the Mode I dominated experiment ($M_1/M_2 = -0.52$), the openings in almost only normal opening; $\Delta_n^* \approx 0$. For the mixed Mode experiment ($M_1/M_2 = 0.50$), the openings increase almost proportional to each other. For both these moment ratios, the ratio $\Delta_r^*/\Delta_n^*$ is so small that the interface dilatation mechanism is not in effect. However, the interface dilatation mechanism is in force for the Mode II dominated experiment ($M_1/M_2 = 0.97$). For small values of $\Delta_r^*$, a normal opening is induced as a result of the interface dilatation. For larger values of $\Delta_r^*$, $\Delta_n^*$ takes a near constant value.

6 Discussion
6.1 Comparison: Experiments and predictions
Comparing the experimental results (Fig. 3) and the model predictions (Fig. 6), we find that the model is capable of reproducing the both the macroscopic $\Delta_n^* - \Delta_r^*$ and the $J_r - \Delta^*$ behaviours reasonably well. Of course, the cohesive law parameters are chosen to fit the experiments, but it is nice to see, for instance that the predicted behaviour for $\Delta_n^*$ and $\Delta_r^*$ (Fig. 6b) looks much like the experimental results (Fig. 3b).

6.2 Comments regarding potential function
As mentioned above, the mixed mode cohesive laws incorporating interface dilatation were derived from a potential function. The implication is that the combined work of separation of the normal and shear tractions is independent of the opening history. According to theory, the work of separation of the cohesive tractions equals the potential function evaluated at the end-opening and end-sliding, $\Phi(\Delta_n^*, \Delta_r^*)$ [12]. A potential function $\Phi(\Delta_n^*, \Delta_r^*)$ for the cohesive traction is given elsewhere [11].

6.3 More advanced cohesive laws
The cohesive law formulation here can be seen as a step toward the formulation of more physical based mixed mode cohesive laws. Analytical mixed mode cohesive laws derived from micromechanical models of observed fracture process zones can be so
mathematically complicated [12] so that it may not be practical to implement them directly in finite element programs; instead phenomenological models of the type presented here may be more feasibly.

7 Conclusion

Finite element simulations that use cohesive elements with a new cohesive law that incorporates interfacial dilatation were found to run stably. Simulations using the new cohesive law with interfacial dilatation were found to give predictions for the end-opening and end-sliding that were in good agreement with experimental findings.

Acknowledgements

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References


Tables

Table 1: Cohesive zone parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<td>$T_n$ (MPa)</td>
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<tr>
<td>$T_t$ (MPa)</td>
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<tr>
<td>$\Delta_n$ (mm)</td>
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<tr>
<td>$\Delta_t$ (mm)</td>
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Fig. 1. Mechanisms that induce normal opening during imposed tangential displacement: a) shear cracks, b) bridging fibers connecting the crack faces in such a manner that they buckle in compression, c) interface roughness. Adapted from [11].

Fig. 2. Schematics of the double cantilever beam specimen loaded with uneven bending moments (DCB-UBM). \( M_1 \), the moment applied to the end of the upper beam of the cracked end, differs from \( M_2 \), the moment that is applied to the lower beam.

Fig. 3. Selected results from the mixed mode experiments from DCB-UBM specimens [4], (a) Fracture resistance as a function of magnitude of opening of end of fracture process zone, and (b) relationship between the normal end-opening and the tangential end-opening.
Fig. 4. Pure tangential displacement induces normal displacement. After a characteristic tangential displacement, $\tilde{\Delta}_t$, the normal opening remains at a constant value $\tilde{\Delta}_n$. From [11].

Fig. 5: The normal traction as a function of normal separation for (a) pure Mode I ($\Delta_t = 0$), (b) evolving interface dilatation ($\Delta_t > \tilde{\Delta}_t$), and (c) interfacial dilatation in full effect.
Fig. 6. Predicted results for the mixed mode experiments of DCB-UBM specimens. (a) Fracture resistance as a function of magnitude of opening of end of fracture process zone, and (b) relationship between the normal end-opening and the tangential end-opening.