1 Introduction
Wind is an attractive and green source for the generation of electricity, but this source will only be practical if its cost can be on par with the currently available popular sources, i.e. coal, gas and hydroelectric. As a result, wind turbine blade manufacturers are constantly looking for cost-effective material systems to reduce the cost of wind energy. Wind farm operators are likewise looking for cost-reducing options, and increasing blade lengths is one means of increasing the energy that can be realized from a wind turbine installation because the output of a wind turbine is proportional the cube of the length of the blades. Thus, if cost-effective wind energy can be achieved, then it will assist in meeting the goal stated in 2008 by the Department of Energy of having 20% of the electricity needs of the U.S. derived from the wind by 2030 [1].

Blade stiffness is a primary concern in the design of a wind turbine blade. The stiffness needs to be such that the shape of the blade is effectively the same for all wind speeds. However, such stiffness needs to be achieved while keeping weight at a minimum, i.e. high specific stiffness. Thus, aligned-fiber reinforced composites are the material system of choice for wind turbine blades and specifically resin infused non-crimp fabrics (NCFs).

To calculate the effective stiffness of the overall blade and the variations in stiffness in local areas of the blade, a complete and credible map of the formed blade needs to be available to the design engineer. Such a high-fidelity map would describe the orientations of the yarns from point to point throughout the blade and also denote the locations of any defects that result from the manufacturing process and thereby compromise the structural integrity of the blade in service (Fig. 1). Such defects can be in the form of (a) out-of-plane waves (wrinkles), (b) folding, (c) in-plane waves, and (d) fabric tearing. The out-of-plane waves and folds can lead to resin-rich pockets. All of these defects will result in compromised load carrying paths.

The current methodology to calculate the effective stiffness of a wind turbine blade uses a zone based approach. This approach divides the model into zones, where each zone has a specified ply stack up and the effective material properties of a zone are calculated using Composite Laminate Theory (CLT). While the zone-based models may be practical on a global scale, they may over- or under-predict the localized stiffness of the structure. They also do not consider the existence of defects that may be present in the composite structure. Hence, the use of knockdown factors is common practice to compensate for any uncertainties in the model, such as fiber misalignment and other defects. An alternative to the zone-based model is to use a ply-based approach. A ply-based model allows entire plies to be added or removed from the structure.

Fig. 2 depicts a four-ply structure which requires eight zones to be properly modeled using a zone based approach. Using the ply-based approach allows plies to be added or removed from the structure with ease. In the zone-based approach, multiple zones may be affected with the addition or removal of a ply. Both methods require the
assumption that the material properties are uniform within a ply or within a zone. Thus, neither method can consider any variations within its defined area and will lead to discontinuities in the material properties between adjacent areas.

Recent work using the finite element method to simulate the composite manufacturing process has shown promise as a robust methodology leading to a high-fidelity map of the formed fabric [2]. The method considers the mechanical behavior of each ply where the fabric is modeled using a mesh comprised of beams and shells. The beams represent the yarns, and the shells account for the shear stiffness of the fabric. As the fabric is formed to the shape of the mold, the beams automatically track the movement of the yarns. The result is a continuous and high-fidelity map of the yarns throughout the part. Such a model removes the need for zone based finite element models of a composite structure because it produces a model that is a continuous description of the effective material properties throughout the structure. The beam-shell model has the additional advantage as a seamless approach to modeling the forming of a composite structure from flat fabric blanks that can subsequently give a finite element model of the formed part that accounts for the changes in the orientations of the fabrics resulting from the forming process.

This paper presents a methodology for finding the respective contributions of the yarns and the resin to the material properties of the cured composite. The process goes beyond using the commonly used Rule of Mixtures, which is an area based approach. In the proposed methodology effective area moments of inertia are used in conjunction with knowing the locations and orientations of the yarns within the structure. With these contributions known, the effective material properties of each formed ply can be included in the finite element model of the cured part for any yarn orientation.

2 Background

Previous work has used the beam-shell model to simulate the manufacturing process of a 9-m CX-100 wind turbine blade (TPI Composites, Warren, RI) for dry fabrics using Abaqus/Explicit [1]. This modeling approach uses beam elements to represent the tensile properties of the yarns and shell elements to represent the shear properties of the dry fabric. Fig. 3 shows a unit cell of such a modeling approach for the case of a 0/90 stitched fabric. Details of the beam-shell model are given in [4].

Fig. 2. Comparison of composites structural finite element techniques [3]

Fig. 4 shows an example step from the simulation of the forming of a wind turbine blade. The fabrics were initially flat before being pushed into the mold. The simulation allows for the automatic tracking of the movement of the yarns. The formation of any defects such as fabric wrinkling, folding or tearing that may occur during the forming process can be observed in this forming simulation. This defect information can be used to guide changes in the manufacturing process so as to minimize the potential of such defects from occurring.
3 Theory

Consider a single-ply fiber-reinforced composite flat plate of thickness \( t \). A finite element model of the plate can be made using a mesh comprised of unit cells as shown in Fig. 3. The respective contributions of the yarns and the cured resin to the effective stiffness of the plate can be backcalculated from flexure test data. The in-plane stiffness will be a function of the respective elastic moduli, and the bending stiffness will be a function of the respective area moments of inertia.

First consider the effective area moment of inertia and elastic modulus of a unit cell of the cured composite plate in one direction. The area moment of inertia, \( I \), of the flat plate in the undeformed state is calculated based on the width of the unit cell, \( w \), and the bending stiffness of the plate, \( D \),

\[
I = wD, \tag{1}
\]

where the bending stiffness is a function of the thickness of the sample, \( t \), and Poisson’s ratio, \( \nu \), such that,

\[
D = \frac{t^3}{12(1-\nu^2)}. \tag{2}
\]

The bending stiffness, \( B \), of the unit cell can be calculated using the test data from a simply-supported plate (Fig. 5) with an applied line load, \( F \), in the middle as,

\[
B = EI = \frac{L^3}{48N}\left(\frac{F}{\delta}\right) \tag{3}
\]

where \( L \) is the length between the two supports, \( F/\delta \) is the slope of the load displacement curve, and \( N \) is the number of yarns that are carrying the load. The effective elastic modulus, \( E \), of the plate is calculated by,

\[
E = \frac{B}{I}. \tag{4}
\]

Knowing the cross sectional area of the plate, \( A \), and using the effective elastic modulus, \( E \), as found from Eq. 4, the rule of mixtures can be used to find the respective elastic moduli of the yarns, \( E_Y \), and the resin, \( E_R \), i.e.

\[
EA = E_YA_Y + E_RA_R, \tag{5}
\]

where \( A = wt \), cross-sectional area of the plate, \( A_Y \) is the cross sectional area of the yarn, \( A_R \) is the cross sectional area of the resin, and the combination of the two areas is equal to the total measured cross-sectional area, \( A \). Because the unit cell of the composite is modeled with a shell that fills a volume that is greater than the actual resin volume in the composite, the resin contribution to the stiffness, \( E_R^* \), is found by the ratio of the resin cross section area to the total cross section area of the unit cell such that,

\[
E_R^* = \frac{E_RA_R}{A}. \tag{6}
\]

Similar to the rule of mixtures as given by Eq. 5, the yarn and resin contributions to the effective area moment of inertia can be calculated by starting with,

\[
EI = E_YI_Y + E_R^*I_R. \tag{7}
\]

where \( I_Y \) and \( I_R \) are the respective area moments of inertia of the yarn and the resin. Eq. 7 can be combined with Eq. 1 and Eq. 2 and rearranged to give,

\[
I_Y = \frac{B-E_R^*I}{E_Y}. \tag{8}
\]

After \( A_Y \) and \( I_Y \) are calculated, their values can be implemented into a general beam cross-section definition in Abaqus/Explicit, and the experimental bending stiffness can be compared to the finite element model bending stiffness.

4 Experimental

Each fabric that is used in the manufacturing of the CX-100 wind turbine blade was tested individually to determine the material properties of the respective composite. The material that will be presented in this paper is an epoxy/resin matrix reinforced with an unbalanced fiberglass NCF sold by Vectorply as E-LT 5500 and is shown in Fig. 6. The fabric is a biaxial NCF. The top layer of fibers runs in the 0° direction while the bottom layer of fibers runs in the
90° direction. It can be seen in Fig. 6 that the 0° yarns are spaced tighter than the 90° yarns. The two layers of fabric are connected by a zigzag polyester stitching. The stitches hold the plies together during the layup process and also define the spacing of the tows of the fabric. This particular fabric architecture results in an unbalanced layup where the stiffness of the material in the 0° direction is greater than the stiffness of the material in the 90° direction.

![Fig. 6. E-LT 5500 fabric (a) 0° (b) 90°](image)

The unit cell definition for this particular fabric is shown in Fig. 7. The dimensions of the unit cell were determined by counting the number of tows, i.e. yarns, contained in a measured sample to calculate the average unit cell size (5.13 mm x 3.62 mm) over for the fabric sample.

![Fig. 7. Unit cell definition for E-LT 5500 fabric (a) 0° (b) 90°](image)

### 4.1 Sample Fabrication

Samples of cured plates were created using one-ply, three-plys, and five-ply of fabric. The fabric was infused with commercially available epoxy resin to create a 55:45 fiber/matrix volume ratio. The samples were cured in a press at a pressure equivalent to atmospheric pressure to simulate the curing conditions of a vacuum bag. The press platens were heated to increase the cure rate of the resin. The samples were then cut into 127 mm x 127 mm plates as depicted in Fig. 8.

![Fig. 8. Manufactured samples for testing](image)

### 4.2 Plate Bend Tests

All plates were tested on the Instron 8511 universal testing machine with a 20-kN load cell on a three-point bend test setup. The cylindrical supports had 25.4-mm diameters and were spaced 101.6-mm apart. The load was applied equidistant from the two supports by a 25.4-mm diameter cylinder attached to the Instron load cell (Fig. 5).

The data from the tests were plotted (Fig. 9) and the force/displacement behavior of the sample was determined. The average of the plates with one-ply was used to calculate the material properties described in Section 3 of this paper. The three- and five-ply samples were used to validate that the effective stiffness of these plates were properly captured in the finite element model. For further validation of the model stiffness, the three-ply plate was rotated 45° on the bend test setup and tested at the rotated configuration.

![Fig. 9. Bend test data](image)
4.3 Impact Modal Tests

To examine how the stiffness of the composite plate was coupled with its mass distribution, impact modal tests were performed to measure the mode shapes and natural frequencies of a three-ply plate. The test setup, shown in Fig. 10, consisted of the plate suspended by fishing line to approximate free-free boundary conditions. An accelerometer mounted to the back of the plate on the bottom right corner and an impact hammer with a force sensor in the hammer head was used to impact the points on a 5 X 5 grid drawn on the plate. The signals were analyzed using a DACTRON data acquisition system and the data was processed using the commercially available software RT Photon.

The frequency response function was calculated as the ratio of the input autopower spectrum from the hammer impact to the cross-power spectrum between the impact and the response measured by the accelerometer. The frequency response function, which can be seen on the computer screen in Fig. 10, is used to determine the natural frequencies and mode shapes. The peaks of the frequency response function occur at the poles of the system, and the poles occur at the natural frequencies of the system. The amplitude of the peaks is proportional to the amplitude of the mode shape at the impact location. By impacting all 25 grid points on the plate, the mode shape of the plate at the natural frequencies can be determined.

5 Finite Element Modeling

A corresponding Abaqus/Standard finite element model was made for each tested plate configuration, i.e. one, three and five plies. Finite element models were created to represent each plate using both a beam-shell model and an orthotropic shell model. The beam-shell model was created using the equations from Section 3 with the bend tests to calculate the effective stiffness of the resin and the yarns in the composite. Each ply was represented by a layer of unit cells as shown in Fig. 3. Multiple-ply plates were created by stacking layers of unit cells on top of each other and using tie constraints to “bond” the plies in the plate.

The orthotropic shell models were created using the composite layup feature in Abaqus/CAE. This feature allows the user to define a lamina material with separate elastic moduli in the 0° and 90° directions. The effective elastic moduli in the 0° and 90° directions were found from the experimental bend test data. The shear modulus in each direction as well as the in-plane Poisson’s ratio is also defined by the user using CLT. The composite layup feature allows the user to define the number of plies defined in a shell element as well as the material orientation of each ply. The material properties used in the model are summarized in Table 1.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>E [GPa]</td>
<td>49.00</td>
</tr>
<tr>
<td></td>
<td>15.50</td>
</tr>
<tr>
<td>E_r [MPa]</td>
<td>2950</td>
</tr>
<tr>
<td>v_r</td>
<td>0.300</td>
</tr>
<tr>
<td>I_r [mm^4]</td>
<td>0.441</td>
</tr>
<tr>
<td></td>
<td>0.625</td>
</tr>
<tr>
<td>E_y [GPa]</td>
<td>60.58</td>
</tr>
<tr>
<td>I_y [mm^4]</td>
<td>0.310</td>
</tr>
<tr>
<td></td>
<td>0.128</td>
</tr>
<tr>
<td>t [mm]</td>
<td>1.100</td>
</tr>
</tbody>
</table>

The beam-shell and orthotropic shell models were then used to simulate the flexure experiments performed on each plate. The finite element results were then compared to the experimental data to examine how well the each model correlated to the observed test results of that plate.

Table 1. Material properties used in models
5.1 Plate Bend Tests

The bend tests were simulated by prescribing boundary conditions to the plate models to be analogous to the roller supports and the loading condition provided by the Instron. A displacement was applied to the middle of the model, and the Force/Deflection behavior for each plate was compared to experimental data. The results for the plates oriented at 0° can be viewed in Table 2.

The beam-shell model and the orthotropic shell model results correlate within a 10% of the values concluded from the flexure tests. Because the material properties defining the model were calculated from the test data, this good correlation was expected.

Table 2. Comparison of finite element and experimental data for 0° bend tests

<table>
<thead>
<tr>
<th>No. of Plies</th>
<th>EXP 0° Bending [N/mm]</th>
<th>Beam-Shell FEA</th>
<th>% Diff</th>
<th>Orthotropic Shell FEA</th>
<th>% Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Ply</td>
<td>29.17</td>
<td>30.78</td>
<td>5.52%</td>
<td>31.26</td>
<td>7.14%</td>
</tr>
<tr>
<td>3 Ply</td>
<td>904.3</td>
<td>824.4</td>
<td>-8.84%</td>
<td>838.5</td>
<td>-7.28%</td>
</tr>
<tr>
<td>5 Ply</td>
<td>3969</td>
<td>3725</td>
<td>-6.13%</td>
<td>3834</td>
<td>-3.41%</td>
</tr>
</tbody>
</table>

The same type of comparison between the beam-shell and orthotropic finite element models and the experimental data for the 90° bend tests can be found in Table 3. There is likewise good correlation between the finite element models and the experimental data.

Table 3. Comparison of FEA and experimental data for 90° bend tests

<table>
<thead>
<tr>
<th>No. of Plies</th>
<th>EXP 90° Bending [N/mm]</th>
<th>Beam-Shell FEA</th>
<th>% Diff</th>
<th>Orthotropic Shell FEA</th>
<th>% Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Ply</td>
<td>10.34</td>
<td>10.51</td>
<td>1.70%</td>
<td>10.35</td>
<td>0.11%</td>
</tr>
<tr>
<td>3 Ply</td>
<td>288.1</td>
<td>285.7</td>
<td>-0.80%</td>
<td>278.4</td>
<td>-3.34%</td>
</tr>
<tr>
<td>5 Ply</td>
<td>1260</td>
<td>1305</td>
<td>3.62%</td>
<td>1279</td>
<td>1.51%</td>
</tr>
</tbody>
</table>

A deflection was applied to the model, and the resulting Force/Deflection behaviors between the tests and finite element models are compared in Table 4. The good correlation between the test data and the finite element models suggests that the finite element models are correctly representing the stiffness of the composite plates.

Table 4. Comparison of FEA and experimental data for 45° bend test of 3-ply plate

<table>
<thead>
<tr>
<th>EXP 45° Bending [N/mm]</th>
<th>Beam-Shell FEA</th>
<th>% Diff</th>
<th>Orth. Shell FEA</th>
<th>% Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>359.3</td>
<td>390.4</td>
<td>8.57%</td>
<td>383.3</td>
<td>6.68%</td>
</tr>
</tbody>
</table>
5.2 Modal Tests

To find the mode shapes of the finite element models, an eigensolution was performed to solve the equation

\[
[K] - [M] \lambda \{x\} = 0
\]  

(9)

where \([K]\) represents the stiffness matrix of the finite element model and \([M]\) represents the mass matrix. The eigenvalues represent the natural frequency of the system, and the corresponding eigenvectors represent the corresponding mode shape of the system [5]. The natural frequencies of the test data and the models are compared in Table 5. The natural frequencies of both models fall within 2% of the experimental values, indicating good correlation.

Table 5. Comparison of experimental and FEA natural frequencies of a 3-ply plate

<table>
<thead>
<tr>
<th>Mode</th>
<th>EXP</th>
<th>Beam-Shell</th>
<th>Ortho. Shell</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FEA</td>
<td>% Diff</td>
<td>FEA</td>
</tr>
<tr>
<td>1</td>
<td>379.2</td>
<td>377.18</td>
<td>-0.48%</td>
</tr>
<tr>
<td>2</td>
<td>617.3</td>
<td>615.63</td>
<td>-0.22%</td>
</tr>
<tr>
<td>3</td>
<td>988.4</td>
<td>973.62</td>
<td>-1.46%</td>
</tr>
<tr>
<td>4</td>
<td>1050</td>
<td>1045.3</td>
<td>-0.45%</td>
</tr>
<tr>
<td>5</td>
<td>1280</td>
<td>1279.4</td>
<td>-0.05%</td>
</tr>
</tbody>
</table>

T=Torsion mode and B=Bending mode

Fig. 12 shows the measured experimental mode shapes in black superimposed on the finite element model mode shapes in gray. The model shows excellent correlation to the test data.

The Modal Assurance Criteria (MAC) were used. The MAC values for the \(i^{th}\) FEA mode, \(u_i\), and the \(j^{th}\) experimental mode, \(e_j\) can be calculated by

\[
MAC_{ij} = \frac{\langle (u_i)^T(e_j) \rangle^2}{\langle (u_i)^T(u_i) \rangle \langle (e_j)^T(e_j) \rangle}
\]  

(10)

For perfect correlation between the experimental test data and finite element data, the MAC matrix will be an identity matrix.

The commercially available software, FEMtools was used to calculate the MAC values between the finite element model and the test data. This software calculates the MAC values as percentages. The graphical representation of the MAC matrix for the beam-shell model to the experimental data is shown in Fig. 13. The matrix is plotted as a bar chart where the location in the matrix is represented by the coordinates of the bar, and the value in the corresponding matrix location is represented by the height of the bar. The MAC matrix appears to be a diagonal matrix, indicating that the correlation between corresponding mode shapes of the beam-shell model and the experimental data is high. The off-diagonal terms in the MAC matrix have a low value, indicating that each mode calculated by the finite element model only correlates with the corresponding mode measured experimentally.

To measure the correlation between the test mode shapes and the finite element model mode shapes,

Fig. 13. MAC matrix for beam-shell model vs. experimental data for a 3-ply plate

The graphical representation of the MAC matrix for the orthotropic shell model to experimental data correlation is shown in Fig. 14. Again the MAC matrix shows a high level of correlation between the corresponding modes of the systems and, as
expected, low correlation between the modes that do not correspond.

The MAC values along the diagonals from Fig. 13 and Fig. 14 are compared in Table 6. It can be seen in this table that the two modeling approaches give equally good MAC values. These values demonstrate a high level of correlation between the experimental results and both models. These model data suggest that both finite element schemes can accurately capture the mass and stiffness distributions of the physical system.

![Fig. 14. MAC matrix for orthotropic shell model vs. experimental data for a 3-ply plate](image)

Table 6. MAC values for experimental and finite element correlations for a 3-ply plate

<table>
<thead>
<tr>
<th>Mode</th>
<th>MAC</th>
<th>Beam-Shell</th>
<th>Ortho. Shell</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1T</td>
<td>97.3</td>
<td>97.2</td>
</tr>
<tr>
<td>2</td>
<td>1B(90°)</td>
<td>96.3</td>
<td>98.0</td>
</tr>
<tr>
<td>3</td>
<td>2T(90°)</td>
<td>95.3</td>
<td>96.3</td>
</tr>
<tr>
<td>4</td>
<td>1B(0°)</td>
<td>94.0</td>
<td>93.9</td>
</tr>
<tr>
<td>5</td>
<td>2T(0°)</td>
<td>90.4</td>
<td>91.7</td>
</tr>
</tbody>
</table>

6 Conclusions

Experimental testing was used to characterize the mechanical properties of composite plates. The plates were characterized in two ways: (1) as a homogenized orthotropic material and (2) as discrete fiber and matrix properties. The discrete properties were implemented into a beam-shell finite element model that can be used to track the orientation of yarns throughout a complex geometry. However, for the current work, the methodology was limited to looking at a rectangular plate as a means of showing the proof of concept.

Composite plates were tested experimentally in bend and modal tests. The bend tests were chosen because the results are directly related to the stiffness of the material in the respective directions of the yarns, and these test data could be used to conclude the effective material properties. In the case of the beam-shell models, these data were used to find the respective area moments of inertia of the resin and the yarns in the cured composite. The modal test was chosen because the results were related to the stiffness of the material as well as the mass distribution throughout the composite material.

Finite element models were built based on the geometry of the constructed plates and the material properties calculated for the respective models. These models were then used to simulate the testing of the physical plates and compared to test data to validate that the stiffness and the mass distribution were modeled correctly.

Good correlation was seen between the experimental data and the model results, leading to the conclusion that both finite element models can be used to correctly represent the cured composite material.

7 Future Work

Future research will be conducted to explore how beam-shell and orthotropic finite element models can represent composite parts that include shear deformation of the fabric prior to curing the matrix material. Introducing shear angles into the fabric causes significant changes in the stiffness of the composite. The beam-shell model has the advantage of being able to track the orientation of the fabric throughout the forming process, while the orthotropic shell model would require either a ply-based or a zone-based approach.
8 Acknowledgements

This research was partially supported by grants from the U.S. Department of Energy (Grant # DE-EE001374) and the National Center for Manufacturing Science (Award Number 201050-130177).

References


