EVALUATING LAYERED FIBER COMPOSITE STRUCTURES ACCOUNTING FOR THE ONSET OF DELAMINATION

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Abstract
In this paper, composite structures are considered which consist of several layers of carbon fiber reinforced plastics (CFRP). For such layered composite structures, delamination constitutes one of the major failure modes and hence predicting its initiation is essential for their design. Evaluating stress-strength relation based onset criteria requires an accurate representation of the through-the-thickness stress distribution, which can be particularly delicate in the case of shell-like structures. Thus, in this paper, a solid-shell finite element is utilized, which allows for incorporating a fully three-dimensional material model, still being suited for application to thin structures. Moreover, locking phenomena are cured by using both the EAS and the ANS concept, and numerical efficiency is ensured through reduced integration. The proposed anisotropic material model accounts for the material’s micro-structure by using the concept of structural tensors. It is validated by comparison to experimental data as well as by application to numerical examples.

1 Introduction
Fiber-reinforced composites are gaining more and more importance in technical applications. Their most beneficial characteristics, the very high Young's modulus and low density, are particularly leveraged in shell-like structures of lightweight constructions. The composites examined in this paper consist of multiple layers, each of which is composed of a woven fabric, with two families of fibers, embedded in a matrix material. Besides this anisotropic structure, the stress-strain behavior of such fiber composite materials is highly non-linear.

The composite’s micro-structure can be taken into account implicitly by use of an hyperelastic formulation, where the anisotropic material behavior is introduced through the concept of structural tensors. The majority of such models accounting for anisotropic material behavior at finite strains were developed in the field of biomechanics. For instance, axisymmetric orthotropic blood vessels were investigated in [1], whereas biological soft tissues were modeled in [2] on the basis of an incompressible transversely isotropic law for moderate deformations. A description of the transversely isotropic behavior of rubber was presented in [3], and orthotropic constitutive equations were provided in [4] for the simulation of human leg impact problems. Noteworthy, a number of anisotropic material models have been proposed for reinforced fiber composites by different authors. An overview can be found e.g. in [5]. For the experimental validation of such models implemented into finite element analyses, the reader is referred to [6], while modeling techniques for layered composites including micro-macro scale transitions are presented in [7]. In the present paper, however, the model proposed by Reese in [8] for fiber reinforced rubber-like composites was adopted, in which the transition from the micro-scale to the macro-scale is formulated in a general manner. Therefore, this model is not restricted to rubber-like materials but also suitable for the carbon fiber-reinforced plastics (CFRP) considered here. Structural collapse in fiber composite structures is caused by the evolution of either matrix transverse cracking, fiber fracture, or delamination. From these different damage modes, the delamination is particularly important, because it drastically reduces the bending stiffness of the structure and promotes
local buckling in case of compressive loads. Including delamination into the computation of composite structures requires the definition of an appropriate criterion for its onset as well as the prediction of its growth after an initial crack has evolved.

For the initiation of delamination, different criteria exist, formulated in dependence of stress-resistance relations, e.g. [9-12]. Noticeably, after onset of delamination, the high stress gradients appearing at the crack front prohibit employing solely stress-based criteria. Even so, the use of on stress-resistance relations is sufficient as long as the crack growth is not considered but only the initiation of the delamination.

Since fiber-reinforced composites are mostly applied in thin shell-like structures, the element formulation demands providing a suitable shape for thin structures while displaying realistically the three-dimensional stress states. Although shell formulations exist, which take into account the through-the-thickness stretching, see e.g. [14-16], the implementation of three-dimensional material models is much simpler in the context of solid elements. On the other hand, the latter typically provide a poor performance when being applied to thin shell-like structures. In particular, there are different locking phenomena to be coped with, which cause an overestimation of the stress state and an underestimation of the deformation.

Using solid-shell elements represents one strategy to overcome this problem by combining the advantages of both solid elements and shell elements at the same time. Further, applying the enhanced assumed strain (EAS) concept eliminates the volumetric locking in case of (nearly) incompressible materials as well as the Poisson thickness locking, which occurs in bending problems of shell-like structures due to the non-constant distribution of transverse normal strain over the thickness. In literature, one can find several solid-shell formulations incorporating the EAS concept, see e.g. [17-19], to name only a few.

To cure the transverse shear locking, which is present in standard eight-node hexahedral elements, the assumed natural strain (ANS) method is applied. In the context of full integration formulations, the ANS can be found e.g. in [20-22], and for reduced integration solid-shell formulations e.g. in [23-26]. The formulation presented in this paper is based on the works of Schwarze and Reese [24-26].

For laminated layered composites, the accurate determination of the through-the-thickness stress distribution was recently investigated by several authors. For instance, in [27] an improved shell formulation was used for this, whereas in [28] and [29] the investigations were based on the solid-shell concept.

For a more elaborate literature overview, the reader is referred to the review papers [30-32] and the references therein.

2 Anisotropic Material Model

The anisotropic material behavior of the considered composites is taken into account by using the meso-mechanically motivated model proposed by Reese in [8]. Therein, the macro-mechanical material response is described on the basis of the concept of structural tensors. Thus, a strain energy density function $W=W(C)$ is defined, which represents the dependence between the second Piola-Kirchhoff stress tensor $S$ and the Cauchy-Green tensor $C$:

$$S = 2 \frac{\partial W}{\partial C}$$

In the case of orthotropic material behavior, the energy function $W(C)$ reduces to an isotropic function of $C$ and the structural tensors $M_1$ and $M_2$, which are defined by the dyadic product of the structural vectors $n_1$ and $n_2$ representing the fiber directions.

$$M_1 = n_1 \otimes n_1$$
$$M_2 = n_2 \otimes n_2$$

Then, the strain energy function $W(C)$ can be represented in dependence of the following invariants:

$$I_1 = \text{tr}(C)$$
$$I_2 = \frac{1}{2} [\text{tr}(C)^2 - \text{tr}(C^2)]$$
$$I_3 = \text{det}(C)$$
$$I_4 = \text{tr}(C M_1)$$
$$I_5 = \text{tr}(C M_2^2)$$
\[ I_6 = tr(C M_2) \]  
\[ I_7 = tr(C M_2^2) \]

In this work, the anisotropic model from Reese [8] is adopted, which assumes that the fibers do not carry any load in case of compression but only in tension, which is not realistic for the carbon fiber reinforced plastics (CFRP) considered here. Therefore, this model is slightly modified, such that the matrix can act as an elastic continuous support for the embedded fibers. Moreover, the fiber volume fractions \( 0 \leq \varphi_1 \) and \( 0 \leq \varphi_2 \) for the two families of fibers are introduced, where \( \varphi_1 + \varphi_2 = 1 \) holds.

Except of this, we adopt the mentioned model and use the following strain energy function:

\[ W = (1 - \varphi_1 - \varphi_2) W_{NH} + \varphi_1 W_{ani1} + \varphi_2 W_{ani2} \]  

Here, \( W_{NH} \) denotes the Neo-Hookean part displaying the isotropic case in the small strain regime. The strain energy function is given by:

\[ W_{NH} = \frac{\mu}{2} (I_1 - 3) - \mu \ln \sqrt{I_3} + A/4 (I_3 - 1 - 2 \ln \sqrt{I_3}) \]  

where \( \mu \) and \( A \) denote the constants of elasticity. The anisotropic behavior is introduced by the parts:

\[ W_{ani1} = K_{ani}^{11} (I_4 - 1)^{\varphi_1} + K_{ani}^{12} (I_5 - 1)^{\varphi_2} \]  

\[ W_{ani2} = K_{ani}^{21} (I_6 - 1)^{\varphi_1} + K_{ani}^{22} (I_7 - 1)^{\varphi_2} \]

Note that in [8] further terms have been introduced e.g. for the coupling of the two directions, which have hardly influenced the results, and therefore are dropped here. The material parameters, which are introduced in the formulation of \( W(C) \) in equations (13) and (14), are determined by comparison with virtual experiments carried out on a representative volume element (RVE) on the meso-scale, as shown in Figure 1.

### 3 Delamination Onset Criterion

The onset of delamination can be determined on the basis of stress-strength relations. In particular, delamination occurs in pure interlaminar tension (mode I), pure interlaminar sliding shear (mode II), and pure interlaminar scissoring shear (mode III), if the corresponding interlaminar stress component exceeds the respective maximum interfacial strength. Here, the interlaminar stress components are denoted by \( \sigma_{13} \), \( \sigma_{23} \) and \( \sigma_{33} \), respectively, where the 3-direction is normal to the considered interface. Then, the respective interfacial strengths are \( Z_{13} \), \( Z_{23} \) and \( Z_{33} \). To account for mixed-mode loading, the formulation of the onset criterion should incorporate the interaction of these modes. In this paper, the approach of Hashin and Rotem [9] and Ye [10] is adopted, in which a quadratic interaction of modes is assumed:

\[ \left( \frac{\sigma_{13}}{Z_{13}} \right)^2 + \left( \frac{\sigma_{23}}{Z_{23}} \right)^2 + \left( \frac{\sigma_{33}}{Z_{33}} \right)^2 \geq 1 \]

Here, the Macauly-brackets are defined as follows:

\[ (x) = \frac{1}{2}(|x| + x) \]

As the formulation presented in this paper is capable of taking into account finite strains, it is important to accurately represent the according stress components. First of all, the stresses calculated by the present solid-shell formulation are expressed by the second Piola-Kirchhoff stress tensor \( \mathbf{S} \), which has to be pushed to the Cauchy stress tensor \( \mathbf{\sigma} \) in the current configuration.

\[ \mathbf{\sigma} = \frac{1}{\det \mathbf{F}} \mathbf{FSF}^T \]

From this, the interlaminar traction \( \sigma_n \) and the interlaminar resultant shear \( \tau_n \) can be achieved by:

\[ \sigma_n = \mathbf{n} \cdot \mathbf{\sigma} \cdot \mathbf{n} \]

\[ \tau_n = \sqrt{\|\mathbf{n} \cdot \mathbf{\sigma}\|^2 - \sigma_n^2} \]

denoting the normal vector of the considered interface by \( \mathbf{n} \). For consistency, the maximum interfacial strength in tension and resultant shear are referred to as \( Z_n \) and \( Z_n \), respectively. Consequently, the condition for delamination onset reads:
This condition has to be checked in each loading step and in each interface of the laminated composite. Thus, the accurate prediction of the stress components in the interfaces is essential for a reliable prediction of the initiation of debonding. Since the layers are usually rather thin, solid elements are not suitable to achieve sufficient accuracy. To overcome this problem, spline approximations for the through-the-thickness stresses can be applied, as proposed in [12]. Even so, using solid elements in the thin shell-like applications should be avoided, and solid-shell elements are preferable.

Further, it should be noticed that this kind of criterion is suitable to predict the delamination onset, but delamination growth is not covered.

### 4 Solid-Shell Finite Element Formulation

Using standard solid elements in thin structures would require a very high mesh density to predict the stress distribution with sufficient accuracy. On the other hand, implementing the described three-dimensional material model into a shell element formulation is not straightforward. Hence, the solid-shell element formulation proposed by Schwarze and Reese [24-26] is used alternatively to avoid inefficient computations.

Furthermore, this solid-shell formulation utilizes a reduced integration scheme within the shell plane (using one integration point), whereas a full integration is used in thickness direction, which allows for choosing arbitrary numbers of integration points (at least two). Thus, all integration points are located on the normal through the center of the element, see Figure 2.

Moreover, several locking phenomena need to be dealt with. In particular, the assumed natural strain (ANS) concept is applied in order to cure transversal shear locking and curvature thickness locking. In addition, to cure Poisson thickness locking as well as volumetric locking, the enhanced assumed strain (EAS) method is used.

### 5 Numerical Examples

The proposed method was applied to a panel with a co-cured stiffener, which had already been investigated in [13]. The panel's length and width were 203 mm and 25.4 mm, respectively, while the stiffener's length was 50 mm at the panel's skin and 42 mm at the uppermost layer. The flange was composed of 10 plies with a lay-up of (45°/90°/-45°/0°/90°), whereas the panel consisted of 14 plies with an (0°/45°/90°/-45°/45°/-45°/0°)s assembly.

All layers were made of unidirectional CFRP, the properties of which are given in Table 1. From these mechanical properties, the non-zero parameters for the described model were calculated, taking into account the fiber volume fraction $\phi_1 = 62\%$, see Table 2. For the interfacial strengths, the values were adopted from [12] as given in Table 3.

The panel was clamped at the left end, while it was loaded by a force in longitudinal direction at its right end. Since the onset of debonding was expected at the tip of the stiffener flange, the mesh was refined in this region as illustrated in Figures 3 and 4.

In order to incorporate delamination, interface elements were located between all layers, which were furnished with material properties of the matrix alone. The latter were assumed to be ten times thinner than the layers. The described solid-shell element was used for both layers and interfaces, leading to a total number of 9,825 solid-shell elements with 25,152 nodes.

The location of delamination initiation is shown in Fig. 5, which corresponds to the time step, in which the delamination onset condition was met first. As one can see, delamination occurred at first at the tip of the stiffener flange, as was expected.

This location of delamination is reasonable and in agreement with the results presented in [13]. Furthermore, experimental data for this problem can be found in [13]. Therein, the maximum displacements from extensometer measurements corresponding to delamination onset are reported to be in the range of 0.11 mm to 0.15 mm. Unfortunately, the exact location of the measurement is not given, which does not allow a quantitative comparison. However, in the current calculation, the computed displacements at delamination onset are in a higher range up to 0.18 mm. This difference can be explained by the
fact that residual stresses are present in the specimens, which have not been taken into account in the present calculation.

6 Conclusions

For many technical applications of fiber-reinforced composites, predicting the onset of delamination is essential for appropriately designing the considered structure. For this, a delamination onset criterion based on stress-strength relations has been suggested in this paper, which requires an accurate representation of the through-the-thickness stress distribution. The proposed solid-shell element is particularly suitable to achieve the required accuracy especially in the thin shell-like applications considered here. The formulation allows for including woven fabrics with two different families of fibers, incorporating a fully three-dimensional, anisotropic, micro-mechanically motivated material model. Concluding, the proposed method is capable of predicting the initiation of delamination of fiber-reinforced composites in shell-like structures accounting for the anisotropic material behavior.

Tables

<table>
<thead>
<tr>
<th>Engineering constant</th>
<th>Value</th>
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<tr>
<td>Tensile modulus fibre [GPa]</td>
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<tr>
<td>Tensile modulus matrix [GPa]</td>
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</tr>
<tr>
<td>Shear modulus matrix [GPa]</td>
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<tr>
<td>Poisson’s ratio matrix</td>
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<tr>
<td>Fiber volume fraction</td>
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Tab.1. Material parameters (engineering constants).

<table>
<thead>
<tr>
<th>Material Parameter</th>
<th>Value</th>
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<tr>
<td>$\Delta$ [MPa]</td>
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</tr>
<tr>
<td>$\mu$ [MPa]</td>
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</tr>
<tr>
<td>$K_{tt}^{11}$ [GPa]</td>
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<tr>
<td>$\beta_1$</td>
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Tab.2. Material parameters for proposed model.

Figures

Fig.1. Representative volume element (RVE).

Fig.2. Reduced integration: Integration points aligned on the normal through the element center.
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Fig.3. Mesh for stiffened panel.

Fig.4. Detailed view on stiffener and panel.

Fig.5. Computed zone of delamination onset.
References


