Abstract
Differences between the geometry of lay-up tools and their composites panels are mainly caused by the material response to the transformations occurring in the cure cycle. The residual stresses resulting from that cycle and causing warpage, are due to different Coefficients of Thermal Expansion of the constituents and different kinds of non-symmetry which might be present, like a non-symmetric lay-up, added doublers and local details. Residual stresses and can be further increased by cutting operations. In this paper a numerical and experimental study is presented for the calculation of warpage (curvature changes) of non-symmetrical hybrid laminates. These changes or deviations from the tooling dimensions can be predicted with good accuracy.

1 Introduction
Laminates of fibre reinforced polymers and hybrids, i.e. combinations of composites and metal alloys, result in structures with high specific properties and are applied in aircraft structures like fuselages and wings. The principle to align high strength fibres in well defined directions, thereby creating highly anisotropic materials, is the basis for the excellent performance of these materials. At the same time the combination of different materials in one laminate and the severe anisotropy also induce phenomena that need to be addressed during design and manufacture. One major phenomenon is the introduction of residual stresses after curing, and the warpage resulting from that. The warpage will increase with the increase of non-symmetry of the component.

The topic of residual stresses in composites and its consequences has been investigated by many researchers. Some researchers [1-5] focused on the development of residual stresses during the cure cycle and their influence on the mechanical properties of the composites like shear properties, impact properties, etc. Others [6-8] have investigated the warpage and proposed solutions for the reduction of the residual stresses like modification of the polymers to reduce cure or glass transition temperatures, adapting the cool rate, or stress relaxation by a kind of heat treatment. Most of these solutions are not applicable for aircraft applications since the trend is to obtain even higher mechanical properties and at higher operational temperatures. Numerical modelling and simulation is also a method to control and predict warpage in composites, using Classical Laminate Theory and its derivatives as well as Finite Element simulations [9-11]. Most papers in this category are rather academic and less applicable to real composite components.

In this paper the focus is on the prediction of the warpage of flat and single curved laminates which represent panels for aircraft fuselage or wing applications. Real panels consist of a skin, doublers at locations for local reinforcement and a number of cut-outs. When such panels are made by lay-up and autoclave curing, the final dimensions, in particular the curvatures (or radiiues), deviate from the curvature of the tooling. These differences should be eliminated: either by adapting the dimensions of the tooling or, during assembly, by forcing the panel into its position. The second option could be feasible for small deviations in thin walled, flexible structures, but will cause significant extra stresses and sometimes damage in thicker and less flexible and tough materials like full composites and hybrid materials. Therefore, the best option is tool compensation for warpage and spring back [12]. This paper discusses one part of a simulation tool development project to compensate tooling dimensions for warpage. This project uses numerical and experimental techniques.
The paper discusses causes for spring back and warpage. Subsequently a research program is presented, followed by some results of that first part of the research.

2 Outline of the problem and research approach

After curing (elastic) material responses occur which result in residual stresses and shape changes or distortions. Two main causes for the material response are: chemical shrinkage of the (thermoset) resin due to cross linking during the cure cycle (which is neglected in this paper), and shrinkage due to temperature changes (from cure temperature to operational temperature, which is usually much lower: -60° to + 70/80° Celsius). Chemical shrinkage is not an issue for thermoplastic matrices.

The shrinkage due to temperature changes is the result of differences in Coefficients of Thermal Expansion (CTE) of the constituents. Fibres have low CTE values and polymers and aluminium, as used in GLARE, have (rather) high CTE values (see Table 1). Upon cooling the resin and aluminium shrink significantly more than the fibres. As result tension stresses are created in the aluminium and resin, and compressive stresses in the fibres. The magnitude of the stresses is influenced by the temperature change and the stiffness of the constituents. Since resins (or pure polymers) have a small E-modulus (1-5 GPa), the residual stresses in a symmetric fibre reinforced composite are limited. When two materials are combined with high E-modulus, the residual stresses will be larger. As long as a flat laminate is symmetrical, the material response is in-plane and there is a build-up of residual stresses, but no shape changes are involved. Once the material build up becomes non-symmetric, part of the accumulated elastic energy is released and geometric deviations or distortions will become apparent (see Fig. 1).

Non-symmetry of a laminate can be created or enhanced in 4 different ways:

1. Non-symmetry in stacking sequence (or lay-up of the laminate). In most applications the laminates will be (nearly) symmetric. However, when the number of fibre layers has an odd number or when special properties are required, a non-symmetric lay-up is used.

2. Non-symmetry in the build-up of the panel. The outside of the skin has a smooth surface but at the inside (= inside of the aircraft), the skin might be locally reinforced by doublers (i.e. round doors, hatches, and windows). These doublers result in non-symmetry.

3. Local non-symmetry due to details, like splices and run-outs of layers. These details usually have a local effect on shape changes, whereas the other 3 ways for non-symmetry have global consequences.

4. Non-symmetry can be further increased (or reduced) by trim lines and cut-outs. Trim lines and cut-outs change the residual stress system in a panel. In case of a non-symmetrical panel,

<table>
<thead>
<tr>
<th>Material</th>
<th>Longitudinal</th>
<th>Transverse</th>
</tr>
</thead>
<tbody>
<tr>
<td>AS-4 Carbon fibre</td>
<td>-0.9</td>
<td>27</td>
</tr>
<tr>
<td>S-Glass fibre</td>
<td>7.1</td>
<td>30</td>
</tr>
<tr>
<td>Aluminium</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>Polyimide</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>Epoxy (HY6010)</td>
<td>62</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Coefficients of Thermal Expansion for different materials \[10^6 \text{m/m/°C}\] \[14\]

Fig. 1. A non-symmetric cross-ply laminate (a) and saddle shape (b) after processing due to residual stresses \[5\].
cutting operations will change the shape (sometimes increase, sometimes reduce warpage).

The research described in this paper is about the non-symmetry due to the stacking sequence (1). Variables that have been investigated are the number of layers in a laminate, the stacking order, the width to length ratio of the laminates and the curvature of the shells. Different configurations have been tested by experiments and by numerical simulations. For the experiments, Fibre Metal Laminates have been made, and for the simulations GLARE and carbon composite shells have been analysed; the carbon shells can have a more pronounced non-symmetry. Since symmetrical laminates don’t warp, although there are internal stresses, the composite and hybrid laminates have been designed and/or manufactured with distinct non-symmetry, e.g. laminates with \([0/90]_4\)-lay-up. Most laminates have been cured on a flat plate, only a few laminates have been cured on a curved tool (see Fig. 2). After curing the curvatures have been measured by laser distance equipment or by the Digital Image Correlation technique.

Tests and simulations

The emphasis in this research has been on the responses of non-symmetric GLARE-3 laminates, a hybrid concept of thin alternating layers of aluminium Al 2024-T3 and glass fibre epoxy prepreg. The details of the benchmark material for this research are given in Table 2.

In the research an analytical model is used to predict the cured shapes of the non-symmetric GLARE laminates. The model uses a nonlinear theory based on polynomial approximations of the displacements, extends the Classical Lamination Theory (CLT) to include geometric nonlinearities (because of the magnitude of the displacements), and uses a Rayleigh-Ritz approach based on total potential energy. The analytical approach on cured non-symmetric laminates is related to simple geometries such as square plates and only free boundary conditions have been considered.

<table>
<thead>
<tr>
<th>Benchmark Laminate</th>
<th>GLARE 3-3/2-0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-symmetric lay-up</td>
<td>AL/0/0/AL/90/90/AL</td>
</tr>
<tr>
<td>Thickness of epoxy prepreg layers</td>
<td>0.127 mm</td>
</tr>
<tr>
<td>Thickness of aluminium 2024-T3 layers</td>
<td>0.3 mm</td>
</tr>
<tr>
<td>Temperature difference</td>
<td>-100º C</td>
</tr>
</tbody>
</table>

Besides the analytical approach, numerical calculations with Finite Element Analysis (FEA) have been carried out. The advantages of finite element approach over an analytical approach are that more complex geometries and lay-ups can be analyzed, more detailed material properties (like thermo elasticity and visco-elasticity) can be incorporated in the model, and more realistic boundary conditions can be simulated. The objective of the finite element analysis is to find a general approach for modelling of the induced geometrical deviations of non-symmetric GLARE laminates with a reasonably accuracy and limited computation time.

In this research ABAQUS is used. The element type used for the models is S4R, a 4-node double curved shell element with reduced integration, but it allows shear deformation. The flat GLARE laminates with different shapes and lay-ups are investigated, and the simulation starts with a perfect square shape of GLARE 3-3/2-0.3 non-symmetric lay-up in order to compare the results with the analytical solution. The cool-down process is simulated by applying an initial temperature of 120 ºC and a final room temperature of 20 ºC to all nodes of the model.
Thermal loads are generated by the temperature difference. The curing process can be considered as a quasi-static process.

In the experimental part of the research seven square GLARE 3-3/2-0.3 panels have been made with different lengths, ranging from 0.1 till 0.6 meters. These dimensions were selected for the verification and validation of the analytical and numerical models. The warpage, expressed in radiuses or curvatures are measured by laser distance or a 3D Digital Image Correlation (DIC) technique. These techniques provide the curvatures which could be compared with the calculated values. Besides flat laminates, also a few curved panels were made (see Fig. 2) to investigate the impact of curvature on warpage.

**Results and Discussion**

In this section the presentation and discussion is aiming for some highlights from the research. The first result is the comparison between the analytical and numerical models. Fig. 3 shows the predictions by both approaches. As becomes clear from the figure, for small square laminates the flat laminate turns into a saddle shape. For larger dimensions the flat laminate becomes cylindrical and the curvature in one of the two orthogonal directions has two possibilities: or there is no curvature in that particular direction (but in the orthogonal direction) or the curvature has a specific value. In Fig. 3 the bifurcation occurs when the side length is ±0.5m; the absolute value of curvature in saddle shape is between 0.15 and 0.65 (1/m). The curvature in cylindrical shape is 0 (zero) or ±0.45 (1/m).

Fig. 3 shows the analytical and numerical results. As can be seen from the plot: the two methods almost predict the same results. Another feature in Fig. 3 is the critical length of the laminate, \( L_{cr} \), which indicate the transition or bifurcation point from saddle shape to cylinder.

![Fig. 3. ABAQUS results versus analytical predictions for GLARE 3-3/2-0.3](image)

Comparing the results in Fig. 3 with results obtained for a T300/5208 carbon epoxy laminate ([0°]_4/[90°]_4)\_L\_ lay-up – results not presented here), shows that the carbon composite had its bifurcation at a much lower value (0.07m) and had more severe curvatures (between 2 and 6 (1/m)). Therefore, it can be stated that GLARE laminates exhibit much less deformation and has its bifurcation point at larger laminate dimensions. This can be explained by the fact that the processing temperature of the carbon...
lamine is higher (180$^\circ$ vs. 120$^\circ$ C) and that the degree of anisotropy is much higher too when it is compared to GLARE (note that GLARE consists of aluminium for more than 60%).

The curvatures of seven cured GLARE laminates are presented in Fig. 4 and compared with the analytical and numerical predictions. For most GLARE laminates, there are two red symbols, representing the values of curvatures in one of the two principle directions.

For the first three GLARE laminates with side-length of 0.1m, 0.2m, 0.3m, the warpage shows a saddle. The values of the curvatures are around 15% lower than predicted by both models. For larger GLARE laminates the curvatures become cylindrical. The GLARE laminate with side-length of 0.35m and 0.4m are almost cylindrical, with curvatures in one direction much larger than that in the orthogonal direction. However, the smaller curvatures are not zero, which means they are still in the transition zone. The last two GLARE laminates namely show full cylindrical shapes with the lower principle curvatures almost zero. The values of the curvatures fit well the model predictions. The critical side-length for fully cylindrical shape with constant curvature (around 0.5m) fit the model predictions as well, although it seems that the bifurcation point have moved to a smaller critical length value.

![Curvature vs. Side Length](image)

**Fig. 4. Experimental results of GLARE 3-3/2-0.3**

With the use of the analytical model, which generates good approximations for the laminate curvatures, the variation of a number of variables of GLARE have been investigated.

Fig. 5 shows the variation in laminate lay-up. As stated before, the reference laminate was a GLARE 3-3/2-0.3 laminate. By increasing the number of layers in the laminate or the laminate lay-up, from 3/2 to 5/4, the curvature after curing becomes smaller and the bifurcation point shifts to larger laminate sizes.

The reduction in curvature can be explained by the fact that the bending moment induced by the residual stresses is proportional to the laminate thickness (t) but that the bending stiffness of the laminate, responsible for resisting the warpage, is proportional to $t^3$.

A similar effect is recorded when the thickness of the metal layers is increased. Also in this case the curvature reduces and the bifurcation point moves to larger panel sizes (see Fig. 6).
Using the analytical model several other parameters have been investigated too, like the effect of the Young’s modulus of the constituents, the effect changes in CTE, and changes in temperature gradients. All results are assembled in Table 3, which show what parameter has been investigated, what changes have been made (e.g. increase of a property by some percentage), and what the results are. The results are expressed as a shift in the bifurcation point (to the left or to the right = to lower or higher values of the laminate dimension) and effect on the curvature (an increase in curvature indicates a smaller radius of the cylinder).

![Curvature in x direction](image1)

**Fig. 5.** Experimental results of different GLARE 3 lay-ups.
Fig. 6. Experimental results of different metal layer thickness in GLARE 3-3/2.

Table 3: Overview of the results of the quantitative parameter study

<table>
<thead>
<tr>
<th>Change in</th>
<th>Action</th>
<th>Consequences</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Change in Parameters</td>
<td>Shift in Bifurcation point</td>
</tr>
<tr>
<td>Elastic modulus</td>
<td>Glass epoxy (E1, E2)</td>
<td>20% ↑</td>
</tr>
<tr>
<td></td>
<td>Aluminium E</td>
<td>20% ↑</td>
</tr>
<tr>
<td>CTE</td>
<td>Glass Epoxy &amp; Aluminium</td>
<td>20% ↑</td>
</tr>
<tr>
<td>Lay-up</td>
<td>GLARE 3-3/2→4/3→5/4</td>
<td>→</td>
</tr>
<tr>
<td>Thickness</td>
<td>Aluminium</td>
<td>33% ↑</td>
</tr>
<tr>
<td></td>
<td>Glass epoxy</td>
<td>100% ↑</td>
</tr>
<tr>
<td>Temp. gradient</td>
<td>ΔT</td>
<td>20% ↑</td>
</tr>
</tbody>
</table>

The results as presented in Table 3 can be easily explained:
- Increasing the Elastic modulus of aluminium has its impact on the bending stiffness and a higher bending stiffness results in less warpage (smaller curvature).
- An increase in the Glass fibre modulus changes the residual stress system in the laminate. In this case warpage will increase as well as the curvature.
- Also an increase in the CTE or an increase in the temperature gradient will increase the warpage (and the curvature). In both cases the residual stresses system induces a higher bending moment.

For curved laminates, it has been discovered that even if the lay-up is balanced and symmetric, after the curing process, there will always be some curvature change. This is referred to as the spring-in effect. The differential strains in the plane and through the thickness cause this change in curvature. For non-symmetric laminates, it could be expected that the unbalanced thermal loads will cause more severe shape distortions. To test and simulate this effect on laminate curvatures, two laminates were defined (see Table 4).

Table 4. Non-symmetric GLARE laminates details

<table>
<thead>
<tr>
<th>Laminate number</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glare Grade</td>
<td>Glare3-3/2-0.3</td>
<td>Glare3-3/2-0.3</td>
</tr>
<tr>
<td>Non-symmetric lay-up</td>
<td>[Al/0/0/Al/90/90/Al]</td>
<td>[Al/90/90/Al/0/0/Al]</td>
</tr>
<tr>
<td>Radius</td>
<td>0.5m</td>
<td>0.5m</td>
</tr>
<tr>
<td>Side-length</td>
<td>0.8m</td>
<td>0.8m</td>
</tr>
<tr>
<td>Temperature difference</td>
<td>-100 ºC</td>
<td>-100 ºC</td>
</tr>
</tbody>
</table>

After curing the panels have been measured by DIC. Both experiments and calculations showed that one panel (1) had spring forward and the other spring back (2), depending on their specific lay-ups. The small effects due to differential strains are negligible. If the mould radius is above a certain critical value \( R_{cr} \), snap-through may occur, when the curvature changes from one axis of the panel to the other.
The value of the curvature in both cases does not change.

Conclusions

Warpage due to non-symmetry in laminated materials like GLARE and carbon composites can be simulated by analytical and numerical tools. In this paper the focus was on non-symmetry induced by lay-up of the laminates. From the calculations and tests the following conclusions can be made:

- Predictions by analytical and numerical simulations are good; the value for the curvature is predicted very well, but the prediction for the position of the bifurcation point needs further improvement.
- Analytical analysis of several parameters showed that the changes in $L_{cr}$ and curvature can be predicted and explained.
- Curved panels have a “natural” warpage, although warpage by non-symmetrical lay-up is much stronger and the direction of warpage depends on the relation between fibre directions and curvature orientations.

References