1 Introduction

Due to the advances in the understanding of composites and development of more efficient core materials sandwich structures have gained increasing importance in the aerospace and marine industries. This can be attributed to their superior strength to weight ratio which allows for low-weight and more fuel efficient design. However, due to their low weight they are also more susceptible to damage due to impact than a similar metallic structure [1]. Thus, in order to facilitate their use in dynamic environments it is necessary to find ways to enhance their capability of attenuating incident dynamic loads.

It has been shown over the past few years that a wave attenuation bandgap can be created by adding local spring-mass resonators to a structure[2-9]. Since this bandgap primarily depends on the properties of the included resonators, it can be more conveniently tailored than the conventional Bragg gap obtained due to the periodic nature of the system. Huang et al. [3] showed that more than one bandgap can be achieved by using multi-resonator mass in mass lattice [4]. Pai [5] further investigated this effect by analyzing a uniform elastic bar with attached spring-mass subsystems and showed that the working mechanism is based on the conventional mechanical vibration absorbers and proposed a design for a broadband vibration absorption system by using multiple vibration absorbers. Using a mass-spring system, Yao et al. [6] experimentally demonstrated the negative effective mass and reported a high transmission loss bandgap in accordance with the theoretical results. Huang et al. [3] further explained the wave attenuation and energy transfer mechanism in the bandgap. They showed that most of the work done by the external force on the system is stored temporarily in the resonators in the form of kinetic energy and is taken out by the external forcing agency in the form of negative work in a cyclic manner.

Based on this work, Sun and Chen [6-8] have proposed the use of such energy absorbing subsystems in sandwich constructions in order to improve their wave attenuation capability. Under the low frequency assumption, it was shown that the behavior of a sandwich beam can be described by an effective Timoshenko beam model. The system was shown to display negative equivalent mass and low frequency bandgaps corresponding to the local resonance frequency were achieved. The attenuation phenomenon was also demonstrated using finite element models and verified by experiments. Subsequently, it was also shown that for a finite system the local resonance attenuation phenomena corresponds to the anti-resonance behavior of a two degree of freedom system [9].

In this study, we continue this work further by analyzing the behavior of sandwich beams with internal resonators under broad spectrum loads such as impact loads. Finite element models of sandwich beams with variously tuned local resonators and without resonators were used to study the wave attenuation behavior of such beams. A typical impact load was applied at the center of the beam and the effect of the resonators on the wave propagation was studied. The performance of resonators tuned to different resonance frequencies was studied. Impact experiments were performed in order to demonstrate the effectiveness of local resonators in attenuating broad spectrum loads. It was shown that addition of resonators helps improve the attenuation characteristics of sandwich beams subjected to impact loads.
2 Numerical analysis

2.1 Method

Finite element simulations were performed to investigate the effect of internal resonators in a sandwich structure subjected to an impact load. Sandwich beams with resonators uniformly distributed throughout the length of the beam were modeled with the load being applied in the form of a concentrated point load at the center of the beam. A schematic of a sandwich beam with resonators and its equivalent homogenous beam is shown in Fig 1. To capture the propagating wave effects and avoid reflections, a 100 m beam was used with symmetry conditions applied at the left end and the right end kept free, thus effectively simulating a 200 m long centrally loaded free-free beam. Planar Timoshenko beam elements with linear interpolation (B21) provided by the commercial code Abaqus were used to model the sandwich beam. Each element was assigned a general beam section with bending rigidity, shear rigidity, rotary inertia, and mass per unit length given in Table 1b. A unit cell length of 0.025 m was chosen and five Timoshenko beam elements were used to model each unit cell. The resonators are discretely attached to the beam element, with the resonator masses modeled as point masses. To avoid round-off errors due to the large number of increments, linear explicit analysis with double precision was carried out using Abaqus/Explicit. It should be noted that no damping was assumed in any of the numerical simulations.

The effect of resonators on attenuating the impact load was assessed by comparing it with a sandwich beam of equivalent mass. An equivalent mass beam can be modeled by either adding the mass of the resonators to the mass per unit length assigned to the beam elements or by replacing the resonators with equivalent point masses. The latter method was chosen since it provides us with an added advantage of replicating the periodicity of the local resonators. The unit size length, beam material properties, mesh size and the boundary conditions were kept the same.

The effect of the local resonance frequency of the resonators on a given impact load was studied by subjecting the beam to a given impact load and varying the local resonance frequency. The impact was modeled as a smooth triangular pulse of duration 1ms. Local resonance frequency was tuned by varying the resonator spring stiffness while keeping the mass constant. The mass was kept constant at 23 grams for all the cases analyzed in this paper. The resonator frequencies analyzed for this case were 600 Hz, 1000 Hz and 2000 Hz. The associated spring constants are given in Table 2.

The effect of a fixed local resonance frequency on impact loads of different durations was also analyzed. The resonators were tuned to a resonance frequency of 600 Hz while three impact loads of duration 4ms, 2ms and 1ms were studied. Impact loads were chosen so that the maximum frequency associated with the impacts are 1000 Hz, 2000 Hz and 4000 Hz, respectively. The impact loads and their frequency spectrums are shown in Fig 2a and 2b, respectively.

2.2 Results and discussion

The effect of local resonators on the impact load attenuating behavior of a sandwich structure as compared to a beam of equivalent mass was studied. The impact consists of a smooth triangular pulse of duration 1ms and maximum frequency content of 4000 Hz, shown as case 3 in Fig 2a. Three different resonator frequencies, 600 Hz, 1000 Hz and 2000 Hz, were analyzed in order to understand the effect of different resonator frequencies on a given impact load. Fig 3a shows the resultant strains in the beams as measured 2m away from the impact location. It was ensured that no reflections are present in the results shown.

The strains in the beam with the resonators can be seen to be reduced as compared to the beam without the resonators. It can be seen that, overall, the 600 Hz resonators provide better attenuation than 1000 Hz and 2000 Hz. It should be noted that since the system is dispersive and due to the difference in wave speeds of the systems, the percent reduction in amplitude is difficult to judge and only a qualitative conclusion can be accurately made. Among the resonators studied, for the given impact load and as measured 2 m away from the impact location, the resonators tuned to 600 Hz offer the best
performance by attenuating the wave by approximately 26%, followed by the 1000 Hz resonators at 21% and the 2000 Hz resonators at about 11%. This can be explained by looking at the frequency transform of the measured strains as shown in Fig 3b. The frequency spectrum of the input wave shows that its lower frequency content is considerably greater than the higher frequency content. From Fig 3b, the bandgaps for the individual resonators can be clearly seen. The 600 Hz resonators attenuate the waves between 535-1131 Hz, the 1000 Hz resonators create a bandgap between 852-1872 Hz, while the 2000 Hz resonators attenuate the waves above 1422 Hz. Thus, due to the nature of the impact load, though the higher frequency resonators offer larger bandgaps, the lower frequency resonator offers a much better performance by attenuating more of the lower frequency content. However, it should be noted that the bandgap width reduces considerably as the resonance frequency reduces and thus simply tuning the resonators to a lower frequency does not give the optimum wave attenuation characteristics. This can be seen in Fig 4 where it can be seen that the 600 Hz resonators reduce the strain more effectively than the 100 Hz resonators. Thus, there exists a particular optimum local resonance frequency which would give the best performance. This will be studied further in future works.

The effectiveness of a given resonator frequency for different impact loads was also studied. Fig 5a, 5b and 5c show the strain measured for a sandwich beam with resonators tuned at 600 Hz, subjected to impact loads of duration 4ms, 2ms and 1ms, respectively. The impact loads and their associated frequency spectrums are shown in Fig 2a and 2b. It can be seen that the 600Hz resonators are more effective in attenuating loads with higher frequency content. This can be explained by the fact that the chosen resonators attenuate a wider spectrum of frequencies for pulses of duration 1ms and 2ms as compared to a 4ms pulse. Thus, while choosing a resonator frequency it is also important to consider the kind of impact loads expected to be encountered by the structure.

3 Experiments

3.1 Experimental set-up and method

Impact experiments were performed in order to demonstrate the effectiveness of internal resonators for attenuating impact loads and to validate the FEM models used in the previous section. A 72 inch long sandwich beam with a rectangular cross-section of height 2 inch and width 1 inch was manufactured. Aluminum 6061 was used for the facesheets while the sandwich core consists of Elfoam 600 [10] with 71 holes drilled through the thickness periodically in order to insert the resonators. The mass consists of 1 inch long copper rods which were turned down using a lathe to a diameter of 0.42 inches. The final mass was measured to be 23 grams. In order to obtain local resonance frequencies of 100Hz and 300Hz, springs of appropriate stiffness were obtained from Century Spring Corp [11], and were bonded to the mass using epoxy. The bonded spring-mass-spring resonators were then inserted into a 2 inch long plastic casing with inside diameter 0.435 in and were enclosed using tight fitting plastic caps. The resonator units were inserted into the foam and the facesheets were bonded to the core using epoxy. In order to maximize the effect of the resonators, they were inserted as a group, i.e., all the 100Hz resonators were inserted into the left half of the beam, while the 300 Hz resonators were inserted into the right half of the beam. The effective shear rigidity of the sandwich beam with the Elfoam foam-core and embedded plastic casings was evaluated using a three point bending test [12]. The dimensions and the material properties are summarized in Table 1a and 1b. The thickness of the face sheet and that of the core are denoted by \( h_f \) and \( h_c \), respectively, and the width of the beam is denoted as \( b \).

To compare the results of the beam with resonators, two more beams were manufactured: one with the same total mass as the beam with resonators, and one without the added mass. For the beam with the same mass, the copper rods were bonded to the plastic caps of the tubes using epoxy and inserted periodically into the beam similar to the beam with resonators. For the beam without the added mass, empty resonator casings were inserted into beam core periodically.
Fig 6a shows the experimental set-up. The sandwich beams were simply supported 1 inch away from both ends. The input force was generated and measured using PCB 086C03 impact hammer. Bending strains were measured using 1000 Ohm strain gauges mounted in a balanced full bridge configuration. In order to avoid a very short impact pulse, a small 0.125 in thick rubber piece was stuck to the upper facesheet at the point of impact. All three beams were impacted 15 inches away from the right-end while the strain signals were measured 42 inches away from the impact point, as shown in Fig 6b. The strain signals were amplified using an amplifier with a voltage gain of 500 and a low-pass filter tuned at 6400 Hz.

The obtained experimental results are compared against FE models. The sandwich beams used in the experiments were modeled according to the method outlined in section 2.1. The experimentally measured input force was used as the input for the FE models while the strains were measured at the same location as the experiments.

3.2 Results and discussion

Fig 7 shows the input force generated by the impact hammer on the three beams. A force of magnitude 340 N was generated for all three cases. The impact for the beam with resonators was 4.2ms long while for the other two cases a shorter pulse of 3.5ms duration was generated. The strains measured from the experiments are compared with the numerical results in Fig 8a, 8b and 8c. It should be noted that due to the length of the beam and the dispersive nature of the waves, there are reflections included in the measured results. For this discussion, we thus restrict ourselves to the initial part of the strain signal which has the least reflections included in it. A good match is observed for the experimentally measured strain signals and the numerical results. There are some discrepancies which are discussed below.

An important difference between the numerical model and the experiments is that no damping was added to the numerical models. The amplitude of the experimentally measured strain signal can be seen to diverge from the numerical result with increasing time. Epoxy was used to bond the facesheets with the foam which would be expected to add to the material damping. Also, the casings enclosing the mass and the resonators are made using plastic and are not perfectly elastic, thus leading to some difference in the wave propagation behavior. For the beam with resonators, the maximum strain measured experimentally can be seen to be approximately 11% greater than that obtained numerically. For modeling purposes, it was assumed that all the local resonators resonate at the expected resonance frequencies. However, in reality this is not the case. Due to slight variation in the individual spring stiffness, and due to the possible friction between the copper mass and the casing interior, the resonance frequencies of the local resonators are in reality spread over a range of frequencies close to the targeted local resonance frequency which reduces their effectiveness.

4 Conclusion

The response of sandwich beams with resonators to broad spectrum loads, such as impact loads, was analyzed. Finite element models were used to show that beams with resonators inserted in the core are more effective in attenuating impact loads than beams without resonators. It was shown that the choice of the resonators depends on the impact load duration and its frequency content. For a given impact, the design of local resonators can be optimized by looking at the frequency content of the input and the bandgap width generated by the local resonators. Transverse impact experiments were performed to verify the numerical results and to demonstrate the effectiveness of the resonators. A good match between the numerical results and the experiments was obtained. Thus, it was shown that sandwich beams with resonators can be used to better attenuate impact loads and applications requiring blast load mitigation.

Acknowledgements

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References


Fig. 1 Schematic of sandwich beam with internal resonators. (a) Unit cell, (b) equivalent homogenous Timoshenko beam with attached resonators.

Fig. 2a. Impact loads used to study effect of resonators.

Fig. 2b. Frequency spectrum of the impact loads.

Fig. 3a. Strain comparison for beams with different resonator frequencies.
Fig. 3b. Frequency spectrum of strains measured 2m from input.

Fig. 4. Strain comparison for beams with 600Hz and 100Hz resonators.

Fig. 5a. Comparison of bending strains for sandwich beams subject to an impact load of duration 4ms.

Fig. 5b. Comparison of bending strains for sandwich beams subject to an impact load of duration 2ms.

Fig. 5c. Comparison of bending strains for sandwich beams subject to an impact load of duration 1ms.
Fig. 6a. Impact experiment set-up.

Fig. 6b. Schematic of impact experiment set-up (not to scale).

Fig. 7. Comparison of the input force generated for by impact hammer.

Fig. 8a. Comparison of strains for beam without additional mass.

Fig. 8b. Comparison of strains for beam with fixed masses.

Fig. 8c. Comparison of strains for beam with resonators.
### Table 1a.
Dimensions and material constants of specimen.

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<th>$h_f$ (m)</th>
<th>$h_c$(m)</th>
<th>b(m)</th>
<th>a(m)</th>
<th>$G$(Mpa)</th>
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### Table 1b.
Effective properties of the sandwich beam.

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### Table 2.
Resonator parameters

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