1 Introduction
Several attempts have been presented to design structures with morphing capabilities. An example is NASA’s research on biologically inspired morphing wings [1]. Hyer and his colleagues [2, 3] established the concept of using smart materials to introduce morphing into bistable panels. In their research they identified the advantage associated with using bistable panels as actuation energy is required to trigger their shape change whereas no energy is needed to maintaining their deformed shape. In [3] Schultz and Hyer used Macro-fiber Composite (MFC) actuators to trigger snap-through in unsymmetric cross-ply laminates. Other researchers [4-6] investigated the same option to trigger snap-through in unsymmetric panels using MFC actuators. A common observation is that bonding MFC actuators to the panel’s surfaces is found to decrease its curvature hence affecting the bistable characteristics of the assembly. The current research investigates possible alternatives aiming at minimizing this loss of curvature. To this end, an analytical model is developed using ABAQUS to predict the loss of curvature due to bonding and snap-through requirements. As a result it was concluded the bonding MFC actuators to the surfaces of a bistable panel in one of its deformed configurations provides an optimum option. The concept was implemented and tested to determine MFC actuation requirements for triggering snap-through behavior. Analytical predictions were found in good agreement with measurements.

2 Morphing panel assembly preparation
A rectangular unsymmetric panel of width 65 mm and length 85 mm is manufactured from Hexcel IM7/8551-7 Graphite/Epoxy prepreg with a stacking sequence of [0/90]. The mechanical properties of Hexcel IM7/8551-7 graphite/epoxy prepreg are provided in Table 1. The maximum curing temperature is 177°C and the room temperature is 21°C. The equilibrium configurations of a rectangular panel are illustrated in Figure 1. A mold is prepared from high density foam with the same curvature as the second equilibrium shape of the panel. Two 40 mm × 10 mm MFC actuators are centrally bonded to the top and bottom surfaces of the panel using Loctite’s E120 HP Epoxy. The actuators are positioned parallel to the shorter dimension of the panel. Adhesive is applied and the actuators are bent to conform to the panel’s curved surface. Vacuum is applied and the mold is placed in an oven for three hours at 60°C. Figure 2 shows the bistable panel before and after the bonding of the actuators for qualitative comparison.

3 Finite element modeling
3.1 Bistable Behavior
In 1945 Koiter [7] showed that failure to account for geometric imperfections is responsible for the large differences between theoretical and experimental results. Ochinero [8] documented the observation of geometric imperfections in an unsymmetric laminate and investigated their effect on the prediction of cured shapes. In [9, 10] Tawfik et al. developed a finite element based methodology to predict the cured shapes of thin unsymmetric cross-ply rectangular laminates and their bistable behavior. This methodology accounts for nonlinear large deformation and geometric imperfections utilizing Koiter’s theory. In this methodology Abaqus finite element software is used to predict the cured shapes performing three steps. In the first step a linear
eigenvalue buckling problem is solved for the laminate under the thermal load associated with the curing cycle. In the second step the geometry of the “imperfect” laminate is obtained by superimposing these eigenmodes to the nodal coordinates of the perfect laminate. Finally, a geometrically nonlinear step is used to model the cooling stage of the curing cycle and results in predicting the cured shape of the imperfect laminate. Also the methodology predicts the critical load required to trigger snap-through behavior in these laminates utilizing loading and unloading nonlinear analysis steps. Further detail of this methodology is provided in [11].

3.2 Piezoelectric Behavior

The finite element methodology developed by Tawfik et al. [9, 10] was later extended to account for piezoelectric actuation [14]. A thermal analogy approach [12, 13] is used in Abaqus commercial software to model piezoelectric behavior. Using this analogy, the applied electric field is modeled as thermal load with equivalent thermal expansion coefficients calculated from the piezoelectric constants. Piezoelectric extension actuation mechanism, i.e. where the electric field and poling is through the thickness, is employed to demonstrate this analogy. Thermo-elastic strain-stress relations in three-dimensional are

\[
\varepsilon = [S][\sigma] + [\alpha]\Delta T
\]

where \([S]\) is the compliance matrix, \({\sigma}\) are the mechanical stresses, \([\alpha]\) are the thermal expansion coefficients, \(\Delta T\) is the temperature difference and \({\varepsilon}\) are the total strains.

In the case of extension actuation mechanism, the only nonzero electric potential is \(\psi_3\) across the thickness, the total strains \({\varepsilon}\) in terms of actuator properties and constants are

\[
\varepsilon = [S][\sigma] + [d]\frac{\psi_3}{t}
\]

where \([d]\) is the piezoelectric strain coefficient vector and \(t\) is the piezoelectric actuator thickness. The thermal analogy is achieved by comparing Equations (1) and (2). Thus, for a given actuator, the equivalent thermal expansion coefficients is calculated from the piezoelectric coefficients using

\[
\alpha_1 = d_{31}t, \quad \alpha_2 = d_{32}t, \quad \alpha_{12} = 0, \quad \Delta T = \psi_3
\]

Tawfik et al. [14] used this approach to model piezoelectric behavior in Abaqus shell elements.

3.3 Piezoelectric Actuator Bonding

Schultz and Hyer presented a novel morphing concept [3] by bonding MFC actuator to the surface of an unsymmetric cross-ply laminates to trigger snap-through behavior. Also Schultz [15] reported good correlation between model and experiment and explained potential reasons for the difference. Another potential reason for the difference can be any uncertainty in the curvature value at the event of actuators bonding in the vacuum bag.

Tawfik [11] performed a detailed study on different options of actuators bonding, e.g. with the laminate being maintained in either its straight or curved configurations. Actuator bonding in the curved configuration of the laminate is advantageous for minimizing the loss of curvature due to bonding. Therefore a curved mold of identical curvature to that of the free laminate is used to support the assembly at a known curvature while bonding. Also, using the curved mold eliminates any uncertainty in the curvature value of the assembly during bonding.

In order to model actuators bonding in the laminate curved configuration concentrated moments are applied at the actuators edges. Moments are calculated using the moment curvature relationship,

\[
M = E I \kappa
\]

for \(M\) is the applied moment, \(E\) and \(I\) are the actuator’s Young’s modulus in bending and its second moment of area, respectively. MFC actuator properties, provided in Table 2 [16], are implemented in the model and the thermal analogy is utilized. Four-noded, reduced integration, doubly curved shell elements (S4R) are used to model both the unsymmetric laminate and the actuators. The unsymmetric panel is modeled using 884 elements, while for each actuator 64 elements are used to model its active portion and 156 elements to model its casing. Three layers of elements representing the unsymmetric panel and the two actuators are held fixed in space at their midpoints by constraining all degrees of freedom. The neutral axis of each layer of elements representing an actuator is offset according to its respective position in the assembly. First, the three-step methodology is used to simulate laminate curing. Then a nonlinear analysis step is performed to simulate actuator bending under concentrated moments at the edges. The analysis was restarted adding tie constraints between the actuators and laminate. A nonlinear analysis is set to
allow the laminate/actuators assembly to relax to its final bonded shape. Figure 3 provides a side-by-side comparison of the equilibrium shapes of the bonded laminate/actuators assembly versus the finite element predictions. Finally, nonlinear analysis steps representing thermal loading and unloading of the actuators are conducted to predict the required voltage to induce snap-through or snap-back of the assembly.

4 Analytical and test results

The test setup, shown in Figure 4, is used to measure the required voltage to trigger snap-through behavior. The laminate/actuators assembly is suspend by a thread from the middle post. The setup consists of a DC power supply, two miniature High Voltage DC converters (HV-DC), a voltmeter and the laminate/actuators assembly. The Agilent E3648A DC power supply is used to generate a 0 to 12V ramp signal which in turn is fed into each of the miniature HV-DC converters for amplification. The AMCO G05 and G15 HV-DC converters linearly amplify a 12V signal to 500V and 1500V, respectively [17]. The voltmeter is used to show the amplified voltage applied to the actuators. The experiment was repeated four times, for both snap-through and snap-back, and the required voltages were averaged and are provided in Table 3. Finite element analysis predicted values are shown in the same table together with their percentage deviations from the measurements. The voltage was predicted with a percentage difference of ~7.9% from the measured value in the case of snap-through and ~9.0% in the case of snap-back. Deviation from measurements could be due to post-curing loss of curvature in the laminate, perfect bonding assumption, possible misalignment of the actuators along the x-direction of the laminate, and/or uncoupled electrical, mechanical and thermal effects during bonding and actuation.

5 Conclusions

The previously developed methodology [9, 10] is extended to account for piezoelectric actuation via a thermal analogy approach. A laminate/actuators assembly is manufactured in a curved mold and the required voltage to induce snap-through is measured. Finite element predictions are compared to measurements and predictions are found to be in good agreement with measurements. This work is the outcome of the comprehensive study presented in [11].

6 Acknowledgements

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References


Table 1. Properties IM7/8551-7 Graphite/Epoxy prepreg

<table>
<thead>
<tr>
<th>$E_{11}$ (GPa)</th>
<th>$E_{22}$ (GPa)</th>
<th>$G_{12}$ (GPa)</th>
<th>$v_{12}$</th>
<th>$\alpha_1 \times 10^{-6}$/°C</th>
<th>$\alpha_2 \times 10^{-6}$/°C</th>
<th>$t$ (µm)</th>
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<tr>
<td>141.18</td>
<td>7.20</td>
<td>4.45</td>
<td>0.30</td>
<td>0.14</td>
<td>30.98</td>
<td>138.75</td>
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Fig. 1. Unsymmetric cross-ply bistable laminate, Equilibrium configurations (I) and (II)

Fig. 2. Bistable panel; before and after actuators bonding

Fig. 3. Panel/actuators assembly equilibrium shapes, actual versus predicted

Fig. 4. The test setup
### Table 2. MFC piezoelectric actuators properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
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<tbody>
<tr>
<td>Thickness</td>
<td>300 µm</td>
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<tr>
<td>**High-field (</td>
<td>E</td>
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<tr>
<td>Piezo Strain Constants $d_{31}$</td>
<td>$460 \times 10^{-12}$ m/V</td>
</tr>
<tr>
<td>Piezo Strain Constants $d_{33}$</td>
<td>$-210 \times 10^{-12}$ m/V</td>
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<tr>
<td>**Low-field (</td>
<td>E</td>
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<tr>
<td>Piezo Strain Constants $d_{31}$</td>
<td>$400 \times 10^{-12}$ m/V</td>
</tr>
<tr>
<td>Piezo Strain Constants $d_{33}$</td>
<td>$-170 \times 10^{-12}$ m/V</td>
</tr>
<tr>
<td>Young’s Moduli $Y^E_1$</td>
<td>30.336 GPa</td>
</tr>
<tr>
<td>Young’s Moduli $Y^E_2$</td>
<td>15.857 GPa</td>
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<tr>
<td>Shear Modulus $G^E_{12}$</td>
<td>5.515 GPa</td>
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<td>Poisson’s ratio $\nu^E_{12}$</td>
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<tr>
<td>Max Operating Voltage $V_{max}$</td>
<td>1500 V</td>
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<td></td>
<td>-500 V</td>
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### Table 3. Required voltages

<table>
<thead>
<tr>
<th>Required voltage (V)</th>
<th>Top actuator</th>
<th>Bottom actuator</th>
<th>(%)</th>
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<tr>
<td></td>
<td>Exper.</td>
<td>FEA</td>
<td>Exper.</td>
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<td>-460</td>
<td>-495.5</td>
<td>1380</td>
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<td>Snap-back</td>
<td>2217</td>
<td>2416</td>
<td>-739</td>
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