AN INTEGRATED XFEM-CE APPROACH FOR MODELING
MATRIX CRACKS AND DELAMINATION INTERACTIONS IN
COMPOSITE LAMINATES WITH ANGLED PLIES

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Abstract
This paper presents a novel integrated approach using the extended finite element method (XFEM) and cohesive elements (CEs) for modeling three-dimensional (3D) delaminations, matrix cracks and their interactions in progressive failure of composite laminates with angled plies. In the proposed XFEM-CE approach, the matrix cracks are explicitly modeled by the XFEM through nodal enrichment and element partition, and the inter-ply delaminations are modeled by cohesive elements through traction-separation law. An XFEM-based cohesive element enrichment scheme is developed in order to properly model the interactions between the delaminations and matrix cracks. It is critical to enrich the cohesive elements at the local juncture where cracks meet, in order to obtain the correct crack path interactions. The results are presented for cross-ply and angle-ply double cantilever beam specimens with strong explicit matrix crack-delamination interactions. This work presents another significant development of a computational platform for realistic modeling of progressive damage in composites.

1 Introduction
Accurate and efficient computational modeling of progressive failure in composite structures with due consideration of the underlying physics is still challenging because of the various complex failure mechanisms such as delamination, splitting, matrix cracking, debonding, etc. [1, 2]. These failure mechanisms occur simultaneously but across vastly different length scales. For example, while matrix cracks may extend several millimeters or centimeters in the plane of the laminate, they are usually constrained by surrounding plies and are fractions of a millimeter in the thickness direction. Similarly, delaminations may extend several centimeters, but their interactions with smaller matrix cracks have profound effects on subsequent damage propagation. The finite element method (FEM) is the most popular computational method for failure analysis of composites. Careful mesh design and appropriate failure theories are usually required for analysis of the complex failures of composites in finite element (FE) models. Special computational schemes such as the extended finite element method (XFEM) have been developed [3-5], where cracks can be more flexibly modeled through nodal enrichments. This method has been applied to interfacial damage and delamination analysis in composites [6, 7]. However, for delamination, the cohesive element (CE) method is preferred [2, 8] and quite convenient because it is now incorporated in most commercial software. Recently, some nodal enrichment schemes have also been adopted in cohesive elements to more accurately model cohesive crack and delamination problems [9-12]. These enrichment schemes, however, are particularly designed for cohesive elements and based on some special enrichment functions such as isotropic analytical functions [9], solutions of isotropic beam on elastic foundation [10, 11] and piecewise linear functions [12]. They are only suitable for modeling the single delamination-type failure problems and interactions between delamination and crack kinking are not considered in these methods. In order to model crack bifurcation problems, an augmented cohesive zone model was proposed in an augmented finite element method framework [13]. In this model, separable mathematical elements instead of nodal enrichment
were adopted to simulate delamination and cracks. This method was used to model two-dimensional (2D) delamination jumping problem in a cross-ply single-cantilever beam.

The authors have proposed an integrated XFEM-CE approach for modeling matrix cracks and delamination interactions in cross-ply laminates [14]. This paper extends the previous work and presents an integrated XFEM-CE approach for modeling 3D complex failures in composite laminates with angled plies. This approach combines XFEM and cohesive elements and adopts a consistent nodal enrichment scheme for both solid elements and cohesive elements. Delamination, matrix cracks and their interactions are modeled explicitly and seamlessly by cohesive elements and solid elements through nodal enrichment in the XFEM framework.

2 The XFEM-CE Approach for Modeling Complex Failures in Composite Laminates

The XFEM-CE approach is based on the XFEM framework to model multiple failures in composites using both solid and cohesive elements. The XFEM is used to model arbitrary matrix cracks through nodal enrichment and element partition. The cohesive elements are used to model interface behaviors and delamination through the traction-separation law. For the interaction between the interlayer delamination and matrix cracks, enriched cohesive elements are adopted to account for the interaction failure behaviors.

2.1 Crack Modeling by the XFEM

2.1.1 Basic Equations in the XFEM

In the XFEM, nodal enrichment with the same shape functions of standard elements is used to implicitly model strong (cracks) and weak (material interfaces) discontinuities. The enrichment functions may vary with the discontinuity types. Two kinds of enrichment functions, namely, the Heaviside function and the tip enrichment functions, are usually used for the crack-type problems. The Heaviside function is a jump function representing the displacement discontinuity across the crack surface. This function is typically used in fully cracked elements, where the crack tip or front is always located at element edges. The tip enrichment functions are a series of bases of asymptotic expansions of displacement fields around the crack tip. These functions are used in partially cracked elements in order to obtain more accurate solutions around the crack tip or front, where the crack tip or front is located within elements. However, these asymptotic expansions may be hard to obtain for some cases such as the 3D crack problems of anisotropic materials. An approach to circumvent this issue in applying XFEM to the 3D crack problems of composite materials is to model the cracks with only fully cracked elements and reasonable element sizes, where only the Heaviside function is used for nodal enrichment.

In this paper, multiple cracks are allowed in the modeling, but it is assumed that a node support or an element can be cut by only one crack and the crack surface remains plane within the element. Crack kinking is also allowed but it is required that the kinking occurs only at element edges. With the fully-cracked-element scheme in XFEM, the displacement approximation \( u(x) \) in an element can be written as:

\[
\begin{align*}
  u(x) &= \sum_{i=1}^{n^e} N_i(x) \left( u_i^e + H(x)a_i^e \right) \quad (1)
\end{align*}
\]

where \( N_i(x) \) is the shape function matrix at node \( i \) and \( n^e \) is the number of nodes in the element; \( u_i^e \) and \( a_i^e \) are the standard and additional (enriched) degree of freedoms (DOFs) at node \( i \) of the element, respectively; \( H(x) \) is the Heaviside function.

For any point \( x \) in an element with enriched nodes, the Heaviside function \( H(x) \) can be determined by position of the point \( x \) to the crack surface as:

\[
\begin{cases}
  n_i \cdot (x - x_c) > 0, & \text{if } n_i \cdot (x - x_c) \neq 0 \\
  0, & \text{if } n_i \cdot (x - x_c) = 0
\end{cases}
\]

(2)

where \( x_c \) and \( n_i \) are respectively a given point and the normal defining the crack surface corresponding to the element.

From Eq. (2), it can be seen that the values of \( H(x) = 1, -1, 0 \) correspond to the point \( x \) above, below and on the crack plane, respectively.
2.1.2 Determination of Nodal Enrichment and Element Partition

In modeling cracks with XFEM, only some nodes around the cracks need to be enriched. The enrichment at a node is determined by the rule that the node or its support is cut by cracks. In order to ensure the continuity around the crack tip or front, all nodes at element edges where the crack tip or front is located will not be enriched (Fig. 1a). A subroutine is coded with FORTRAN to determine the node list for enrichment and then automatically implement the partition of elements cut by cracks. This is illustrated with a simple 3D example in Fig.1.

Depending on the crack position and nodal enrichment, there are usually three types of elements in XFEM: 1) standard elements (continuous and un-enriched), 2) cracked elements (discontinuous and enriched) which are cut by cracks into two parts, and 3) affected elements (continuous and enriched) which are not cut by any crack but include at least one enriched node, as shown in Fig.1(a). For the standard and affected elements, regular numerical integration methods (e.g. the Gaussian quadrature method) can be used to calculate the element stiffness. For the cracked elements, however, special integration schemes are used because of the discontinuity [4]. This paper adopts an element partitioning method for the numerical integration of cracked elements. The element partition is based on the triangulation of 2D polygons or tetrahedralization of 3D polyhedrons and automatically implemented with the developed subroutine, where all cracked elements are partitioned into 2D triangular or 3D tetrahedral sub-elements as shown in Fig.1(b). Then the numerical integration of the cracked elements is implemented in the sub-elements through regular numerical integration methods (e.g. the Hammer quadrature method). Auxiliary points will be generated to form the sub-elements in partitioning cracked elements (Fig.1b). It should be noted that these auxiliary points are simply used in the numerical integration of the cracked elements but not involved in the node-based system equations. In other words, the integration of the cracked elements is implemented in the sub-elements through regular numerical integration methods but the element stiffness assembly is still based on the nodes of the cracked elements. The element stiffness assembly in XFEM follows the same procedure as that in standard FEM, except that the size of the stiffness matrix of an element with enriched nodes is expanded due to the additional DOFs.

2.1.3 Failure Criterion for Solid Elements

With the nodal enrichment and element partition, arbitrary cracks can be modeled explicitly in XFEM. A failure criterion is required to determine crack initiation. There have been many failure criteria for composite materials in literature [15, 16]. For simplicity and illustrative purposes, but without loss of generality, the maximum stress criterion is used in this paper to determine the failure initiation of solid elements. The maximum stress criterion can be expressed as:

$$\sigma_i^T \leq \sigma_i \leq \Sigma_i^C$$

where $\sigma_i (i = 1, 2, \cdots, 6)$ are stress components in the material directions (local stress components); $\Sigma_i^T$ and $\Sigma_i^C$ are the tensile and compressive strengths in the direction $i$, respectively.

Once the failure criterion is satisfied in a solid element, the element will be cut by a crack plane though its centroid. If the failure is caused by normal
stress, the normal of the crack plane is aligned with the direction of the maximum normal stress; otherwise if the failure is caused by shear stress, the normal of the crack plane is perpendicular to the direction of the maximum shear stress. This element is assumed fully cracked. Then the nodal enrichment is activated and element partition is implemented. The XFEM-based element partition is performed only for solid elements; a different scheme is used to treat failure in cohesive elements, as described below.

2.2 Delamination Modeling by Cohesive Elements

2.2.1 Cohesive Elements and Failure Criteria
In a cohesive finite element, the stiffness $K^e$ can be written as:

$$K^e = \int_{\Omega} N^T D N \ d\Omega$$  \hspace{1cm} (4)

where $\Omega$ corresponds to the domain in the mid-plane of the cohesive element; $D$ is the material stiffness matrix of the cohesive element; $N_u$ is the deformation matrix of the cohesive element and for 3D problem

$$N_u = \begin{bmatrix} N_h & -N_h \end{bmatrix}$$  \hspace{1cm} (5)

$$N_h = \begin{bmatrix} N_1 & N_2 & \cdots & N_{n/2} \end{bmatrix}$$  \hspace{1cm} (6)

$$N_i = N_i I, \quad i = 1, 2, \cdots, n/2$$  \hspace{1cm} (7)

where $I$ is the unit matrix; $N_i$ is the value of shape function at node $i$ of the cohesive element.

In the progressive failure analysis with cohesive elements, traction-based and energy-based criteria are adopted to determine the failure initiation and evolution of cohesive elements, respectively. These two criteria can be written as:

$$\left( \frac{t_n}{S_n} \right)^2 + \left( \frac{t_s}{S_s} \right)^2 + \left( \frac{t_i}{S_i} \right)^2 \leq 1$$  \hspace{1cm} (8)

$$G_f \leq G_{f_c} = \left( \frac{m_n}{G_{f_c}} + \frac{m_s}{G_{f_{lc}}} + \frac{m_i}{G_{f_{ilc}}} \right)^{-1}$$  \hspace{1cm} (9)

where $t_n$, $t_s$, and $t_i$ are the normal and two shear tractions, respectively, and $\{ \}$ denotes the positive value; $S_n$, $S_s$, and $S_i$ are cohesive strengths along the normal and two shear directions, respectively; $m_n$, $m_s$, and $m_i$ are three energy-based mode ratios; $G_f = G_n + G_s + G_i$ is the total deformation energy of an cohesive element, which is a summation of the deformation energies along the normal and two shear directions; $G_{f_c}$, $G_{f_{lc}}$ and $G_{f_{ilc}}$ are the critical fracture energies corresponding to the three fracture modes.

2.2.2 A Zig-Zag Evolution Model for Progressive Failure of Cohesive Elements

For mix-mode deformation of cohesive elements, define an equivalent traction $t$ and an equivalent separation $\delta$ as:

$$\begin{cases}
\delta = \sqrt{\left( \frac{t_n}{S_n} \right)^2 + \left( \frac{t_s}{S_s} \right)^2 + \left( \frac{t_i}{S_i} \right)^2} \\
t = \sqrt{\left( \frac{t_n}{S_n} \right)^2 + \left( \frac{t_s}{S_s} \right)^2 + \left( \frac{t_i}{S_i} \right)^2}
\end{cases}$$  \hspace{1cm} (10)

where $\delta_n$, $\delta_s$, and $\delta_i$ are the normal and two shear separations, respectively.

Fig. 2. Zig-zag evolution model based on linear traction-separation law.

Based on the linear traction-separation law, the separations corresponding to failure initiation and total failure of a cohesive element, $\delta_0$ and $\delta_f$, can be respectively calculated as:
\[
\begin{align*}
\delta_0 &= t_0 / E_0 \\
\delta_f &= 2G_{tc} / t_0 
\end{align*}
\] (11)

where \( E_0 \) is the initial elastic modulus of the cohesive element; \( t_0 \) is the maximum traction determined from Eq. (8), as shown in Fig.2.

In the traction-separation plotting as shown in Fig.2, the zig-zag evolution model consists of a number of zig-zags instead of linear segment in the linear evolution mode. For the \( k \)th zig-zag, the separation increment \( \Delta \delta_k \) is divided into two parts \( \Delta \delta_k^0 \) and \( \Delta \delta_k^a \), i.e.:

\[
\begin{align*}
\Delta \delta_k &= \delta_k - \delta_{k-1} = \Delta \delta_k^0 + \Delta \delta_k^a \\
\Delta \delta_k^0 &= \delta_k - \delta_{k-1} \\
\Delta \delta_k^a &= \delta_k - \delta_k^a
\end{align*}
\] (12)

where \( \delta_{k-1} \) and \( \delta_k \) are the separations after \( k-1 \) and \( k \) zig-zags, respectively; \( \delta_k^a \) is an intermediate value between \( \delta_{k-1} \) and \( \delta_k \), which needs to be determined in the \( k \)th zig-zag.

In Fig.2, two energies \( \Delta G_k^+ \) and \( \Delta G_k^- \) are defined with \( \Delta \delta_k^0 \) and \( \Delta \delta_k^a \) as:

\[
\begin{align*}
\Delta G_k^+ &= \frac{1}{2} \Delta \delta_k^0 \left( t_k^e - t_k^a \right) \\
\Delta G_k^- &= \frac{1}{2} \Delta \delta_k^a \left( t_k^a - t_k^e \right)
\end{align*}
\] (13)

where the tractions \( t_k^e, t_k^a \) and \( t_k' \) can be calculated from \( t_0 \) as:

\[
\begin{align*}
t_k^e &= E_k^0 \delta_k^a = t_0 \delta_k^a \left( \delta_f - \delta_{k-1} \right) / \delta_{k-1} \left( \delta_f - \delta_0 \right) \\
t_k^a &= t_0 \cdot \delta_f - \delta_k^a / \delta_f - \delta_0 \\
t_k' &= E_k^\prime \delta_k^a = t_0 \cdot \delta_k^a \left( \delta_f - \delta_k \right) / \delta_k \left( \delta_f - \delta_0 \right)
\end{align*}
\] (14) - (16)

In the zig-zag evolution model (segment \( ABCD \)) as shown in Fig.2, energy balance requires \( \Delta G_k^+ = \Delta G_k^- \), where the dissipated energy equals to that in the linear evolution model (segment \( AD \)).

With \( \Delta G_k^+ = \Delta G_k^- \), the separation \( \delta_k^a \) can be determined as:

\[
\delta_k^a = \sqrt{\delta_{k-1} \delta_k}
\] (17)

By defining a separation ratio \( c_k = \Delta \delta_k^a / \delta_{k-1} \), the separations \( \delta_k^a, \delta_k, \Delta \delta_k^a \) and \( \Delta \delta_k \) are rewritten as:

\[
\begin{align*}
\delta_k^a &= (1 + c_k) \delta_{k-1} \\
\delta_k &= (1 + c_k)^2 \delta_{k-1} \\
\Delta \delta_k^a &= c_k \delta_k^a = (c_k + c_k^2) \delta_{k-1} \\
\Delta \delta_k &= (2c_k + c_k^2) \delta_{k-1}
\end{align*}
\] (18) - (19)

From Eqs. (14) – (19), once one of the values of \( c_k \) or \( \Delta \delta_k \) is known, all other parameters required to define the \( k \)th zig-zag can be determined from \( \delta_{k-1}, \delta_0, \delta_f \) and \( t_0 \).

In the progressive analysis, \( \Delta \delta_k \) can be determined from a pre-defined number of zig-zags, \( N_z \), i.e.:

\[
\Delta \delta_k = (\delta_f - \delta_0) / N_z
\] (20)

Then the parameters defining all the zig-zags \( (k = 1, 2, \ldots, N_z) \) can be determined from Eqs. (14) – (19).

As shown in Fig.2, two stiffnesses \( E_k^0 \) and \( E_k^a \) in each zig-zag are used and they are determined as:

\[
E_k^0 = E_{k-1} \\
E_k^a = (1 - d_k) E_0 = t_0 \cdot \delta_f - \delta_k / \delta_k \delta_f - \delta_0
\] (21) - (22)

where \( d_k \) is a degradation factor in the \( k \)th zig-zag and can be written as:

\[
d_k = \delta_f - \delta_k / \delta_f - \delta_0
\] (23)
with the initial conditions $E_1^0 = E_2^0 = E_0$ or $d_0 = 0$.

It can be seen that $E_k^a$ and $E_k^a$ are constant in $\Delta \sigma_k^0$ and $\Delta \sigma_k^0$, respectively.

### 2.3 Delamination-Crack Interaction Modeling by Enriched Cohesive Elements

When an XFEM-based matrix crack encounters a cohesive element at an interface between two plies, two different failure modes interact. In this paper, the cohesive elements connecting the matrix crack are not partitioned but enriched to account for the interaction, as shown in Fig.3.

![Fig.3. Interaction between cohesive elements and partitioned solid elements in XFEM.](image)

From Eqs. (1) and (5) – (7), for a general XFEM-based enriched cohesive element, the deformation matrix corresponding to the enrichment can be formulated as:

$$N_u = H(x)N_a$$  \hspace{1cm} (24)

With the normal and enriched deformation matrices in Eqs. (5) and (24), the element stiffness for an enriched cohesive element can be formulated as:

$$K^e = \begin{bmatrix} K_{uu}^e & K_{ua}^e \\ (K_{ua}^e)^T & K_{aa}^e \end{bmatrix}$$  \hspace{1cm} (25)

where

$$K_{\alpha\beta}^e = \int_{\Omega} N_{\alpha}^T D N_{\beta} \, d\Omega$$  \hspace{1cm} (26)

where the subscripts $\alpha$ and $\beta$ take the symbols of $u$ or $a$, and $K_{\alpha\beta}^e = (K_{\alpha\beta}^e)^T$.

In Eq. (25), $K_{uu}^e$ is the stiffness corresponding to the standard DOFs, which is the normal cohesive element stiffness (without enrichment) as shown in Eq. (4); $K_{ua}^e$ is the stiffness corresponding to the enriched DOFs, and $K_{aa}^e$ is the stiffness corresponding to the interaction between the standard and enriched DOFs.

With Eq. (25), the equilibrium equation of an enriched cohesive element can be written as:

$$\begin{bmatrix} \mathbf{f}_u^e \\ \mathbf{f}_a^e \end{bmatrix} = \begin{bmatrix} K_{uu}^e & K_{ua}^e \\ (K_{ua}^e)^T & K_{aa}^e \end{bmatrix} \begin{bmatrix} \mathbf{u}^e \\ \mathbf{a}^e \end{bmatrix}$$  \hspace{1cm} (27)

where $\mathbf{f}_u^e$ is the nodal force corresponding to the additional DOF $\mathbf{a}^e$.

### 3 Example

This section demonstrates the application of the proposed XFEM-CE approach to delamination analysis of composite laminates with angled plies. A carbon/epoxy (AS4/3501-6) composite laminate with layup [0/45]s loaded as a double-cantilever beam (DCB) is used to illustrate failure analysis with strong matrix crack-delamination interactions.

The material properties of AS4/3501-6 are listed in Table 1. The elastic modulus of cohesive elements is assumed to be 4.0GPa which is a typical value for epoxy resin. The cohesive strengths and fracture energies for three modes are listed in Table 2. The length and width of the laminate are $L=125mm$ and $W=20mm$, respectively. Each ply and cohesive layer thicknesses are 1mm and 0.005mm, respectively. The fibre direction refers to the length direction (X-axis) of the laminate. The 0/45 and 45/0 interfaces are modeled with cohesive elements.

An initial delamination is placed at part of the top interface (0/45) and allowed to propagate and kink into the 45° layer with increased load. The initial length of the delamination is $c_0=10mm$ and throughout the laminate width (Fig. 4). The crack front is a straight line perpendicular to the fibre direction in the top 0° ply. Uniform and ramping out-of-plane tensile loads are applied on both top
and bottom surfaces of the laminate near the end part with the pre-existing interfacial crack, and the other end of the laminate is fixed.

Table 1: Material properties of AS4/3501-6 [17]

<table>
<thead>
<tr>
<th>Material properties</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal modulus $E_1$ (GPa)</td>
<td>147</td>
</tr>
<tr>
<td>Transverse moduli $E_2 = E_3$ (GPa)</td>
<td>10.3</td>
</tr>
<tr>
<td>In-plane shear moduli $G_{12} = G_{13}$ (GPa)</td>
<td>7.0</td>
</tr>
<tr>
<td>Shear modulus $G_{23}$ (GPa)</td>
<td>3.7</td>
</tr>
<tr>
<td>In-plane Poisson’s ratios $v_{12} = v_{13}$</td>
<td>0.3</td>
</tr>
<tr>
<td>Poisson’s ratio $v_{23}$</td>
<td>0.5</td>
</tr>
<tr>
<td>Longitudinal tensile strength $X_T$ (MPa)</td>
<td>2280</td>
</tr>
<tr>
<td>Longitudinal compressive strength $X_C$ (MPa)</td>
<td>1725</td>
</tr>
<tr>
<td>Transverse tensile strength $Y_T$ (MPa)</td>
<td>57</td>
</tr>
<tr>
<td>Transverse compressive strength $Y_C$ (MPa)</td>
<td>228</td>
</tr>
<tr>
<td>In-plane shear strength $S_{12} = S_{13}$ (MPa)</td>
<td>75</td>
</tr>
</tbody>
</table>

Table 2: Cohesive strength properties of AS4/3501-6 [18]

<table>
<thead>
<tr>
<th>Material parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohesive normal strength $S_n$ (MPa)</td>
<td>60</td>
</tr>
<tr>
<td>Cohesive shear strengths $S_t = S_l$ (MPa)</td>
<td>68</td>
</tr>
<tr>
<td>Mode I $G_{IIc}$ (N/mm)</td>
<td>0.075</td>
</tr>
<tr>
<td>Modes II and III $G_{IIc} = G_{IIIc}$ (N/mm)</td>
<td>0.547</td>
</tr>
</tbody>
</table>

The FE model of the laminate is built with 8-node hexahedral solid elements in the 0° and 45° plies and 8-node hexahedral cohesive elements at the 0/45 and 45/0 interfaces, with one element through the thickness per ply. The cohesive elements share nodes with the solid elements in the adjacent plies, as shown in Fig. 4.

The failure progression is illustrated in Fig. 5. The delamination at the top 0/45 interface propagates first, and the delamination front shape is similarly a straight line as its initial shape. After the delamination propagates to a certain length, matrix cracks initiate and the delamination starts to kink. A matrix crack first initiates at one of the edges in the 45° plies and then quickly propagates throughout the thickness and width of the 45° plies to form a major crack in the plies. After the 45° plies are totally cracked, delamination initiates and propagates at the bottom 45/0 interface. The second delamination initiates at the edge opposite to that where the matrix crack initiates and then propagates throughout the width of the laminate.
AN INTEGRATED XFEM-CE APPROACH FOR MODELING MATRIX CRACKS AND DELAMINATION INTERACTIONS IN COMPOSITE LAMINATES WITH ANGLED PLIES

Fig. 5. Failure progression: (a) delamination progression at the top 0/45 interface and matrix crack initiation, (b) matrix crack progression, (c) fully cracked 45° plies and delamination initiation at the bottom 45/0 interface, (d) delamination progression along the crack at the bottom 45/0 interface and (e) delamination spreading throughout the width at the bottom 45/0 interface.

Fig. 6. Delamination progression at the bottom 45/0 interface after spreading throughout the width.

Fig. 7. Deformations: (a) the laminate and (b) the 45° plies.

After the delamination at the bottom 45/0 interface spreads throughout the width of the laminate, the delamination will further propagate along the length direction of the laminate, as shown in Fig. 6. The
deformations of the laminate and the cracked 45° plies are shown in Fig.7, where the delaminations and the major crack are explicitly modeled by the failed cohesive elements and the partitioned solid elements, respectively.

4 Conclusions
An XFEM-CE approach for modeling delamination and matrix crack interactions in composite laminates with angled plies is presented in this paper. In this approach, matrix cracks are modeled by XFEM with the maximum stress criterion for determining crack initiation in matrix. Delaminations and their interactions with matrix cracks are modeled by cohesive elements with a zig-zag evolution model based on the traction-separation law. Enriched cohesive elements are also presented in the XFEM framework to better account for the interactions between delaminations and matrix cracks. A composite laminate with the layup [0/45], was loaded as DCB to demonstrate the application of the proposed approach. Progressive failures of the major matrix crack in the 45° plies and the extensive delamination at the 45/0 interface were illustrated. This approach provides a potential numerical simulation tool for complex failures of composite structures.

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