1 Introduction
Microdamage which during service life develops in plies of laminated composites is caused by complex combinations of thermo-mechanical and environmental loads. Since the transverse strain to failure of unidirectional composites is lower than other failure strain components, intralaminar cracking in layers of laminates, caused by combined action of transverse tensile stress and shear stress, is the first mode of damage. The crack is usually well defined, it runs parallel to fibers in the layer and the crack plane is transverse to the laminate middle-plane.

Due to progressing damage, macroscopic thermo-mechanical properties of the laminate are degraded. Two basic approaches are used in modeling, see review for example in [1][2]: either continuum damage mechanics or perturbation stress analysis based (called micromechanics models). Most of the latter models are focused on an approximate description of the local stress distribution in the repeating element between two cracks. The stress field is further used to determine certain thermo-elastic constant of the laminate with damage (most often the axial modulus or the Poisson’s ratio). The simplest calculation schemes used are based on shear lag assumption [3] or variational principles [2-6]. Most of the analytical solutions are applicable to cross-ply type of laminates with cracks in internal 90-layers only. With increasing complexity of the analytical model it actually becomes numerical and the most accurate semi-numerical routines based on Reissner’s variational principle in [7] belongs to this group.

An interesting approach to analyze general laminates using a modified shear lag model and “equivalent constraint model” to determine the effective properties of the damaged layer was introduced in [8]. The laminate properties degradation due to cracks in layers is uniquely related to the relative displacements of the corresponding points on both crack surfaces (opening and in-plane sliding displacements). As long as points on both surfaces coincide (relative displacements are zero) the thermo-mechanical properties of the laminate are not affected by the presence of crack. The opening and sliding of crack surfaces reduce the average strain and stress in the damaged layer, thus reducing the portion of the load carried by this layer leading to reduction of the laminate thermo-elastic properties. Reduced average stress in a layer can be expressed also in terms of reduced effective elastic properties of the damaged layer, the ply-discount model being the most extreme way how to account for it. Thus the crack opening displacement (COD) and crack face sliding displacement (CSD) together with the number of cracks per unit length in the layer (crack density) are the micromechanical parameters governing the macroscopic stiffness reduction. In a linear elasticity they are proportional to the applied load, ply thickness and, therefore, the COD and CSD have to be normalized to be used in stiffness modeling.

The normalized COD dependence on geometrical parameters of the laminate and on the material properties was studied experimentally using optical microscopy of loaded damaged specimens in [9][10].

A theoretical framework (called GLOB-LOC approach) expressing the damaged laminate thermo-elastic properties via number of cracks in layers
(linear density of cracks) and the microdamage parameters COD and CSD was developed in [11][12]. It was shown that only the average values of these quantities enter the stiffness expressions. Certainly macro-properties are in an exact and explicit form related also to composite properties and geometrical characteristics of the laminate.

FE studies were performed to understand which material and geometry parameters affect the COD and CSD and simple empirical relationships (power laws) were suggested in [11-13]. The developed model accounts also for crack interaction. If the crack density is high the stress perturbations of two neighboring cracks overlap and the average stress between cracks at the given applied load is lower. It means that the COD and CSD of interacting cracks are smaller than for non-interactive cracks. This effect was found experimentally in [10] and analyzed theoretically in [14].

In the present paper we investigate the bending stiffness of cross-ply laminates with damaged surface layers. It is suggested that simplified analysis can be used where first the effective stiffness of the layer with a given crack density is calculated and then the classical laminate theory (CLT) is used to find the bending curvature relationship as dependent on the damage state. Two different methods, one using FEM and another one using the GLOB-LOC model [13] are used to back-calculate the effective layer thickness from the damaged laminate and the undamaged laminate stiffness matrices. This approach is validated using FEM by modeling the four-point-bending test.

2 Bending resistance of a damaged laminate

2.1 Approach

General unsymmetric but balanced laminate is subjected to bending, for example in 4-point bending test where a region with constant applied moment \( M_x \) exists leading to constant curvature \( k_x \). Possible shear nonlinearity of the layers with off-axis orientation is not considered: the stress analysis is linear elastic. The following boundary conditions in CLT were assumed to describe the laminate deformation between both loading zones: constant moment \( M_x \) and \( M_y = M_{xy} = 0 \). The response is constant average curvatures \( k_x, k_y \).

Due to the initial lack of symmetry the B-matrix is not zero and some midplane strains \( e_{x0}, e_{y0} \) are also present. When intralaminar cracking with local delaminations develop in some layers the laminate A, B and D-matrices change in a different way but the laminate is still unsymmetric and also unbalanced. For example, due to different damage in different off-axis layers the \( \gamma_{xy} \) and \( k_{xy} \) are not zero anymore.

The simplest way of presenting the change of the bending resistance with damage development is to plot the bending moment to create unit curvature versus the crack density in a layer \( (M_x / k_x) \sim \rho \). Other parameters to consider could be delamination length normalized with respect to the crack size (ply thickness); ply thickness ratios, etc. It has to be noted that at these boundary conditions the introduced ratio is not equal to the bending stiffness \( D_{11} \).

Alternative boundary conditions representing a multilayer beam are: applied \( M_x \) and \( k_y = k_{xy} = 0 \) as well as y-direction midplane strain is zero. In this case the relevant CLT equations are

\[
0 = A_{11} e_{x0} + B_1 k_x \\
M_x = B_1 e_{x0} + D_{11} k_x
\]

From here

\[
M_x = \frac{C_{11} k_x}{C_{11}} = \left( D_{11} - \frac{B_{11}^2}{A_{11}} \right)
\]

Prediction and numerical determination of the \( C_{11} \) dependence on crack density and other parameters is the main objective of this study.

Two approaches will be used and compared:

a) Direct FEM simulation of the 4-point bending test for laminate with damaged layers, calculating the curvature for a given applied force (moment)

b) Replacing the damaged layer with homogenized layer with effective stiffness constants. The curvature corresponding to applied bending moment is calculated using CLT with effective properties of layers.

2.2 Effective stiffness of the damaged layer

An extreme case of effective stiffness of the damage layer is obtained using the ply-discount model
modification relevant for intralaminar cracking: transverse modulus $E_T = 0$ and the inplane shear modulus $G_{LT} = 0$ whereas the longitudinal modulus $E_L$ and Poisson’s ratio $v_{LT}$ are not changed. This case corresponds to infinite number of intralaminar cracks in the layer. With increasing crack density the effective stiffness of the damaged layer has to approach to this asymptotic value.

Even if the initial laminate is unsymmetric it is convenient to “make” it symmetric (it is always possible by putting some additional layers in CLT analysis or symmetry conditions if the analysis is FEM based). The benefit of this is that for symmetric laminate the $[A]$ matrix represents the stiffness matrix $[Q]^\text{LAM}$ of the laminate, which defines the elastic constants of the laminate. The effective stiffness matrix of any layer due to damage can be back-calculated from the difference in the undamaged and damaged laminate extensional stiffness matrix, $[Q]^\text{LAM}_0$ and $[Q]^\text{LAM}$ respectively. This technique is described more in detail in [15]. If the laminate would be unsymmetric before or after the damage introduction, the $[B]$, $[D]$ matrices have to be included in the calculation routine.

If the layer with index $k$ is damaged its effective stiffness is changing from $[Q]_k$ to $[Q]^\text{eff}_k$ (the overbar is used to denote the stiffness of the layer in the global system of coordinates). Then the damaged laminate extensional stiffness matrix can be calculated using CLT as

$$[Q]^\text{LAM}_k = 
\frac{t_k}{h} [Q]^\text{eff}_k + 
\sum_{i=1}^{k-1} \frac{t_i}{h} [Q] + \sum_{i=k+1}^{N} \frac{t_i}{h} [Q].$$

In (4) $N$ is the number of layers. The undamaged laminate stiffness matrix can be written as

$$[Q]^\text{LAM}_0 = 
\frac{t_k}{h} [Q]_k + 
\sum_{i=1}^{k-1} \frac{t_i}{h} [Q] + \sum_{i=k+1}^{N} \frac{t_i}{h} [Q].$$

Subtracting (4) from (5) we obtain

$$[Q]^\text{LAM}_0 - [Q]^\text{LAM}_k = \frac{t_k}{h} [Q]^\text{eff}_k - \frac{t_k}{h} [Q]^\text{eff}_k$$

From (6) the effective stiffness matrix of the damaged $k$-th layer in global axes is found

$$[Q]^\text{eff}_k = [Q]_k - \frac{h}{t_k} \{[Q]^\text{LAM}_0 - [Q]^\text{LAM}_k\}$$

If due to the symmetry and the applied in-plane loading the damage is also symmetric and the damaged layer is not the middle layer, two damaged layers have to be considered which is achieved by adding factor 2 to the first term on the left in (4), (5). This is the case for the example laminates with 90-layers on the surface analyzed in this paper.

Equation (7) can be transformed to the local coordinate system using expression

$$[Q]^\text{eff}_k = \{Q\}^\text{eff}_k \{Q\}^T$$

In the next step the effective compliance matrix of the damaged layer in local axes is calculated

$$[S]^\text{eff} = (\{Q\}^\text{eff}_k)^{-1}$$

Effective engineering constants of the damaged layer (index $k$ is omitted) follow from

$$E_{11}^{\text{eff}} = \frac{1}{S_{11}^{\text{eff}}}, \quad E_{22}^{\text{eff}} = \frac{1}{S_{22}^{\text{eff}}},$$

$$-\nu_{12}^{\text{eff}} = \frac{E_{11}^{\text{eff}}}{S_{12}^{\text{eff}}}, \quad G_{12}^{\text{eff}} = \frac{1}{S_{66}^{\text{eff}}}$$

The described back-calculation is unique only if damage is considered explicitly in one layer at a time. This means that interaction between cracks in different layers is not accounted for. However, it may be accounted for in an indirect way by reducing elastic constants of the rest of damaged layers. Generally speaking even the damage related effective properties in layers should be found in an iterative way. However, it was shown in [15] that the transverse effective modulus of the damaged layer, which is the main parameter in the bending analysis is not sensitive with respect to the elastic properties of the neighboring layers.

It is interesting to note the following result of many back-calculations for large number of material
systems and laminate lay-ups: only the effective transverse modulus $E_2^{\text{eff}}$ and the shear modulus $G_{12}^{\text{eff}}$ of the layer are reduced due to intralaminar cracks. The longitudinal modulus is $E_1^{\text{eff}} = E_1$ and $\nu_{12}^{\text{eff}} = \nu_{12}$. As expected the effective $E_2^{\text{eff}}$, $G_{12}^{\text{eff}}$ approach to zero when the crack density in layer approaches to infinity.

3 Effective stiffness of the damaged laminate

3.1 FEM model

As described in Section 2, the damaged layer may be replaced by homogenized effective properties and then CLT can be used to calculate the bending stiffness of the laminate.

The effective properties of the damaged layer can be calculated using Equations (4)-(10), however, the stiffness matrix of the damaged laminate $[Q]^{\text{LAM}}$ is needed in order to perform the back-calculation. The $[Q]^{\text{LAM}}$ matrix needs to be calculated for every crack density value. In this paper 2 methods were used to calculate $[Q]^{\text{LAM}}$ as a function of crack density: 1) FEM calculations using representative volume element (RVE) model; 2) GLOB-LOC approach described in Section 3.2. It is important to note that the back-calculation procedure in Equations (4)-(10) is valid for symmetric laminates only. Thus symmetric damage has to be considered with both FEM RVE model and GLOB-LOC approach.

The FEM model of the RVE is schematically shown in Figure 1. A 3-D FEM model was generated using FEM software ANSYS [16]. Taking advantage of the symmetry conditions, the FEM model in Fig.1 consists of a 90 degree layer of thickness $t_{90}$ and a 0 degree layer of thickness $t_0$, the length of the model is $l_k$, which is the half distance between two transverse cracks in the 90 layer. Thus, parameter $l_k$ defines the crack density ($\rho_k = 1/(2\cdot l_k)$) and can be parametrically varied to calculate $[Q]^{\text{LAM}}$ for different crack densities. The width of the FEM model of RVE was in all calculations equal to $w = 0.2\cdot h$ where $h = 2(t_0 + t_{90})$ is the total thickness of the laminate.

On the top surface of the FEM model ($z = t_0 + t_{90}$) symmetry boundary conditions were used. At $x = 0$, symmetry boundary conditions were applied on the 0 degree layer while the surface of the 90 degree layer is free representing the transverse crack surface. At $x = l_k$ a uniform displacement in the x axis direction was applied. On each of the side edges of the model ($y = 0$ and $y = -w$) displacement coupling in the y axis direction was used.

Elastic modulus $E_{k}^{\text{LAM}}$ and Poisson’s ratio $\nu_{0k}^{\text{LAM}}$ of the damaged laminate were calculated from post-processing using reaction forces and resulting displacements.

For the calculation of the transverse modulus $E_{k}^{\text{LAM}}$ of the damaged laminate the FEM model with the same geometry as in Fig.1 was used with different boundary conditions: uniform displacement was applied in the positive y axis direction, symmetry boundary conditions were applied on nodes at $y = -w$, displacement coupling in x axis direction was applied on the nodes with coordinates $x = l_k$.

3.2 GLOB-LOC model

We consider $N$- layer laminate which is symmetric before damage and also in the damaged state. The $k$-th layer of the laminate is characterized by thickness $t_k$ and fiber orientation angle $\theta_k$. Direction 1 is fiber direction and direction 2 is transverse to fibers. We denote by $h$ the total thickness of the laminate, $h = \sum_{k=1}^{N} t_k$. Dependent on loading any layer may contain certain number of intralaminar cracks which is characterized by crack density in the $k$-th layer $\rho_k$ defined as the number of cracks per mm length measured transverse to the crack plane. The crack density in a layer is $\rho_k = 1/(2\cdot l_k \sin \theta_k)$, where $2\cdot l_k$ is the average distance between cracks in the layer measured on the specimen edge. Dimensionless crack density $\rho_{kn}$ in layer is introduced as

$$\rho_{kn} = t_k \rho_k \quad (11)$$

The macroscopic stiffness matrix of the damaged laminate is $[Q]^{\text{LAM}}$ and the stiffness of the undamaged laminate is $[Q_0]^{\text{LAM}}$, related to laminate
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compliance matrix. For example, \( [S_0]^{LAM} = \left( [Q_0]^{LAM} \right)^{-1} \). The strain response to applied laminate stress \( \{\sigma\}_0^{LAM} \) is denoted \( \{\varepsilon\}_0^{LAM} \) for undamaged laminate and \( \{\varepsilon\}_0^{LAM} \) for laminate with intralaminar cracks. Hook’s law for the damaged laminate is

\[
\{\sigma\}_0^{LAM} = [Q]^{LAM} \left( \{\varepsilon\}_0^{LAM} - \{\alpha\}_0^{LAM} \Delta T \right)
\]

or more explicitly

\[
\begin{align*}
\sigma_{x0}^{LAM} &= \sum \left( \frac{E_{xy}^{LAM}}{E_{xy}^{LAM}} \right) \sigma_{y0}^{LAM} \\
\sigma_{y0}^{LAM} &= \sum \left( \frac{E_{xy}^{LAM}}{E_{xy}^{LAM}} \right) \sigma_{x0}^{LAM}
\end{align*}
\]

\[
\sigma_{xy0}^{LAM} = \sum \left( \frac{E_{xy}^{LAM}}{E_{xy}^{LAM}} \right) \sigma_{xy0}^{LAM}
\]

For damaged laminate both laminate stiffness matrix and the laminate thermal expansion coefficients depend on the damage state.

Exact expressions for macroscale thermo-elastic constants of the damaged laminate RVE were derived in [11],[12]. The results are briefly repeated here. The approach is called “GLOB-LOC” to emphasize the established exact link between global thermo-elastic properties and local stress state characteristics. Expressions for general symmetric laminate with intralaminar cracks in plies were obtained. The integral effect of the crack caused stress perturbation is expressed in terms of crack density and crack face opening and sliding displacements. The expressions for stiffness are as follows

\[
[Q]^{LAM} = \left( [I] + \sum_{k=1}^{N} \rho_{kn} \frac{L_k}{h} [K]_k [S_0]^{LAM} \right)^{-1} [Q_0]^{LAM}
\]

The 3x3 matrix-function \( [K]_k \) for a layer with index \( k \) is defined as

\[
[K]_k = \frac{2}{E_2} [\bar{Q}]_k [T]^T [\bar{Q}]_k [T]^T [\bar{Q}]_k
\]

The stiffness matrix of the damaged laminate, \( [Q]^{LAM} \), is symmetric as requested for any material by thermodynamics considerations. These matrix expressions for thermo-elastic properties contain elastic ply properties, details of laminate lay-up and dimensionless density of cracks in each layer. The influence of each damage entity is represented by the 3x3 displacement matrix in (15) which contains the normalized average COD, \( u_{2an}^k \) and normalized average CSD, \( u_{1an}^k \) of the crack surfaces in \( k \)-th layer. It is assumed that all cracks in the same layer are equal: they have the same crack face displacements and the crack distribution is uniform.

In (14) \([I]\) is identity matrix.

When the distance between cracks in a layer is much larger than the crack size, the stress perturbations of two neighboring cracks do not overlap and cracks in this region are called non-interactive. The normalized average COD and CSD in this crack density region are independent on the value of the crack density. Superscript 0 is used to indicate values in this region, \((u_{1an}^{0k}, u_{2an}^{0k})\).

Parametric analysis of \( u_{1an}^{0k} \) and \( u_{2an}^{0k} \) using FE was performed in [11],[12] to identify the most significant geometrical and material constants affecting crack face displacements. In result simple and rather accurate fitting expressions were obtained to calculate \( u_{1an}^{0k} \) and \( u_{2an}^{0k} \) as a function of neighboring layer properties. These expressions are considered to be sufficiently general to be used for cracks in any laminate. If so, there is no need to use FEM in any of simulations presented in this paper. Sliding displacements are of no relevance for the considered bending of cross-ply laminates. Cracks are in the surface 90-layers only. Expressions for COD in surface layers are briefly repeated below.
\[ u_{2an}^0 = A + B \left( \frac{E_z}{E_x} \right)^n \]  

(16)

In (16) \( E_x \) is the Young’s modulus of the support layer measured in the x-direction. For a crack in a surface layer

\[ \begin{align*}
A &= 1.2, \quad B = 0.5942 + 0.190 \left( \frac{t_{90}}{t_s} \right)^2 - 1 \\
B &= -0.5229 \left( \frac{t_{90}}{t_s} \right)^2 + 0.8874 \left( \frac{t_{90}}{t_s} \right) + 0.2576
\end{align*} \]

(17)

When the distance between cracks decreases (high dimensionless crack density) the stress perturbation regions of individual cracks overlap and the normalized average COD and CSD start to decrease. In [14] the \( u_{2an}^0 \) has been related to COD of noninteractive cracks, \( u_{2an}^{0k} \) by relationship

\[ u_{2an}^{0k} = \lambda_k (\rho_{an}) u_{2an}^0 \]  

(18)

The crack interaction function \( \lambda \) is a function of the crack density in the layer and generally speaking it depends on material and geometrical parameters of the cracked layer and surrounding layers. For noninteractive cracks \( \lambda = 1 \). Detailed analysis of the effect of different parameters on interaction function was performed in [14] using FEM

Weak interaction (2.5%) is observable at normalized spacing \( 2t_{90}/t_s = 2.5 \). Further decrease of spacing (increase of crack density) leads to dramatic drop of the values of the interaction function. The calculated values of the interaction function where fitted by an empirical relationship with an origin in a simple shear lag model. The interaction function according to the shear lag model is

\[ \lambda_k = \tanh \left( k / \rho_{an} \right) \]  

(19)

The shape function (19) was used to obtain the \( k \) value from the best fit of FEM results. The best fit leads to \( k_{CF} = 1.12 \) for CF laminates and to \( k_{GF} = 0.84 \) for GF cross-ply laminates with damage.

At present the crack interaction effect on \( u_{2an} \) in surface layer has not been studied. Therefore, we suggest using for COD in surface layers the same interaction function (19) as for COD of internal cracks.

4 Direct calculation of bending stiffness using FEM

3-D laminate scale FEM model was generated using finite element software ANSYS version 13.0 [16]. The FEM model is schematically shown in Fig.2. The 3-D model consists of 3 layers representing the given cross-ply lay-ups [90/0]_L. The total length of the laminate L was 150 mm with the equal distance between the loading points being 50 mm. The width of the laminate beam was 0.8 mm. The laminate was subjected to 4-point bending loading scheme by applying constant displacement of 3 mm on the top surface points illustrated in Fig.2 in the negative z axis direction. The supports were added as for a simply supported beam: the nodes corresponding to coordinates \( x = 0, z = 0 \) were constrained against displacement in x and z axis directions; the node at coordinate \( x = 0, z = 0 \), \( y = -0.5 \cdot w \) was additionally constrained against displacement in the y axis direction; the nodes corresponding to coordinates \( x = L, z = 0 \) were constrained against displacement in the z axis direction and the node corresponding to coordinate \( x = L, z = 0, y = -0.5 \cdot w \) was additionally constrained against displacement in y axis direction.

Displacement coupling in the y axis direction was applied on the nodes of the side edges of the laminate beam.

It was assumed that the top 90 degree layer remains undamaged since it is in the compressive part of the loaded laminate (cracks in this layer are closed). Also the 0 degree layer remains undamaged. The bottom 90 degree layer, however, is in the tensile side of the laminate therefore transverse cracks will form in this layer and preexisting cracks will be opened. Hence, the bending stiffness of the laminate will be reduced. In the FEM model shown in Fig.2 the number of transverse cracks was parametrically varied from 0 to 33 cracks. Only uniform crack spacing was studied.

It was assumed that the transverse cracks form only in the maximum bending moment zone, i.e., in the distance between the displacement application points as shown in Fig.2.
Apart from transverse cracks the effect of delaminations on bending stiffness was also investigated. Delamination cracks were introduced between the damaged 90 degree layer and 0 degree layer as shown schematically in Figure 3. We assumed that delaminations initiate from transverse cracks and are always symmetric with respect to transverse crack plane. The delamination length \( t_d \) was parametrically varied from 0 to 0.5 \( \cdot t_{90} \). The effect of delaminations on bending stiffness of the laminate was evaluated by comparing with FEM calculations for laminate without delaminations. Bending stiffness from the described laminate scale FEM model shown in Fig.2 was calculated using (3) where the bending moment \( M_s \) was calculated from the reaction forces and the curvature \( k_s \) was calculated from displacements in the constant bending moment zone, assuming that the curvature follows a circular line.

5 Results and discussion

5.1 Test cases and material properties

CLT calculations using effective stiffness approach described in Section 2 were compared with direct FEM calculations of bending stiffness of damaged laminate described in Section 4. Two carbon fiber/epoxy cross-ply laminate lay-ups were studied: \([90/0]_2s\) and \([90/2]_2s\). The elastic properties of the unidirectional undamaged ply are presented in Table 1. In all calculations the thickness of one ply was assumed equal to 0.6 mm, thus the total thickness is 3.6 mm for \([90/0]_2\), laminate and 4.8 mm for \([90/2]_2\), laminate and the respective thicknesses of the 90 degree layer, \( t_{90} \), are 0.6 and 1.2 mm.

Since we consider only \([90/0]_s\), cross-ply lay-ups, the shear modulus of the ply is not affecting the bending stiffness of the laminate.

5.2 Effective properties of the damaged layer

The calculated effective transverse modulus of the damaged layer \( E_{2eff} \) changed with the crack density, while the longitudinal effective modulus \( E_{1eff} \) and the Poisson’s ratio \( \nu_{12eff} \) were exactly equal to undamaged ply properties listed in Table 1 for all crack densities. The results plotted in Fig.4 show coinciding results between FEM and GLOB-LOC methods at low crack densities, however at higher crack densities exceeding 0.4 cracks/mm GLOB-LOC model gives lower values of effective modulus \( E_{2eff} \). As mentioned above, the crack interaction in surface layers in the GLOB-LOC model is taken as for inside layer cracks, which is a rough approximation.

5.3 Bending stiffness

In Figures 5 and 6 results for bending stiffness parameter \( C_{11} \) are plotted for lay-ups \([90/0]_2\) and \([90/2]_2\), respectively. Direct FEM calculations are compared with two CLT calculations using effective properties of damaged layer from FEM RVE (denoted as “CLT FEM eff” in Figures 5 and 6) and GLOB-LOC models (denoted as “CLT GLOB-LOC”). The presented results correspond to transverse cracking only (delaminations are not present). Ply discount model results (assuming near zero properties of the damaged layer) are also presented in Figures 5 and 6 for comparison.

It can be seen that direct FEM results and CLT calculations using FEM RVE model for effective stiffness are in very good agreement in the whole range of studied crack densities \( \rho_k \). CLT calculations using GLOB-LOC model are, however, in good agreement only in the region of small crack densities and at high crack densities the prediction of bending stiffness is below the direct FEM calculations (overestimated transverse modulus reduction in the crack interaction region results in overestimation of the bending resistance reduction). These trends are more pronounced for the \([90/0]_2\), lay-up results plotted in Fig.6. It can even be seen that CLT calculations using GLOB-LOC for effective layer properties result in values below the ply-discount model. These results confirm again that functions (19) in the GLOB-LOC model describing the surface crack interaction in the surface layer have to be improved. The direct FEM results and CLT calculations using FEM RVE approach ply-discount values at high crack densities.
Comparing the agreement between direct FEM calculations and CLT calculations using FEM RVE for effective layer properties in Figures 5 and 6 it is evident that the agreement is better for [90/0/2] lay-up. This is because for thinner 90 degree layers the axial stress distribution across the ply thickness is more uniform, which better corresponds to the tensile calculation case used to determine effective constants of the damaged layer.

In Figures 7 and 8 bending stiffness results obtained using direct FEM calculations only, reveal the influence of delaminations for [90/0/2] and [90/2/0] lay-ups respectively. The bending resistance of the laminate with cracks is additionally reduced by the presence of delaminations initiating from cracks. As expected, the reduction is much larger due to longer delaminations. The effect of the delamination seems to be relatively smaller (comparing to the cracking effect) for laminates with a thick cracked layer, see Fig. 8.

### 6 Conclusions

In this paper the resistance to bending of cross-ply laminates with intralaminar cracks in surface 90-layers and delaminations was analyzed. Direct FEM simulation of a four-point bending specimen was compared with two classical laminate theory calculations that use homogenized effective properties of the damaged layer. The homogenization of the damaged layer in CLT calculations was done using the difference in the stiffness matrix of the damaged and undamaged laminate. FEM representative volume element (RVE) model and the analytical GLOB-LOC method were used to find damaged laminate stiffness. The results indicate very good agreement between the calculation methods at low crack densities. At high crack densities, however, it was shown that only FEM RVE model gives good agreement with direct FEM calculations, while GLOB-LOC based model may underestimate the bending stiffness even below the ply-discount value.

| Table 1. Elastic properties of unidirectional CF/EP composite |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| $E_1$  | $E_2$  | $\nu_{12}$  | $G_{12}$  | $G_{23}$  |
| [GPa]  | [GPa]  | [-]  | [GPa]  | [-]  |
| 104.00  | 6.14  | 0.40  | 5.00  | 0.45  |

Fig. 1. Schematic representation of FEM model of representative volume element (RVE)

Fig. 2. Schematic representation of laminate scale FEM model

Fig. 3. Schematic representation of delamination cracks between 90 and 0 layers
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Fig. 4. Effective modulus $E_{2}^{\text{eff}}$ of the damaged 90 degree layer as a function of crack density.

Fig. 5. Bending stiffness of $[90/0_{2}]$, laminate with transverse cracks

Fig. 6. Bending stiffness of $[90_{2}/0_{2}]$, laminate with transverse cracks

Fig. 7. Bending stiffness of $[90/0_{2}]$, laminate with transverse cracks and delaminations. The delamination length is scaled with respect to the cracked layer thickness.

Fig. 8. Bending stiffness of $[90_{2}/0_{2}]$, laminate with transverse cracks and delaminations. The delamination length is scaled with respect to the cracked layer thickness.

References


