OPTIMIZED EXPERIMENTAL CHARACTERISATION OF PVC FOAM USING DIC TEST AND THE VIRTUAL FIELDS METHOD

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Keywords: Full-field measurements, Digital Image Correlation, Virtual Fields Method, Polymeric foam, Experimental design

1 Introduction

Cellular polymer closed cell foams are broadly used as the core material of lightweight sandwich structures. Common polymer closed cell foams include PVC, PMI, PU or PET foams. Ideally, polymer foam core materials are considered as homogenous isotropic materials. However, in practice most polymer foams perform both heterogeneous and anisotropic material behaviour due to the density variations and directionality of foam cells developed during the manufacturing process [1]. A previous study has characterized the material properties of an orthotropic PVC foam material [2]. However, a significant amount of time and effort was spent on designing the different test specimen shapes needed to reach a uniform stress/strain state in the gauge area. Therefore an efficient experimental methodology is proposed in the present work to identify all the material stiffness parameters of a PVC foam material in one single test using Digital Image Correlation (DIC) and the Virtual Fields Method [3]. The study focuses on optimizing the test configuration in terms of the accuracy of identifications. The idea is to simulate a series of synthetic images as the ones generated from actual experiments. Then Digital Image Correlation and the Virtual Fields Method will be implemented into a routine to process the synthetic images. By introducing several different error sources, such as subset sizes, smoothing levels, measurement area, noise, the identified results can be evaluated to identify the optimized test configuration.

2 DIC testing setup

In the present study, a modified Arcan fixture was used to characterize the constitutive parameters of PVC foams (shown in Fig. 1). The fixture has been developed recently to identify orthotropic material parameters [2]. By connecting different loading holes on the fixture, more complex loading conditions can be introduced compared with the conventional Arcan fixture. Two cameras with a resolution of 2048 x 2048 pixel\textsuperscript{2} were placed on opposite sides of the specimen to capture the images on both sides simultaneously. The cameras were rotated according to the loading angle of the specimen so that the displacements and strains were computed along the global coordinate direction of the specimen. A small cubic H100 PVC foam specimen [4] with 20x20 mm\textsuperscript{2} measurement area were bonded to aluminum tabs and fixed into the fixture.

Fig.1 DIC test set-up
3 VFM

The Virtual Fields Method is an effective technique to process full-field measurements to characterize the material properties. It expresses the condition of global equilibrium of the tested specimen using the principle of virtual work and solves the inverse problem directly. One of the most important issues in the VFM is to choose a suitable test configuration. Since the heterogeneous stress/strain fields play an important role in the identification procedure, it is very important to have a test configuration that activates all the sought constitutive parameters of the materials under investigation. Optimization of the test configuration for VFM identification was firstly proposed by Pierron et al. (2007) [5]. Recently, a refined test configuration design procedure was proposed by Rossi and Pierron [6]. The study used the grid method as the full-field technique and simulated the whole measurement and identification chain, including image forming and grid method algorithm.

4 Optimized test design of PVC foam identification

The quality of the VFM identification depends on various factors, such as test configurations, measurement areas, smoothing techniques and so on. In the previous paper [7], using the strain fields from FE simulations, the effect of different test configurations on the VFM identified results was studied.[7]. Optimized test configuration was sought that could activate all the stress components to provide a balanced identification of all stiffness. This selected optimal test configuration resulted in a significant improvement of the VFM identification compared with other test configurations. However, the actual experimental validations indicated that there were still some differences between the identified parameters and the reference values (obtained from conventional mechanical testing methods). Thus, in this study, a complete measurement process is simulated based on the procedure proposed in [6] to include the error introduced by the DIC measurement techniques, such as the effect of the spatial resolution when high deformation gradients have to be measured. By track all possible sources of bias, regularizing parameters (subset size, smoothing…) can be chosen in a more rational way.

4.1 Simulating the DIC measurements

The DIC measurement presented in Fig.1 is reproduced by simulating a real acquisition of speckle patterns with DIC. The reference image used here is a random white light speckle captured from the previous DIC measurements. Then the grey level of the reference image is deformed according to the displacement fields calculated using FE simulations. The deformation process is performed numerically using an interpolation routine. The unreformed and deformed synthetic images are processed by DIC software [8] to calculate the displacement and strain maps. This speckle deformation procedure has been validated on a uniform strain map. The reference and deformed synthetic images of pure shearing loading condition are shown in Fig.2. Fig.3 (a) indicates the strain maps obtained from the simulated synthetic speckle patterns processed by DIC. Fig.3 (b) is the strain maps obtained from the FE model. Two sets of results display clear consistency, especially at the areas with high strain concentrations. These are mainly caused by the spatial resolution of the DIC technique. The stimulated DIC strain maps have lower strain concentration compared with the FE strain maps due to the low pass filtering effect of the DIC measurements. This is an important source of error included in this study using this simulated DIC measurement process.

(a) Reference synthetic image (b) Deformed synthetic image
4.2 Study the optimized test configuration without noise

Two design variables are selected here to identify the optimized test configuration. First one is the loading angle which can be adjusted by connecting to different holes of the modified Arcan fixture. Another one is the material principal direction which can be varied by cutting the specimen in different directions within the foam slab. Finite Element analyses were conducted using ANSYS version 13.0 along with the ANSYS APDL language, to create various simulated displacement fields with different combinations of the two design variables. The material principal direction was selected from 0° to 90° with increments of 5°. The load angle was selected from 0° to 90° (pure shear to pure tension) with increments of 15° according to the real design of modified Arcan fixture. By imposing the FE displacement fields on the reference speckle pattern, synthetic deformed images with different test configurations are produced. The displacement fields are processed by the DIC routine to calculate the strain fields of different test configurations. At the end, a subroutine of VFM is used to identify the material stiffness parameters. Several error functions will be defined to evaluate the overall performance of VFM identifications and find the best test configuration. Initially, the optimized test configuration will be studied without introducing noise. An error function $C_1$ is defined in eq. (1).

\[
C_1(\alpha, \theta) = \frac{1}{N} \sum_{i=1}^{N} \sqrt{\sum_{j=1}^{4} \left( \frac{Q_{ij}^{ref} - Q_{ij}}{Q_{ij}} \right)^2}
\]

where $\alpha$ is the loading angle, $\theta$ is the off-axis angle of material principal direction, $Q_{ij}^{ref}$ is the reference values of the four identified material stiffness parameters inputted into the FE model, and $Q_{ij}$ is the identified parameters using the simulated DIC strain fields. This error function calculates the average value over $N$ repetitions. Since noise effect is not considered in this study, $N$ is equal to 1. The error function $C_1$ involves bias (mean) and scatter (standard deviation). Here, objective is to figure out the bias due to the spatial resolution of DIC, hence the separation between mean and standard deviation. The error function $C_1$ is plotted as a contour map with respect to the design variables. The previous work has indicated that the missing data on the upper and bottom free edges of the specimen has a significant influence of the identification results [7]. Therefore, missing data on the free edges will be reconstructed in this study by copying the nearest data points to the missing data positions. The plots of error function $C_1$ after and before the reconstruction are presented in Fig. 4 and Fig. 5. The results indicate the similar trend with the previous work [7] using FE strain fields. The tensile loading test configuration gives the most stable identification when there are some missing data points on the free edges. After reconstructing the missing data, the identified results from several other test configurations also get a significant improvement. Meanwhile, much larger overall identification error can be observed in Fig.4 compared with the contour map calculated from FE simulated strain fields (shown in Fig. 6). The reason is that the additional error from low filter effect of DIC process has been included into this study. As can be noted from Fig.4, the configuration for $\theta=5^\circ$ and $\alpha=60^\circ$ displays extremely high identification errors. By comparing the strain components $\varepsilon_{yy}$ of this test configuration with the one of the optimal test configurations ($\theta=15^\circ$ and $\alpha=90^\circ$), it can be noted that the optimal test configuration induce more uniform strain distribution. For the bad situation, the areas with strain value above 0.4% (marked with black lines in Fig.7 (a)) only locate at the corners of the specimen. During the DIC process, these small areas with extremely high strain concentrations might be
collapsed and bring dramatic bias into the VFM identification. This finding shows that the source of error from the DIC process has an important effect on the accuracy of VFM identification. A detailed study of the influence parameters of DIC measurements (Subset size, smoothing effect) will be presented in the section 4.4. From the present study of contour map C1, the most stable test configuration is selected around the tensile test configuration with the off-axis angles around 15 degrees.

Fig. 4 Error function C1 after reconstructing missing data on the edges (using simulated DIC strain fields)

Fig. 5 Error function C1 before reconstructing missing data on the edges (using simulated DIC strain fields)

Fig. 6 Error function C1 after reconstructing missing data on the edges (using FE strain fields)

The relatively error of each identified stiffness component is defined in eq. (2):

\[
E_{rel}(Q_{ij}) = \frac{|Q_{ij} - Q_{ijref}|}{Q_{ijref}}
\]

where \(Q_{ij}\) is the stiffness parameter identified from VFM routine and \(Q_{ijref}\) is the reference stiffness values. The plots of relative error for each stiffness parameter are shown in Fig. 8. The results indicate that \(Q_{12}\) is the most difficult one to identify. The identification of \(Q_{11}\) and \(Q_{22}\) mainly depend on the off-axis angles of the specimens. When the loading direction is aligned with the stiffness component direction (\(\theta=0^\circ\) for \(Q_{11}\) and \(\theta=90^\circ\) for \(Q_{22}\)), the best identifications of these two components can be obtained.

Fig. 7 (a) \(\varepsilon_{yy}\) (\(\theta=5^\circ\) and \(\alpha=60^\circ\)) (b) \(\varepsilon_{yy}\) (\(\theta=15^\circ\) and \(\alpha=90^\circ\))
4.3 Study the optimized test configuration with noise

The second error function $C_2$ defined here is the standard deviation of the identified stiffness parameters by considering the noise effect. The noise is simulated by adding a standard Gaussian white noise to the grey level value of the synthetic images. The amplitude of noise is obtained from actual measurements by capturing two stationary images and evaluating the standard deviation $s$. 20 repetitions are used for each configuration. Since the orders of magnitude of $Q_{ij}$s are different for anisotropic materials, the coefficients of variation are used here instead of the standard deviations. These coefficients are defined by the ratios of the different standard deviations by their corresponding stiffness parameters. The error function $C_2$ for the sum of the coefficients of variation of four identified parameters is defined in eq. (3):

$$
C_2(\alpha, \theta) = \frac{1}{\sum_{\alpha} \sum_{\theta} \sigma_{\alpha \theta}} = \sum_{\alpha} \sum_{\theta} \left( \frac{1}{N} \sum_{k=1}^{N} \left( \frac{Q_{ij}^{(k)} - \bar{Q}_{ij}}{Q_{ij}^{ref}} \right) \right)
$$

where $\alpha$ is the loading angle, $\theta$ is the off-axis angle of material principal direction. $Q_{ij}^{ref}$ is the reference values of the four identified material stiffness parameters, and $Q_{ij}^{(k)}$ is the identified parameters at the $k$th repetition. $\bar{Q}_{ij}$ is the mean value of 20 repetitions. Fig.9 shows the plot of error function $C_2$. The highest standard deviation is located at the area where the $Q_{22}$ component is aligned with the tensile loading direction (loading angle $\alpha =90^\circ$, off-axis angle $\theta=90^\circ$). This can be expected as the specimen has much lower stiffness along the in-plane direction ($Q_{22}=70.43\text{MPa}$) compared with the through-thickness direction ($Q_{11}=146.26\text{MPa}$). If the specimen is loaded along the direction $Q_{22}$, the strain component in the direction $Q_{11}$ will be extremely low and easily be influenced by noise. As can be inferred from Fig.9, the favorable test configurations with less standard deviation are located around the off-axis shearing and off-axis tensile positions. It is reasonable as the off-axis angle between material principle direction and loading direction leads to more balanced results of three strain components and makes the overall identification less sensitivity to noise. The standard deviation of each stiffness component is plotted separately in Fig. 10. The contour map indicates the similar trend as for error function $C_1$ without noise. For the identification of $Q_{11}$ and $Q_{22}$, the lowest standard deviation is obtained when the loading direction is along the stiffness component direction. The identification of $Q_{66}$ is the most stable. This is due to that the relative low stiffness of this component ($Q_{66}=32\text{MPa}$) can result in a relative large shear strain component which is less sensitive to the noise. Combined with the previous finding in Fig.4, the optimal test configuration is selected as the off-axis tensile configuration ($\alpha=90^\circ$, $\theta=15^\circ$).
In the next part of the study, the error function \( C_3 \) is plotted to study the mean value of 20 repetitions. The error function \( C_3 \) is based on eq. (1) by adding noise and calculate the average stiffness over 20 repetitions (N=20).

\[
C_3(\alpha, \theta) = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{Q_{ij}^{ref} - Q_{ij}}{Q_{ij}^{ref}} \right)
\]  

The result is plotted in Fig.11. The contour map displays almost the same result as the one in Fig.4 without adding noise. It reveals that the measurement noise might be eliminated by recording multiple images and averages them. The effect of image averaging will be studied in the future.

4.4 Selection of subset sizes and smoothing levels

As being discussed before, the identified results from VFM routine are dependent on the effect of DIC process. Therefore different subset sizes (from 10 to 50) are adopt here to study the optimum choice of this parameter based on the optimal test configuration (\( \alpha=90^\circ \), \( \theta=15^\circ \)) selected from above results. The overlap is 50% for all the subset sizes. Moreover, the smoothing function are normally introduced when derive the strain field from the displacement field. This smoothing process will also bring some bias of VFM identification. The smoothing techniques adopt here is a pointwise local least squares fitting technique stated in [9]. The basic idea is to select a square “strain window” containing N x N discrete displacement data points around the computed points. The displacement values in these selected regions can be approximated as a surface. The linear least squares method is used to find the analytical expression of the surface. Finally, the strains at the center of this “strain window” can be computed based on the analytical expression. In the next part, the overall identification error of four identified stiffness parameters is plotted according to different smoothing levels (strain calculation window size) and subset sizes. The results will be compared with the reference values (\( Q_{11}=146.26 \), \( Q_{22}=70.43 \), \( Q_{12}=28.12 \), \( Q_{66}=32.00 \)). Then the optimum choice of these test parameters can be determined. The error function used in this study is similar with eq. (1). The only difference is
that the design variables used here are subset size and strain calculation window size.

\[
C_i(sub, win) = \sqrt{\sum \left( \frac{Q_{11} - \bar{Q}_{11}}{\bar{Q}_{11}} \right)^2}
\]  

(5)

The results are plotted in Fig. 12. It shows relatively high identification error when using small subset size and no smoothing technique as raw strain data are deteriorated by high levels of noise. As can be observed from the plot, the small subset size (20 or 30) with smoothing strain calculation window of 10 or 20 gives the optimum identification of all the stiffness parameters due to a significant reduction of noise. For the identification using larger smoothing window (40-60), the identification errors are increased again, especially for the results using large subset sizes (30-50). This is mainly caused by the reduction of heterogeneous deformation fields using large smoothing area. It can be noted that a smaller subset size (marked in the Fig.12 with red lines) gives more stable identifications which are less dependent on various smoothing level compared with large subset sizes. It can be expected as higher spatial resolution will give a better description of heterogeneous deformation. Meanwhile, the smaller smoothing area results in the identifications which are less dependent on the variation of subset sizes (marked in the Fig.12 with black lines). In order to balance these two effects, the optimum choice of test condition is the subset size of 30x30 pixels and smoothing window size of 10.

5. Conclusions

This work presents a procedure to optimize the test design of PVC foam characterization using digital image correlation and virtual field method. The optimal test configurations and other measurement parameters (subset size and smoothing levels) has been sought to give the most stable identification of all the elastic constitutive properties from one single mechanical test. Based on the simulated deformed speckle patterns and DIC process, the systematical error of the characterization of PVC foam specimens using modified Arcan fixture and VFM has been evaluated by defining the error function \( C_1 \). The results indicate a significant influence from the low pass filtering effect of DIC process. Therefore, highly strain concentration areas should be avoided when choosing the optimal test configuration. Furthermore, the random error of measurements has been investigated by introducing Gaussian white noise into our simulated speckle patterns and run the whole simulated measurement process with 20 repetitions. Combined the random error with previous systematical error, the confidence intervals of each identification can be obtained. The mean value over 20 repetitions with noise displays the similar result with the one without adding noise. It indicates that it is possible to record many images and average them to eliminate camera noise in real measurements. For the future work, the effect of image averaging will be studied to determine the optimal number of images. Then the confidence interval of this measurement will be calculated. Experimental validation will be implemented to check the results of the current study.

Acknowledgement

The research reported was sponsored by the Danish National Advanced Technology Foundation through the project “Advanced Thermal Breaker”, which is carried out in close collaboration with Fiberline Composites A/S. The financial support is gratefully acknowledged.

References


