OPTIMIZATION OF VARIABLE ANGLE TOW PLATES WITH ONE FREE EDGE USING LAMINATION PARAMETERS

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1 Introduction
Advanced tow placement techniques allow the fibre (tow) to be placed curvilinearly within a lamina, and the composites manufactured using this concept are described as variable angle tow (VAT) composite laminates. As such, tow steering enables the designer to tailor the local stiffness and fully take advantages of the directional properties of composite laminates, resulting in lightweight structures with superior performance for aerospace applications [1,2,3,4,5,6]. Buckling resistance is often considered as primary design criterion of aero-structures. It has been reported in the literature that, the buckling load carrying capacity of a rectangular VAT plate can be substantially improved when the in-plane prebuckling stresses that result from the variable stiffness are redistributed in a benign way [1, 2, 3, 4]. The aim of this work is to optimize the VAT configuration of a long plate with one free edge and others simply supported for the maximum buckling load. A long plate with one free edge is commonly used in aerospace structures, such as a stringer or half flange of an I-section [2].

On the contrary to the benefits offered by VAT, the optimal design of VAT laminates is a challenging task due to the increased design flexibility and point-wise stiffness tailoring feature. Design of VAT laminates involves large number of design variables as one has to determine the layup sequence at each point in the structure. In addition, the objective function defined in terms of fibre angle orientations or the fibre trajectories is highly non-convex and the optimization process is likely to be trapped in local optima. To overcome these problems, the approach of using lamination parameters as the design variables was shown to be an effective way to solve the optimization problem of the VAT laminates [5, 6]. Using lamination parameters to represent the lamination layups not only results in a reduction of design variables to 12 but also offers possibly the largest design space. In addition, the stiffness tensors (A, B, D matrices) of a laminate based on the classical lamination theory are a linear function of 5 material invariants and 12 lamination parameters [8]. However, the values of 12 lamination parameters are not completely independent and are linked by a particular layup. Therefore, constraints that define the design space (feasible region) of lamination parameters are necessary for the optimization process. The feasible region of the lamination parameters has been proven to be convex, for variable stiffness composites [7].

For buckling or vibration problem, the objective functions expressed with respect to the lamination parameters are much less ill-conditioned (even concave [7]) than using the layer angles as the design variables. The simplified objective function together with the convex design space effectively reduces the complexity and computational time/efforts for a mathematical programming approach to search the global optima. Several works have successfully demonstrated the merits of using lamination parameters as the design variables to optimize the variable stiffness composite laminates [5, 6]. Nevertheless, these works rely on finite-element modelling, in which the local lamination parameters (stiffness) are associated with each element/node. Such element-based design method may suffer from the increasing number of design variables and non-smooth distribution of the lamination parameters.

In this work, the spatial variation of lamination parameters (stiffness) is defined in the form of B-Spline curves/surfaces. A given degree B-Spline curve/surface is determined by a set of control points and a prescribed knot vector. Compared with the finite element discretised approach, using the B-Spline functions to represent the variation of
lamination parameters needs less number of design variables and also results in a continuous and smooth distribution naturally. In addition, the whole region of the spatially varying lamination parameters is fully constrained inside the feasible region provided that the control points are inside the feasible region. This feature is due to the convex hull property of B-Spline functions, and is important for the implementation. It enables us to avoid dealing with infinite number of points over the variation of laminate parameters for the feasible region constraints.

To model the behaviour of VAT plates efficiently, a semi-analytical approach using the Rayleigh-Ritz method is applied to solve the prebuckling and buckling problem [2]. The in-plane elasticity (prebuckling) problem is formulated in terms of Airy’s stress function, which can effectively model the pure or mixed stress boundary conditions and capture the highly non-uniform stress distribution accurately [4].

The optimum process is carried out using a two-level optimization strategy, which was initially proposed by Yamazaki [13-16]. In the first step, the VAT configuration is defined in terms of the B-Spline representation of laminate parameters and a gradient-based method is used to determine the optimum variation of laminate parameters across the plane of the VAT plate. The convergence of the optimization process is studied by varying the number of the control points to define the laminate parameters distribution. In the second step, the realistic distributions of fibre orientation angles for a certain stacking sequence are retrieved from the obtained laminate parameters based on a genetic algorithm (GA) [4]. In general, the most time-consuming process in the optimal design of composite laminates is the mechanics analysis (in this work, the buckling analysis). Also the two-level optimization method requires much less computational times when compared to a direct search approach using the GA or other meta-heuristics algorithms. Consequently, the two-level optimization strategy provides an efficient means to solve the optimization problem, especially for VAT laminates. Furthermore, the laminate parameters guided design process allows us to determine the possible best laminate configuration subject to the studied problem, both theoretically (first-level) and practically (second-level).

Finally, the optimal layouts are demonstrated and compared with previous published results. The mechanism of applying the VAT design concept to improve the buckling resistance of laminated plates with free edge is discussed.

2 Lamination Parameters

Considering classical lamination theory, the constitutive equations of a VAT plate is given by,

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A(x,y) & B(x,y) \\ B(x,y) & D(x,y) \end{bmatrix} \begin{bmatrix} \xi \\ \eta \end{bmatrix}$$ (1)

where A(x,y), B(x,y) and D(x,y) are the in-plane, coupling and bending stiffness matrices and they are functions of x and y for VAT plates. The stiffness matrices are expressed as a linear function in terms of laminate parameters and material invariants. In the present study, only the specially orthotropic VAT laminates are considered in this study. In other words, there exists no in-plane and out-of-plane coupling (B=0), no shear-extension coupling (A_{16}=0, A_{26}=0) and no flexural-twisting coupling (D_{16}=0, D_{26}=0). As a result, two in-plane ($\xi^A_1, \xi^A_2$) and two out-of-plane ($\xi^D_1, \xi^D_2$) laminate parameters are sufficient to define the stiffness matrices as,

$$\begin{bmatrix} A_{11} \\ A_{22} \\ A_{12} \\ A_{66} \end{bmatrix} = h \begin{bmatrix} 1 & \xi^A_1 & \xi^A_2 & 0 & 0 \\ 1 & -\xi^A_1 & \xi^A_2 & 0 & 0 \\ 0 & 0 & -\xi^A_2 & 1 & 0 \\ 0 & 0 & -\xi^A_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix}$$ (2)

$$\begin{bmatrix} D_{11} \\ D_{22} \\ D_{12} \\ D_{66} \end{bmatrix} = \frac{h^3}{12} \begin{bmatrix} 1 & \xi^D_1 & \xi^D_2 & 0 & 0 \\ 1 & -\xi^D_1 & \xi^D_2 & 0 & 0 \\ 0 & 0 & -\xi^D_2 & 1 & 0 \\ 0 & 0 & -\xi^D_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix}$$ (3)

where $U_1, U_2, U_3, U_4, U_5$ are the matrices of material invariants. The four laminate parameters are defined by,

$$\xi^A_{1,2} = \int_{-1}^{1} \left[ \cos(2\theta(z)) \cos(4\theta(z)) \right] d\bar{z}$$

$$\xi^D_{1,2} = \frac{3}{2} \int_{-1}^{1} \left[ \cos(2\theta(z)) \cos(4\theta(z)) \right] \bar{z}^2 d\bar{z}$$ (4)
where $\theta(z)$ is the layup function in the thickness direction of the plate. For variable angle tow (VAT) composites, lamination parameters are continuously varying with respect to the x and y coordinates. The corresponding nonlinear constraints for the lamination parameters should be satisfied strictly from point to point. Failures to satisfy the constraints will either give rise to an unstable optimization process or result in an infeasible optimal lamination configuration. Hence, to define an accurate boundary of the feasible region of lamination parameters is vital to the optimization of VAT laminates. The explicit closed-form relations between the 12 coupled lamination parameters are generally unknown. Nevertheless, the feasible region of these four lamination parameters can be well-bounded by a set of closed-form expressions [6], which were derived from Bloomfield et al.’s work [8].

\[
5(\xi^A - \xi^D)^2 - 2(1 + \xi^A - 2(\xi^A)^2) \leq 0 \quad (5)
\]
\[
(\xi^A - 4t\xi^A + 1 + 2t^2)^3 - 4(1 + 2|t| + t^2)^2(\xi^D - 4t\xi^A + 1 + 2t^2) \leq 0 \quad (6)
\]
\[
(4t\xi^A - \xi^A + 1 + 4|t|)^3 - 4(1 + 2|t| + t^2)(4t\xi^A - \xi^D + 1 + 4|t|) \leq 0 \quad (7)
\]

where $t=[-1,-0.75,-0.5,-0.25,0,0.25,0.5,0.75,1]$. Eqs. (5)-(7) contain 19 nonlinear inequities, which can accurately define the boundary of the feasible region and also allows us to effectively deal with the constraints.

### 3 Buckling Analysis of VAT Plates

Fig. 1 illustrates the analysis problem of a long VAT plate with one free edge and the rest simply-supported subject to a uniform displacement compression. The transverse edges are allowed to move but kept straight. The aspect ratio (a/b) of the plate is chosen to be 20 to simulate a (effectively infinite) long effect.

Prior to buckling, the distribution of non-uniform in-plane stresses needs to be determined [1]. The general prebuckling problem of VAT plates is solved through minimizing the complementary energy [2,10], which is expressed in terms of Airy’s stress function.

\[
\Pi_c = \Pi_s - \int_{c_2} (u_0N_{xw} + v_0N_{yw}) ds \quad (8)
\]

Airy’s stress function is defined in the following double series form for capturing the highly non-uniform stress resultants,

\[
\Phi(x,y) = \Phi_0(x,y) + \Phi_1(x,y)
\]
\[
\Phi_0(x,y) = \sum_k c_k \psi_k(y) + \sum_k d_k \psi_k^D(x) \quad (9)
\]
\[
\Phi_1(x,y) = \sum_{pq} \Phi_{pq} x_p y_q \quad (10)
\]

where $\Phi_0(x,y)$ represents the boundary stress part, while $\Phi_1(x,y)$ satisfies the stress free boundary conditions. The unknown coefficients for Airy’s stress function are determined from,

\[
\begin{bmatrix}
U \\
U^T \\
C \\
C^T \\
D \\
D^T
\end{bmatrix}
\begin{bmatrix}
\Phi \\
c \\
\phi
\end{bmatrix}
= 
\begin{bmatrix}
O \\
P_{\infty} \\
O
\end{bmatrix}
\]

In particular, when the fibre orientation (stiffness) is varying along y direction only, the transverse stress ($N_y$) and shear stress ($N_{xy}$) are zero and the compressive stress has a closed-form solution [1].

\[
N_x(y) = \left[A_{11}(y) - \frac{A_{12}(y)}{A_{22}(y)} \right] \frac{\Delta_0}{a} \quad (11)
\]

Subsequently, the buckling problem of VAT plates is solved by applying the Rayleigh-Ritz method to the minimum potential energy functional that takes account of the state of in-plane non-uniform stress resultants. The transverse deflection $w(x,y)$ is expanded in a series form,

\[
w(x,y) = \sum_{mn} A_{mn} x_m(x) y_n(y) \quad (12)
\]

In this work, Legendre polynomials are used for the series expansion of the admissible functions in Eqs. (9) and (11). Legendre polynomials capture localised behaviour and result in a fast convergence rate than the Fourier series [11].

The buckling load is then determined by solving the following standard eigenvalue problem,

\[
([K] + \lambda[L])[A] = 0 \quad (13)
\]
To facilitate layup comparison, the buckling load is normalised with respect to the corresponding value of a homogeneous quasi-isotropic laminate [2].

\[ K_x^* = \frac{(N_x^c)_{vat}}{(N_x^c)_{iso}} \]  

(14)

where \((N_x^c)_{vat}\) is the average compressive load

\[ N_x^c = \int_a^b N_x(y)dy \]  

(15)

4 Two-Level Optimization Strategy

The buckling optimization procedure of VAT plates is divided into two steps. At the first step, a gradient-based mathematical programming is used to determine the optimum distribution/variation of laminate parameters which give the maximum buckling load. At the second step, a GA is employed as an optimizer to obtain the realistic layups (stacking sequence and fibre orientations) from the target variation of laminate parameters.

4.1 First Level Optimization

The distribution of the four laminate parameters \((x_1^A, x_2^A, x_1^D, x_2^D)\) for establishing an orthotropic VAT laminate configuration are represented in terms of the B-Spline surfaces,

\[ x_{1,2}^{A,D}(x,y) = \sum_{mn} \Gamma_{mn}^{(\tau)} B_{m,k}(x)B_{n,k}(y) \]  

(16)

where \(\Gamma_{mn}\) is the coefficient of the series expansion and equal to the assigned value of a particular laminate parameter at a pre-defined control point \((P_{mn})\). \(B_{m,k}(x)\) and \(B_{n,k}(y)\) are the \(m^{th}\) and \(n^{th}\) \((k - 1)\) order B-Spline basis functions varying along \(x\) and \(y\) axis, respectively. \(\tau = 1,2,3,4\) denotes these four different laminate parameters. When the laminate parameters (stiffness) are only varying along one principal direction, for example the \(y\)-axis, the variation is defined by the B-Spline curves,

\[ x_{1,2}^{A,D}(y) = \sum_{mn} \Gamma_{mn}^{(\tau)} B_{n,k}(y) \]  

(17)

The distribution of the four laminate parameters \((x_{1,2}^{A,D})\) along \(y\)-axis. In this example, the knots vector is \(Z = [0,0,0.1/3,2/3,1,1]\), for which the B-Spline basis functions are piece-wise quadratic variation \((k = 3)\). Due to the local support property of B-Spline, each control point only affects a segment of this curve. This feature allows us to separate the design problem and integrate this B-Spline based design scheme into a gradient-based mathematical programming. By adjusting all the values of the four laminate parameters \((x_{1,2}^{A,D})\) associated with these five control points within the feasible region, a group variation of the laminate parameters is generated and it forms a truncated feasible design space. The complete design space can be approximately achieved by gradually increasing the number of control points.

At the first level, a gradient based method is used to determine the optimum distributions of the laminate parameters for the long VAT plates to achieve the maximum buckling load. The optimization problem is formulated as,

Maximise: \(K_x\)

Design Variables: \(\Gamma_n\)

Subjected to: \(-1 \leq \Gamma_n^T \leq 1\)

\[ g_j(\Gamma_n^T) \leq 0 \]

where \(\Gamma_n^T\) is the laminate parameters associated with each control point. \(g_j(\Gamma_n^T)\) are the nonlinear constraint functions that define the relations between the four different laminate parameters, given by Eqs. (5)-(7).

4.2 Second Level Optimization

The objective of the second level optimization process is to retrieve the realistic stacking sequence and the spatial variation ofibre orientation angles from the optimum distributions of laminate parameters. It was demonstrated that the relation between the laminate parameters and the layups is not unique and is complicated [6]. Hence, it is not possible to directly convert the optimum laminate parameters into a realistic layups using some explicit formulations. To accomplish this task, the objective is altered to search a laminate configuration that closely matches the target laminate parameters.
using a sophisticated optimization algorithm, namely the second level optimization process.

In this work, an unsymmetrical 16-layer stacking sequence \([+\theta_1/\pm \theta_2/\pm \theta_2/\pm \theta_2/\pm \theta_1/\mp \theta_1]\) is used extensively, due to its specially orthotropic property \([B] = 0, A_{16}, A_{26} = 0, D_{16,26} = 0\). In this stacking configuration, there are two VAT design layers \(\theta_1(x,y), \theta_2(x,y)\).

The design flexibility for the through-the-thickness stacking rearrangement can be extended by increasing the number of design layers.

For each VAT layer, the spatially varying fibre orientation angles are described by a general definition for the nonlinear variation of fibre orientation angles. The nonlinear variation (NLV) of fibre orientations is defined and is based on a set of \(M \times N\) pre-selected control points in the plate domain, as illustrated in Fig. 3. Lagrangian polynomials are used to interpolate the prescribed fibre angles at the control points and construct a nonlinear distribution of fibre angles, given by the following series form,

\[
\theta(x,y) = \sum_{m,n=0}^{M,N} T_{mn} \prod_{m \neq i}^{N} \left( \frac{x-x_i}{x_m-x_i} \right) \prod_{n \neq j}^{N} \left( \frac{y-y_j}{y_n-y_j} \right) \tag{10}
\]

As such, this formulation parameterises each VAT layer in terms of few number of fibre orientation angles at the pre-selected control points. It was also observed that, for a flat VAT plate, only 3–5 grid points along each direction are usually needed to approach convergence. In addition, this formulation gives a continuous, smooth distribution for the fibre orientations, which are suitable to be converted into practical tow trajectories when the manufacturing constraints need to be considered. Fig. 3 demonstrates two VAT configurations using 5 uniform spaced control points along each direction.

To accomplish the second level optimization process, we first select a VAT laminate format, which refers to the stacking sequence (number of design layers) and the control points (number and positions) for defining the nonlinear variation of fibre orientation angles. Subsequently, a GA is used to determine the fibre orientation angles at all the control points within each design layer which can lead to the distributions of lamination parameters matching with the desired results as closely as possible.

The fitness function is then expressed in terms of the least square distance between the obtained lamination parameters and the target lamination parameters [14]. For the VAT distribution, the distance is measured as a mean value of a group of grid points over the plate domain. The optimization problem is formulated as,

Minimize:

\[
\Delta \xi = \frac{1}{N_p} \sum_j \Delta \xi_j \tag{11}
\]

\[
(\Delta \xi_j) = \sum_{i=1}^{2} w_i^A (\xi_i^A - \hat{\xi}_i^A)^2 + \sum_{i=1}^{2} w_i^D (\xi_i^D - \hat{\xi}_i^D)^2
\]

Design Variables: \([T_{k}^{1}, \ldots, T_{k}^{n}, \ldots, T_{k}^{k}]\)

where \(T_{k}^{n}\) is the fibre angle at the control point for the \(k\)th ply. \(w_i^A\) and \(w_i^D\) are the weightings to distinguish the relative importance between \(\xi_i^A\) and \(\xi_i^D\). \(N_p\) is the total number of grid points.

The search for fibre angles at each control point is performed using a GA. As the evaluation process of lamination parameters is very efficient, it allows us to use a sufficiently large population and generation for the GA to provide convergent solutions. Based on our trial-and-error experiences, the population size was set to be at least 50 times the number of design variables, while the generation is usually set to 150–200 depending on the population size. The crossover and mutation probabilities were chosen to be 0.7 and 0.04.

5 Results and Discussion

This section presents the numerical results of applying this two-level design strategy to optimize a long VAT composite plate with one free edge for the maximum compressive buckling load. The material properties and the geometry of VAT plates are chosen to be the same with previous works [1,2]. The lamina properties for the graphite-epoxy composite are given by \(E_{11} = 181\) GPa, \(E_{22} = 10.273\) GPa, \(G_{12} = 7.1705\) GPa, \(v = 0.28\). The length and width of plate are \(a=5.08\) m, \(b=0.254\) m, respectively. The tow thickness is 0.127 mm. The thickness variation of the VAT plate due to the manufacture process is not considered in the present
study and the ply-thickness is assumed to be constant.

5.1 Optimal Lamination Parameters (1st Level)

It was observed that [2], for this case, the majority of lateral compressive load redistributed away from the free edge is likely to improve the critical buckling load significantly. Thus, the case of transversely varying fibre orientation is initially considered in the optimization and the four lamination parameters are varied along the y direction $y_{1,2}(y)$. To examine the convergence of the first-level optimization process, the number of control points (as illustrated in Fig. 2) is gradually increased from 5 to 11. In each optimization run, all the control points are uniformly distributed across the plate width and the uniform quadratic B-Spline basis functions are used for constructing the variations of lamination parameters. Therefore, for the 5, 7, 9 and 11 control points, the positions of the control points are defined as,

\[
y_{1.5} = \left[ \begin{array}{c} b \ 2 \ 3b \\ 4 \ 5 \ 4 \ b \end{array} \right] \]

\[
y_{1.7} = \left[ \begin{array}{c} b \ b \ b \ 2b \ 5b \\ 6 \ 3 \ 2 \ 3 \ 6 \ 1 \ b \end{array} \right] \]

\[
y_{1.9} = \left[ \begin{array}{c} b \ b \ b \ 2b \ 3b \ 5b \ 3b \ 7b \\ 8 \ 4 \ 8 \ 2 \ 8 \ 4 \ 8 \ b \end{array} \right] \]

\[
y_{1.11} = \left[ \begin{array}{c} b \ b \ b \ 2b \ 3b \ 2b \ 3b \ 7b \ 4b \ 9b \\ 10 \ 5 \ 10 \ 5 \ 2 \ 5 \ 2 \ 5 \ 10 \ 5 \ 10 \ b \end{array} \right] \]

and the corresponding knots vectors are,

\[
\xi_5 = \left[ \begin{array}{c} 0 \ 0 \ 0 \ 1 \ 2 \end{array} 3 \ 1 \ 1 \ 1 \end{array} \right] \]

\[
\xi_7 = \left[ \begin{array}{c} 0 \ 0 \ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 1 \ 1 \ 1 \ 1 \end{array} \right] \]

\[
\xi_9 = \left[ \begin{array}{c} 0 \ 0 \ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 1 \ 1 \ 1 \ 1 \end{array} \right] \]

\[
\xi_{11} = \left[ \begin{array}{c} 0 \ 0 \ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 1 \ 1 \ 1 \ 1 \end{array} \right] \]

Fig. 4 shows the convergence trends of the first-level optimization process using different (5, 7, 9 and 11) control points to construct the variation of lamination parameters, respectively. All of these four optimization processes exhibit rapid convergences within a few iterations (less than 10). It is observed that, when more number of control points is used, a larger optimal buckling load is obtained. The curves for the 9 and 11 control points are nearly coincident, which shows that the optimal maximum buckling load is ultimately convergent with increasing number of control points. This also proves that the full design space can be approximately achieved by increasing the number of control points for defining the B-Spline form variation of lamination parameters. Nevertheless, for long VAT plate with one free edge, no further improvement of buckling load was observed when the lamination parameters (stiffness) are allowed to vary along both axes.

The obtained optimal variations of the four lamination parameters are plotted in Fig. 5, for which the maximum buckling coefficient ($K_{ct}$) is 4.12 (the buckling load is 3549N). This value is 9% larger than the optimal results obtained from a direct search using the genetic algorithm [2].

Fig. 6 illustrates the load redistribution along the y-axis. It shows that the majority of compressive load is re-distributed to a narrow region near the supported edge, whereas the free edge and the central part of the plate bear very little amount of load. It demonstrates again that the load redistribution (towards to the supported edges) induced by variable stiffness is the main contribution to improve the buckling resistance of VAT laminates.

5.2 Optimal VAT Layups (2nd Level)

In this section, realistic variation of fibre orientation angles (or the tow trajectories) for the VAT lamination layups are retrieved from the optimal lamination parameters which are shown in Fig. 5. As aforementioned, the stacking sequence is fixed to be a 16-layer unsymmetric specially orthotropic laminate with two VAT design layers. In the optimization, at each VAT design layer, the number of control points for defining the NLV of fibre orientation angles (Eq. 10) is gradually increased to obtain the convergent results.

Table 1 lists the second-level optimization results with respect to use different number of control points, in which the resultant minimum distance (Eq. 11), the buckling load given by the retrieved VAT layups and the differences between the obtained maximum buckling loads and the target value (1st level result) are compared. It shows that 5 control points are sufficient to yield converged results. The
final maximum buckling load obtained is 3594 N ($K'_{cr} = 4.00$), which is 4.5% less than the target value ($K'_{cr} = 4.19$). Note, only two design VAT layers were used for the design flexibility through the thickness. Closer matching result may be achieved if more design layers are used.

Nevertheless, without adding design layers, the optimization results could be further improved by adopting sensitivity-based objective functions [6] or applying an enhanced second-level optimization strategy [16]. Additionally, the manufacturing and other realistic design constraints also need to be considered in this second-level optimization process in future work.

Table 2 compares the optimal layups for the maximum buckling load of the long VAT SSSF specially orthotropic laminated plates, which are obtained using two different optimization approaches. One is a direct GA search based on the definition of NLV of fibre orientation angles to parameterise the VAT layups [2] and the other one is the two-level optimization strategy presented in this paper. For a clear comparison, 5 control points along y-axis are used to define the NLV of fibre angles for both VAT layups. The determined optimal variation of fibre angles using these two methods are slightly different, but gives nearly identical buckling loads. The optimized VAT plates demonstrate nearly 300% improvement on the buckling load over that of quasi-isotropic laminated plate. The evaluation of buckling load for each design is also validated by the FEM results (Abaqus).

A direct GA search approach requires many (population size $\times$ the number of generations) buckling analyse of VAT plates. The computational effort increases considerably when more design layers and more control points are used for the VAT layups design. Nevertheless, this issue is avoided in the two-level optimization strategy. For this problem, it needs less than 10 iterations (=times of buckling computation) to achieve the convergent result. In addition, the theoretically possible maximum buckling load of the VAT plate (constant thickness) problem is determined. The computational expense involved in the process of retrieving realistic layups from the resultant lamination parameters is much less and is not high when the design flexibility is extended.

Fig. 7 plots the spatially varying fibre angles of the two optimal VAT layers determined by the two-level design scheme (the 5 control point). Overall, the fibre orientation is monotonically increased from the bottom to the top. This variation of fibre angles lead to the compressive load redistributed towards the bottom edge (supported edge). It is also interesting to note that the fibre orientations are all approximately 0 degree near the bottom (simply-supported) edge for the inner layers($\theta_2$). The 0 degree fibres are useful for strengthening the plate, since the majority of compressive load is spread in this region.

### 6 Conclusions

The buckling resistance of a long variable angle tow (VAT) composite plate with one free edge (the others are simply-supported) is optimized using an effective two-level design approach. At the first-level, the VAT plate is designed with respect to the lamination parameters and at the second-level the optimal distributions of fibre angle for each layer are determined from the target lamination parameters. As different from the finite-element based approach, the distribution of spatially varying lamination parameters and fibre angles are both characterized by a set of pre-defined control points over the plate domain. The B-Spline basis functions and the Lagrangian polynomials are used to mathematically define the variations of lamination parameters and fibre angles, respectively. The control points based description scheme requires fewer design variables than finite-element approach and naturally results in smooth, continuous distributions.

Moreover, it was observed that both of two level optimization processes exhibit rapid convergences with few number of control points. Finally, a 16-ply optimal VAT laminate layups for the maximum buckling load is determined and compared with the results given by a direct GA search approach. In future work, the two-level design approach will be applied to optimize the buckling performance of VAT plates under different boundary conditions and load cases.
References


Table 1. Second-level GA optimization results using different number of control points for the definition of the NLV fibre orientation angles

<table>
<thead>
<tr>
<th>Number of control points</th>
<th>Minimum distance (Δξ)</th>
<th>Buckling load (N) (coefficient $K_a^*$)</th>
<th>Difference to target</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.458</td>
<td>3354 (3.73)</td>
<td>11%</td>
</tr>
<tr>
<td>5</td>
<td>0.3496</td>
<td>3549 (3.95)</td>
<td>5.7%</td>
</tr>
<tr>
<td>7</td>
<td>0.2828</td>
<td>3580 (3.98)</td>
<td>5.0%</td>
</tr>
<tr>
<td>9</td>
<td>0.2583</td>
<td>3594 (4.00)</td>
<td>4.5%</td>
</tr>
</tbody>
</table>

Table 2. Optimal layups for the maximum buckling load of a long SSSF 16-layer specially orthotropic laminates. The FEM results (Abaqus) are provided in parentheses for comparison.

<table>
<thead>
<tr>
<th>Optimal approach</th>
<th>Layups</th>
<th>Buckling load (N)</th>
<th>$K_a^*$</th>
<th>Increase (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>Quasi-Iso</td>
<td>897.85</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>[±45/±45]$_{AS}$</td>
<td>1527.6</td>
<td>1.70</td>
<td>70%</td>
</tr>
<tr>
<td>Direct GA</td>
<td>$\theta_1:T_{0.4} = [-11.5, 41.5, 56, 58, 65.5]$</td>
<td>3539 (3483)</td>
<td>3.94</td>
<td>294%</td>
</tr>
<tr>
<td></td>
<td>$\theta_2:T_{0.4} = [4, -20, -58, -67, -70]$</td>
<td>3549 (3489)</td>
<td>3.95</td>
<td>295%</td>
</tr>
<tr>
<td>Two-level</td>
<td>$\theta_1:T_{0.4} = [17.5, 36.5, 52.5, 56, 64]$</td>
<td>3539 (3483)</td>
<td>3.94</td>
<td>294%</td>
</tr>
<tr>
<td></td>
<td>$\theta_2:T_{0.4} = [-5, -11, -51, -65, -68]$</td>
<td>3549 (3489)</td>
<td>3.95</td>
<td>295%</td>
</tr>
</tbody>
</table>

Fig 1. Loading cases and geometry of a long VAT plates with one free edge

Fig 2. An illustration of using the B-Spline curve to represent the spatial variation of lamination parameters along y-axis
Fig 3. The uniformly distributed control points for defining the nonlinear variation (NLV) of fibre orientations. Left: varying along y-axis; Right: varying along both axes.

Fig 4. Convergence trends of the first level optimization process using different number of control points for constructing the B-Spline form variation of lamination parameters along y-axis

Fig 5. Optimal variations of the four lamination parameters ($A^{D}_{31,2}$) for the maximum buckling load of VAT plates with one free edge

Fig 6. The normalized lateral compressive load redistribution along y-axis

Fig 7 A segment of the NLV of fibre orientation angles: top $\theta_1(y)$ and bottom $\theta_2(y)$