CUMULATIVE FATIGUE DAMAGE PREDICTION OF COMPOSITE STRUCTURES

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Abstract
The Progressive Fatigue Damage Model (PFDM) developed by Shokrieh and Lessard employs a simplified cumulative damage rule. This paper investigates the use of Miner’s rule and a novel Degraded Strength Based Model for cumulative damage.

1 Introduction
Finite element analysis (FEA) has been widely used in composite structure design. The incorporation of the prediction of fatigue life in FEA is a logical step in the development of a general fatigue model for the prediction of fatigue life/durability of 2D and 3D composite structures. Several research groups [1-3] have reported the development of FE based integrated fatigue design tool. However, there were very limited validations. Furthermore, none of the tools has been verified by a second party.

Of the modeling strategies reported, the Progressive Fatigue Damage Model (PFDM), developed by Shokrieh and Lessard [1,4], promises to be the most architecturally independent for modeling laminated composite structures. Based on the classic laminate theory (CLT), the model uses the stiffness and strength degradation data generated with unidirectional (UD) composites to predict the property degradation in composite structures made of general laminates. The model parameters are available for a carbon/epoxy composite AS4/3501-6.

To examine the ability of PFDM, the model has been implemented in ABAQUS through its user material subroutine – UMAT [5]. Fatigue experiments have been conducted with center holed specimens made of AS4/3501-6 laminates of different lay-ups. The digital image correlation (DIC) technique was used to monitor the surface strain evolution during fatigue experiment [6]. The predicted surface strain evolutions with fatigue cycles compared well with the DIC data in earlier cycles before DIC measurement was disabled due to surface damage. In the later stage, however, the predicted compliance increases deviated from the experimental results [6]. We observed that while the PFDM provides a useful framework to model the gradual degradation of composite structures with arbitrary geometries and laminate lay-ups under fatigue loadings, it has a few key limitations: (1) it lacks a proper algorithm to account for the effect of fatigue damage accumulation on cycle life under variable amplitude loading conditions, i.e. a cumulative damage rule; and (2) it lacks the capability to model the delamination, an important fatigue damage mechanism in laminated composites.

In the PFDM, the gradual degradation empirical rules were based on UD data which only considered constant stress cycles. The original PFDM lacked the capability of accounting for cumulative damage under fatigue loading with a variable amplitude.

The CLT based failure prediction assumes that a lamina in a laminate would behave the same as it were a UD composite. This method was extended to fatigue failure prediction in PFDM. Fatigue damage processes in laminated composites have been investigated extensively. Reifsnider [7] has developed a diagram illustrating a 5-stage damage mode evolution during fatigue of laminated composites, in which the 3rd and 4th stages are dominated by delamination initiation and growth. The PFMD did not consider delamination or residual stresses, which significantly alter the initial stress state of the laminated composite.
This work looks to improve the PFDM by addressing the cumulative damage rule.

2 Model Descriptions

Figure 1 presents the flowchart of the PFDM model. The model is based on the progressive damage model for composite laminate with Hahsin Criteria.

![Flowchart of PFDM](image)

Table 1. Mechanical Properties of AS4/3501-6

<table>
<thead>
<tr>
<th>Static Material Constants</th>
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<tr>
<td>E&lt;sub&gt;xx&lt;/sub&gt;</td>
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<td>4.20E+07</td>
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</table>

2.1 Failure Criteria

For 3D composites, the following seven failure modes were considered:

Mode I, Fiber Tension (FT)

\[
\left( \frac{\sigma_{xx}}{X_T} \right)^2 \geq 1
\]

Mode II, Fiber Compression (FC)

\[
\left( \frac{\sigma_{xx}}{X_C} \right)^2 \geq 1
\]

Mode III, Fiber-Matrix Shear-out (FMS)

\[
\left( \frac{\sigma_{xx}}{X_C} \right)^2 + \left( \frac{\sigma_{xy}}{S_{xy}} \right)^2 + \left( \frac{\sigma_{xz}}{S_{xz}} \right)^2 \geq 1
\]

Mode IV, Matrix Tension cracking (MT)

\[
\left( \frac{\sigma_{xy}}{Y_T} \right)^2 + \left( \frac{\sigma_{yx}}{S_{xy}} \right)^2 + \left( \frac{\sigma_{yz}}{S_{yz}} \right)^2 \geq 1
\]

Mode V, Matrix Compression cracking (MC)

\[
\left( \frac{\sigma_{xy}}{Y_C} \right)^2 + \left( \frac{\sigma_{yx}}{S_{xy}} \right)^2 + \left( \frac{\sigma_{yz}}{S_{yz}} \right)^2 \geq 1
\]

Mode VI, Normal Tension (NT)

\[
\left( \frac{\sigma_{zz}}{Z_T} \right)^2 + \left( \frac{\sigma_{xz}}{S_{xz}} \right)^2 + \left( \frac{\sigma_{yz}}{S_{yz}} \right)^2 \geq 1
\]

Mode VII, Normal Compression (NC)

\[
\left( \frac{\sigma_{zz}}{Z_C} \right)^2 + \left( \frac{\sigma_{xz}}{S_{xz}} \right)^2 + \left( \frac{\sigma_{yz}}{S_{yz}} \right)^2 \geq 1
\]
In FE analysis, each element is examined against the failure criteria. If one of the seven criteria is met, the elastic property in that particular material direction will be degraded to 0.5% of its initial value. The element will be deleted if failure occurs in all directions. The mechanical properties of AS4/3501-6 is provided in Table 1.

2.2 Gradual Degradation

With the results of the initial loading step, the stress amplitude $\sigma_a$, mean stress $\sigma_m$ and stress ratio $R$ of each element will be determined at each material direction. With these values, the fatigue life $N_f$ for the element will be predicted for each direction from the master fatigue curve:

$$ u = \frac{\ln(a/j)}{\ln((1-q)(c+q))} = A + B \log N_f $$

(8)

where $a = \frac{\sigma_u}{\sigma_f}$, $q = \frac{\sigma_m}{\sigma_f}$, and $c = \frac{\sigma_c}{\sigma_f}$, $\sigma$ is the tensile strength, $\sigma_c$ is the compressive strength of the composite, A and B are constants obtained by curve fitting of experimental data.

Table 2 PFDM model parameters for AS4/3501-6 [1,4]

<table>
<thead>
<tr>
<th></th>
<th>Residual Stiffness Parameters</th>
<th>Residual Strength Parameters</th>
<th>Fatigue Life Parameters</th>
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<tbody>
<tr>
<td></td>
<td>$\lambda$</td>
<td>$\gamma$</td>
<td>$\epsilon_f$</td>
</tr>
<tr>
<td>Longitudinal Tensile</td>
<td>14.57</td>
<td>0.3024</td>
<td>0.0136</td>
</tr>
<tr>
<td>Longitudinal Compressive</td>
<td>14.57</td>
<td>0.3024</td>
<td>0.0136</td>
</tr>
<tr>
<td>Transverse Tensile</td>
<td>14.77</td>
<td>0.1155</td>
<td>0.0068</td>
</tr>
<tr>
<td>Transverse Compressive</td>
<td>14.77</td>
<td>0.1155</td>
<td>0.0068</td>
</tr>
<tr>
<td>In-Plane Shear</td>
<td>0.7</td>
<td>11</td>
<td>0.101</td>
</tr>
<tr>
<td>Out-of-Plane Shear</td>
<td>0.7</td>
<td>11</td>
<td>0.101</td>
</tr>
</tbody>
</table>

The analysis will then proceed with a predefined incremental number of cycles $\delta n$. The elastic moduli and strengths for the material volume of the element will be degraded according to the following empirical degradation rules:

$$ E^d = \left[ 1 - \left( \frac{\log(n) - \log(0.25)}{\log(N_f) - \log(0.25)} \right)^\lambda \right]^{\frac{\gamma}{\lambda}} (E' - \frac{\sigma}{\epsilon_f}) + \frac{\sigma}{\epsilon_f} $$

(9)

$$ R^d = \left[ 1 - \left( \frac{\log(n) - \log(0.25)}{\log(N_f) - \log(0.25)} \right)^\lambda \right]^{\frac{\beta}{\lambda}} (R' - \sigma) + \sigma $$

(10)

where $\alpha, \beta, \gamma, \lambda$ are obtained by curve fitting of experimental data. The values are given in Table 2.

After the gradual degradation for all elements in every direction has completed, the stress analysis will be updated with the new material information. The element stress will be compared with the failure criteria with the degraded strength values. This process continues until the structure is failed catastrophically or the overall compliance degrades below a certain value.

2.3 Load History Considerations

The empirical degradation rules as implemented in the PFDM [6] utilized the predicted fatigue life and current number of fatigue cycles, $n$, to determine the degraded material properties. As shown in Figure 1, the PFDM will utilize the stresses determined the Stress Analysis step, and assuming no sudden failure, the material is degraded according to
equations (9) and (10) using the total number of cycles $n_i$, instead of $\delta n$. For a material point under constant stress for all $n_i$ cycles, the above process (using the master fatigue curve and empirical degradation rules) are accurate in determining the material response. When modeling structural geometries that contain stress concentrations such as holes or notches, however, the progressive degradation and/or failure of elements around the feature will cause areas of increasing/decreasing stresses at local material points. Thus, it is imperative to investigate the means of incorporating the effects of load history. Numerous models have been developed in literature for predicting the response under variable amplitude loadings [10]. Two algorithms, one based on Palmgren-Miner’s linear degradation rule (Miner’s Rule) [11] and another novel scheme and implemented into the PFDM.

2.4 Palmgren-Miner Implementation

The Palmgren-Miner linear degradation rule, or Miner’s Rule, was one of the earliest developed damage accumulation models [12,13] and was chosen due to its simplicity. Miner’s Rule assumes the material accumulates damage according to a linear summation of the percentage of fatigue life ($N_i$) accumulated for a variety of stress levels [11]

$$D = \sum \frac{n_i}{N_i}$$

(11)

According to the relation, the material has reached its fatigue life when $D$ has reached a critical value, assumed to be one. The implementation of Miner’s Rule into the PFDM essentially provides a means of incrementing through the gradual degradation routines even when changing the state of stress between FE increments. At the beginning of each step, the ratio of fatigue life is computed as a unique state variable for each material direction. The state variable $N_{XX\_ratio}$, where $XX$ dictates material direction, is used to determine the “effective” number of cycles experienced under the current state of stress:

$$n_{i\_eff} = N_{XX\_ratio} \cdot N_f$$

(12)

where $N_{XX\_ratio}$ is updated at the beginning of each increment according to

$$(N_{XX\_ratio})^{i+1} = (N_{XX\_ratio})^i + \left(\frac{\delta n_{i\_eff}}{N_{f\_eff}}\right)$$

(13)

The residual strength curve plotted in Figure 2 shows an effective number of cycles achieved from the previous, $n_{i\_eff}$. The residual strength is now computed by advancing by $\delta n$ along the residual strength curve. Thus, $n$ in equations (9) and (10) is replaced by $n_{i\_eff} + \delta n_{i\_eff}$ for the next $i+1$ increment.

Fig.2 Example of the transverse residual strength curves as a function of the normalized number of cycles for the AS4/3501-6 composite showing the “effective number of cycles computed for a previous stress state.

2.5 Degraded Strength Based Model

An alternative method is proposed that allows one to leverage the inherent benefits of the PFDM and the residual properties computed using the gradual degradation rules and is referred to as the Degraded Strength Based Model. Whereas the Miner’s Rule implementation computed the “effective” number of cycles according to the ratio of fatigue life encumbered in the previous step, this method will determine the “effective” number of cycles by enforcing continuity of the residual strengths across cycles of varying stress levels. By solving equation (10) for $n$ and setting $R$ equal to the value at the end of the previous increment, the “effective” number of cycles converted to the current stress space is computed:
In this method, no additional state variables are required, since the residual strength from the previous increment is already stored in the PFDM and is all that is required in equation (14). Note that although continuity is constrained with respect to the residual strength, continuity with respect to the moduli is not preserved. Thus, the predicted stiffness at the end of \( n_i \) cycles may not be equivalent to the stiffness predicted at \( n_i \) effective cycles. The choice of enforcing strengths was chosen arbitrarily and will be under consideration in future works. Similar to the Palmgren-Miner implementation, \( n_i \) in equations (9) and (10) is replaced by \( n_i \) effective + \( \delta n \) for the next \( i+1 \) increment.

\[
n_{i, \text{eff}} = \exp[(1 - \frac{R_d^0 + \sigma}{R^0 + \sigma})^\gamma \beta (N_{i+1}) \ldots (1 - \frac{R_d^0 + \sigma}{R^0 + \sigma})^\gamma \beta \ln(0.25)] \cdot 0.25
\]

\[\text{(14)}\]

3 Results and Discussion

The effects of accounting for load history by utilizing the two methods discussed previously are explored here numerically. Future experimental work is currently under development to validate the model. A single cubic element, C3D8, is used as a verification of the Palmgren-Miner and Degraded Strength Based Model. Two states of uniaxial stress are utilized to compare the two methodologies against the original PFDM. The first is uniaxial tension in the matrix, or transverse, direction. Two sets of load blocks are explored, one with increasing stress and another with decreasing stress. A total of 100,000 cycles are applied with load blocks changing every 20,000 cycles. The percentage of ultimate tensile strength (UTS) of the material used in the tensile loading are listed in the figure descriptions.

Figure 3 presents the results for the residual transverse tensile strength of a single integration point in the C3D8 element as a result of uniaxial transverse tension. As expected, the PFDM (Original) under-predicts the number of cycles to failure in comparison to the two other methodologies. Before fatigue failure, however, the residual strengths surprisingly do not vary between all three curves. The first result, premature failure in the PFDM, was expected, since the empirical degradation routines are utilizing the total number of cycles, rather than an effective number of cycles when stepping through increments. It should be noted that based on the last load cycle, the DSBM model still had 30,000 cycles remaining before reaching fatigue failure, while the PFDM had already reached failure prior to the stop time of 100,000 cycles.

Fig.3 Residual strength curves for transverse tension of the single element verification for increasing stress load blocks (43% to 60% UTS)

Fig.4 Residual stiffness curves for transverse tension of the single element verification for increasing stress load blocks (43% to 60% UTS)

Figure 4 presents the results from the transverse residual stiffness curves. The Miner’s Rule implementation under-predicted the degradation of the transverse tensile modulus, whereas both the original PFDM and DSBM had similar degradation curves prior to the premature failure of the PFDM.
For the case of decreasing stress load blocks shown in Figures 5 and 6, the Miner’s Rule implementation was found to have pronounced degradation not seen in that of the PFDM and SBDM. This was the case for both the residual stiffness and residual strength. The discrepancy was found to be due to the lack of continuity in stiffness and/or strength across the load blocks, which caused the Miner’s Rule to move further along the degradation curves.

The results for the single element test under 1H2 shear loading for increasing load blocks is shown in Figures 7 and 8. Due to the differences between the transverse tensile degradation curves and the 1H2 shear degradation curves, the trends observed in the material degradation from Figures 3 and 4 no longer hold. For example, the Miner’s rule implementation now over-predicts the material degradation in both residual strength and residual stiffness. Similar to the results from the transverse tensile cases, the original PFDM reached levels of critical stiffness/strength degradation sooner than the DSMB. Again, this phenomenon is attributed to the degradation of the shear degradation rules being less sensitive with respect to decreasing load blocks than the transverse tensile direction.
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4 Conclusion

The results from the single element verification have found that the material degradation obtained from the PFDM corresponded well to that of the DSBM which considers load history. The PFDM, however, would tend to under-predict fatigue life in the case of increasing material point stresses with increasing cycles. The Miner’s Rule implementation was found to either under-predict or over-predict the material degradation depending on the shape of the degradation curves (which vary across between material directions). For low cycle fatigue, which may fail from material stresses reaching the sudden failure criterion, the PFDM as implemented may be adequate for full coupon testing including stress concentrations. For high cycle fatigue, which will induce fatigue failure (reaching $N$) at a material point, the PFDM will tend to cause pre-mature fatigue failure. The Strength Based Degradation Model showed promise for the case of increasing material stresses during the fatigue cycling, while the effects are more pronounced in certain material directions than other.

The resulting changes to the PFDM will better allow for finer incrementation in the FE fatigue analysis, without introducing the effects of low-high local load cycles due to present stress concentrations in the structural geometry. Future work will hope to validate the improvements through the use of experimental fatigue testing, under a variety of variable amplitude fatigue loading for the AS4/3501-6 composite. Additionally, delamination modeling and residual stress considerations are currently underway.

5 Acknowledgements

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References


