UNBALANCED AND SYMMETRIC LAMINATES: NEW PERSPECTIVES ON A LESS COMMON DESIGN RULE.

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Keywords: Warp-free, Tapered, Non-symmetric, Extension-Shearing Bending-Twisting Coupled Laminates

1 Abstract

The definite list of extension-shearing and bending-twisting coupled composite laminates, with up to 21 plies, is presented. The listings comprise of individual stacking sequences, which are characterized in terms of angle- and cross-ply subsequence symmetries as well as the blend-ratio of un-balanced angle-plies. Dimensionless parameters are provided, from which the extensional and bending stiffnesses are readily calculated for any fibre/matrix system and angle-ply orientation. Comparisons are made between the structural response of the new extension-shearing and bending-twisting coupled laminates and others possessing both extension-shearing coupling only or bending-twisting coupling only, which typically arise in un-balanced and symmetric or balanced and symmetric designs, respectively. These comparisons involve the buckling interaction of infinitely long plates, which serve to isolate the effects of each form of coupling behaviour and highlight the detrimental effects of bending-twisting coupling on compression buckling strength as well as the beneficial effects on shear buckling strength. Finally, the scope for laminate tapering is investigated, using single or multiple ply terminations.

2 Introduction

Symmetric stacking sequences are ubiquitous in modern composite laminate design practice, for the simple reason that their use guarantees the laminate remains flat, or the desired shape, after high temperature curing. Non-symmetric laminates are commonly associated with, or often (incorrectly) used to describe, configurations that warp extensively after high temperature curing, for which the deformed shape is also difficult to predict reliably: requiring non-linear analysis techniques. Balanced stacking sequences guarantee that extension-shearing coupling, or anisotropy, is eliminated by using matching pairs of angle-ply layers; such that when the material and structural coordinate systems are coincident, each positive angle-ply orientation, $\theta^\circ$, is matched by a negative angle-ply orientation, -$\theta^\circ$. Unbalanced stacking sequences, with different numbers of positive and negative angle-ply orientations, will, and without exception, give rise to extension-shearing coupling. This article will demonstrate that there exists a vast and unexplored design space containing non-symmetric laminates that do not possess thermal and/or mechanical coupling behaviour, as common design rules lead us to believe, and that mechanical coupling can be achieved without thermal warping distortions. To this end, new laminate configurations are derived with immunity to (out-of-plane) thermal warping distortions due to high temperature curing and in-service temperature fluctuations. Additionally, results are presented for tapered laminate designs with immunity to changes in mechanical and thermal behaviour resulting from ply terminations. The results presented here, bring to a conclusion a study on four very specific laminate classes, shown in Fig. 1, all of which are immune to thermal warping distortions by virtue of the fact that their coupling stiffness properties are null; as would be expected from symmetric laminate configurations. The first two classes contain balanced angle-ply layers, leading to uncoupled extensional stiffness properties. The Simple laminate in the first column is also uncoupled in bending [1], whilst the laminate class in the second column possesses bending-twisting coupling [2]. The final two laminate classes
possess unbalanced angle-ply layers, leading to extension-shearing coupling properties. The laminate class in the third column is uncoupled in bending [3], whereas the laminate class in the fourth column, representing the subject of the current article, has both extension-shearing and bending-twisting coupling, as would arise from unbalanced and symmetric laminates; a less well-known and rather specialized design rule. It should be emphasized that unbalanced laminates, otherwise referred to as extension-shearing coupled laminates, remain warp free for all solutions derived here, irrespective of the number of layers in the laminate. This is in marked contrast to other recent studies [4], where approximate solutions have been derived, which convergence towards thermo-mechanically curvature-stable behaviour only when the number of layers in the laminate becomes large. The results of this article, as with the related articles that have preceded it, demonstrate that the notion of symmetric laminates is not only a simplifying assumption, but a significant oversight of the potential design space, in which symmetric stacking sequences are the exception rather than the rule.

Recent research on thin laminates with up to 21 plies, has demonstrated that it is possible to meet the industrial requirements for the Simple laminate class shown in Fig. 1, with non-symmetric designs. Indeed, symmetric laminates were found to offer no benefit over their non-symmetric counterparts in terms of tapering possibilities [5], buckling strength [2] or damage tolerance [6]. In fact there is an increasing realization within the aerospace industry that the balanced and symmetric design rule produces sub-optimal laminate designs with: reduced compression buckling strength due to presence of bending-twisting coupling and; restricted design freedom, particularly in the context of tapered laminates where each angle-ply termination requires three additional angle-ply terminations to retain the balanced and symmetric form of the laminate.

For these reasons, un-balanced laminate designs are beginning to attract interest, particularly in the context of thickness (hence weight) reduction in wing-box skins; the hypothesis being that the extension-shearing coupling effects that arise in such designs can be eliminated by careful matching, or balancing, of the upper and lower skin properties of the wing-box. By contrast, mismatching these properties can induce bending-twisting behaviour at the wing-box level: which may serve as a passive load alleviation or drag reduction mechanism, see Fig. 2.

3 Derivation of stacking sequences

The key features of the extension-shearing and bending-twisting coupled laminate class is that they are decoupled, i.e. \( B_{ij} = 0 \); hence in-plane and out-of-plane behavior can be treated separately. The constitutive relations therefore simplify as follows:

\[
\begin{align*}
\{N_x\} &= \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{22} & A_{26} & 0 \\ A_{66} & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \\
\{N_{xy}\} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
\{M_x\} &= \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{22} & D_{26} & 0 \\ D_{66} & 0 & 0 \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}
\end{align*}
\]  

(1)

where the elements of the stiffness matrices are derived from the well know relationships:

\[
\begin{align*}
A_{ij} &= \sum_{k=1}^{n} Q_{ij}^r(z_k - z_{k-1}) \\
B_{ij} &= \sum_{k=1}^{n} Q_{ij}^r(z_k^2 - z_{k-1}^2)/2 = 0 \\
D_{ij} &= \sum_{k=1}^{n} Q_{ij}^r(z_k^3 - z_{k-1}^3)/3
\end{align*}
\]  

(2)

in which: the summations extend over all \( n \) plies; \( Q_{ij}^r \) are the transformed reduced stiffnesses (i.e., \( j = 1, 2, 6 \)) and; \( z \) represents the distance of the \( k^{\text{th}} \) ply from the laminate mid-plane.

In the derivation of the stacking sequences that follow, which assume (but are not restricted to) combinations of standard fibre angle orientations, i.e. \( 0, 90 \) and/or \( \pm 45^\circ (= \pm \theta^\circ) \), the general rule of symmetry is relaxed. Cross plies, as well as angle plies, are therefore not constrained to be symmetric about the laminate mid-plane.

The derivation involves the added restrictions that each layer in the laminate:

- has identical material properties;
- has identical thickness, \( t \), and;
- differs only by its orientation, \( \theta \).
For compatibility with the previously published data [1]-[3], similar symbols have been adopted for defining all stacking sequences, i.e., O, ●, + and − are used in place of standard ply angle orientations 0, 90, +45 and −45°, respectively.

The resulting sequences are characterized by sub-sequence symmetries using a double prefix notation, the first character of which relates to the form of the angle-ply sub-sequence and the second character to the cross-ply sub-sequence. The double prefix contains combinations of the following characters: A to indicate Anti-symmetric form; N for Non-symmetric; and S for Symmetric. Additionally, for cross-ply sub-sequence only, C is used to indicate Cross-symmetric form.

To avoid the trivial solution of a stacking sequences with cross plies only, all sequences have an angle-ply (+) on one outer surface of the laminate. As a result, the other outer surface may have an angle-ply of equal (+) or opposite (−) orientation or a cross ply (O), which may be either 0 or 90°.

### 3.1 Non-dimensional parameters

Non-dimensional parameters allow the extensional and bending stiffness properties to be readily calculated for any fibre/matrix system and angle-ply orientation and provide a compact data set alongside each extension-shearing and bending-twisting coupled laminate stacking sequence derived.

The development of non-dimensional parameters is demonstrated, by way of an example for a 15-ply non-symmetric laminate stacking sequence [+/−/+O/+O/+O/+/−/+−/++/+/−/+/+/−/+/+/−/+/+/−/+/+/−/+/+/−/+/+/−/+/+/−/+/+/−/+/+/−], in Table 1. The first two columns of Table 1 provide the ply number and orientation, respectively, whilst subsequent columns illustrate the summations, for each ply orientation, of (z_k − z_k−1), (z_k^2 − z_k−1^2) and (z_k^3 − z_k−1^3), relating to the A, B and D matrices, respectively. Here, the distance from the laminate mid-plane, z, is expressed in terms of ply thickness t, assumed to be unit value.

The non-dimensional parameters arising from the tabular summations are as follows. For the extension stiffness matrix [A]: n_O, the number of (0° or 90°) cross plies (= λΣ_O) = 3, n_, the number of negative angle plies (= λΣ_) = 3, n_+ the number of positive angle plies (= λΣ_+) = 9. The coupling stiffness matrix [B] summations confirm that B_0 = 0 for this laminate. For the bending stiffness matrix [D]: ζ_n, the bending stiffness parameter for (0° or 90°) cross plies (= 4 × Σ_O = 4 × 169.25) = 675, ζ_−, the bending stiffness parameter for negative angle plies (= 4 × Σ_− = 4 × 169.25) = 675, ζ_+, the bending stiffness parameter for positive angle plies (= 4 × Σ_+ = 4 × 506.25) = 2025, where n^3 = 15^3 = ζ = ζ_− + ζ_+ + ζ_+ = 3375.

This, and all other laminates presented here, satisfy the following non-dimensional parameter criteria:

\[ n_+ ≠ n_− \]

\[ ζ_O (= ζ_+) = ζ_− = 0 \]

\[ ζ_− ≠ ζ_+ \]

These non-dimensional parameters, together with the transformed reduced stiffness, Q_{ij}, for each ply orientation of constant ply thickness, t, facilitate simple calculation of the elements of the extensional, coupling and bending stiffness matrices from:

\[ A_{ij} = \{ n_O Q_{ij}^O + n_− Q_{ij}− − n_+ Q_{ij}+ + n_+ Q_{ij}+ \} t \]

\[ B_{ij} = 0 \]

\[ D_{ij} = \{ ζ_O Q_{ij}^O + ζ_− Q_{ij}− + ζ_+ Q_{ij}+ + ζ_+ Q_{ij}+ \} t^2/12 \]

### 4 Results

#### 4.1 Stacking sequences and sub-sequence symmetries

The total (Σ) number of extension-shearing and bending-twisting or E-S-B-T coupled laminate stacking sequences for each ply number grouping, n, with up to (n =) 16 plies, are given in Table 2, together with the number is each sub-symmetric group. Example stacking sequences, representing the minimum number of plies, n, within each sub-symmetric group, are given in the footnote to Table 2.

An abridged listing of stacking sequences is presented in Table 5 together with non-dimensional parameters from which the A and D matrices are readily derived using Eqs (4). Here, (n =) 15 ply laminates were chosen because this is the lowest ply number grouping to contain all possible forms of sub-sequence symmetry. The sub-sequence symmetries for AC, AN and AS laminate stacking sequences with 15 plies are noteworthy because all possess approximately equal bending stiffness contributions from both positive
and negative angle plies. They therefore share similar properties to the exact Extension-Shearing or $E-S$ coupled laminates, derived previously [3], for which only even ply number groupings were found to exist. This close approximation can be explained by the fact that odd ply number groupings with anti-symmetric angle-ply sub-symmetries all contain angle plies at the laminate mid-plane.

**4.2 Blend ratio**

Blend ratio was defined previously [3], for the characterization of $E-S$ coupled laminates, as the proportion of positive angle plies with respect to the total number of angle plies:

$$n_p/n_k$$

expressed as a percentage. Only three blend ratios were identified in laminates with up to 21 plies $E-S$ coupled laminates. The same definition is applied to $E-S-B-T$ coupled laminates, but in this case, a very broad and varied range of blend ratios are present. The abridged listing of stacking sequences in Table 5 have been ordered by ascending blend ratio and then by ascending values of the non-dimensional bending stiffness parameter ($\zeta$), corresponding to the angle plies and finally, but the non-dimensional bending stiffness parameter ($\zeta$), corresponding to the positive angle plies. The justification for this ordering follows from the simple laminate class [1], where the highest bending stiffness parameter ($\zeta$) corresponds to the highest compression buckling strength within each ply number grouping. However, due to the unbalanced nature of the $E-S-B-T$ laminate, this ordering also accounts for shear buckling strength, as seen in the buckling interaction curves that follow.

It should be noted that a blend ratio for bending could be defined as:

$$\zeta_p/\zeta_k$$

thus providing a ready measure of the bending stiffness contribution of bias angle plies to that from both angle plies. However, this is unrevealing in the assessment of buckling strength without an addition parameter ratio:

$$\zeta_p/\zeta_k$$

providing a measure of the bending stiffness contribution of angle plies to the total bending stiffness; found to be proportional to the compression buckling strength in uncoupled laminates. In fact the bending stiffness properties of bias angle plies have a significant effect on buckling performance under shear, especially where load reversal is a design constraint.

Blend ratio ($n_p/n_k$) provides an assessment of the magnitude of the laminate level Extension-Shearing coupling, which can be used to induce Bending-Twisting coupling at the wing box level, see Fig. 2. It has no relevance to assessment of buckling strength, particularly in view of the fact that two laminates with the same blend ratio may have very different bending stiffness properties.

**4.3 Buckling interaction**

Buckling interaction curves of Fig. 3 permit an assessment of the beneficial effects of mechanical coupling behaviour under various combinations of shear and compression loading. The results correspond to an infinitely long plate with simply supported edges, thus providing a lower-bound solution, useful in preliminary design. Laminate stacking sequences have been carefully chosen to isolate specific forms of mechanical coupling and with precisely matching stiffness properties between each comparator. In Fig. 3(a) an 18-ply quasi-homogeneous orthotropic laminate was chosen as a datum laminate, since it represents the simplest form of uncoupled laminate, i.e.:

$$D_{ij} = A_{ij}H^2/12$$  

and since it becomes a fully isotropic ($\pi/3$) laminate ($A_{ijkl}D_{ijkl}$) when angle plies (+/-) are changed from +45°/–45° to +60°/–60°, thus permitting normalization of the buckling load with respect to $D_{iso}$, which corresponds to $D_{11}$ of the datum laminate when symbols +/– represent +60°/–60°/0°, respectively.

For laminates with other ply number groupings, and for which no fully isotropic laminate exist, normalization is performed using the following equation:

$$D_{iso} = E_{iso}H^3/(1 - V_{iso}^2)/12$$

where

$$E_{iso} = 2(1 + V_{iso})G_{iso},$$

$$G_{iso} = (Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66})/8,$$

$$V_{iso} = (Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66})/(3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}).$$
and $Q_i$ are the reduced stiffnesses.

For ease of comparison with other buckling studies in the literature, the laminate stiffness properties are presented in non-dimensional form using the following parameters \([8]\):

$$\beta = \frac{(D_{12} + 2D_{66})/(D_{11}D_{22})^{1/2}}$$

$$\gamma = \frac{D_{16}/(D_{11}D_{22})^{1/4}}$$

$$\delta = \frac{D_{26}/(D_{11}D_{22})^{1/4}}$$

These parameters are presented in Table 3 for each of the laminate stacking sequences that follow, together with selected buckling factor results for benchmarking purposes.

Datum laminate:

\[ [+/-|-]/O/O/O/+/-/O/+/-/-/-/-/-/O/O/+ ]_T \]

\( (A_3 B_3 D_3) \)

Two laminates with Bending-Twisting coupling were chosen with identical stiffness (A and D) to the quasi-homogeneous laminate \((A_3 B_3 D_3)\), except \(D_{16} = D_{26} \neq 0\), and for their maximum and minimum Bending-Twisting coupling magnitudes, respectively, within this ply number grouping.

**B-T coupled laminates:**

\[ [+/-/+]/O/-/-/+/O/-/-/-/-/-/+/O/+ ]_T \]

\( (A_3 B_3 D_3 #1) \)

\[ [+/-|-]/O/-/-/+/O/-/-/-/-/-/+/O/+ ]_T \]

\( (A_3 B_3 D_3 #2) \)

The buckling envelopes consist of a number of intersecting curves, each corresponding to a specific buckling mode with half-wave \( \lambda \). For the fully isotropic laminate \((A, B, D)\), the compression buckling factor, \( k_c = 4.00\), corresponds to \( \lambda = h\), i.e., the plate width, whereas the shear buckling factor, \( k_s = 5.34\), corresponds to \( \lambda = 1.25h\).

These comparators serve to isolate the effect of Extension-Shearing and Bending-Twisting coupling, and demonstrate the beneficial effects on shear buckling strength when the dominant angle-ply orientation and the principal compressive stress direction are perpendicular. Additional comparisons are presented in Fig. 3(b) where the Extension-Shearing coupled laminate has identical stiffness (A and D) to the Extension-Shearing and Bending-Twisting coupled laminate, except that \( D_{16} = D_{26} \neq 0\) in the latter. A Simple laminate with matching bending stiffness (D) only provides the Simple or fully uncoupled datum laminate.

**E-S coupled laminate:**

\[ [+/-|-]/-/-/+/O/-/-/+/O/-/+ ]_T \]

\( (A_3 B_3 D_3) \)

**E-S-B-T coupled laminate:**

\[ [+/-|-]/-/+/-/O/-/-/+/O/-/+ ]_T \]

\( (A_3 B_3 D_3) \)

Simple laminate:

\[ [+/-]/O/O ]_\lambda \]

\( (A_3 B_3 D_3) \)

These comparators serve to isolate the effect of Extension-Shearing, previously shown \([3]\) to give rise to virtually identical bending-twisting coupling response at the wing-box level, but here demonstrating that this has no influence on buckling strength; the simple laminate with matching bending stiffness properties, shares exactly the same buckling envelope. Hence the differences in the buckling envelopes of the **E-S** and **E-S-B-T** coupled laminates are entirely due to the effect of laminate level Bending-Twisting coupling, which gives rise to similar beneficial effects, in shear, seen in the B-T coupled laminates of Fig. 3(a), but also the detrimental effects in compression buckling strength, cf. Table 3.

### 4.4 Laminate Tapering

Tapered designs are certified for symmetric laminate construction, the majority of which possess bending-twisting coupling, but such designs have a severe design constraint, i.e., a single angle-ply termination requires a further three angle-ply terminations to maintain balanced and symmetric construction. This section investigates the extent to which this restriction can be overcome using extension-shearing and bending-twisting coupled laminates.

Tapered laminate designs have been developed in a two stage process, using the new definitive stacking sequence listing. The first stage can be described as a top down process, in which each ply number grouping \((n)\) is algorithmically filtered against ply number groupings with fewer plies, i.e., representing one \((n-1)\), two \((n-2)\), etc., ply terminations. The first, or upper surface ply is assumed to be continuous at the end of the tapering process, but no restriction is made on other plies, including the last, or lower surface ply. The results from this first stage provide a reduced design space of compatible stacking sequences for each ply number grouping. The second stage, which can be described as a bottom up process, begins with the compatible stacking sequences representing the minimum ply...
number grouping of interest. These sequences are then algorithmically filtered against higher ply number groupings, in turn, where only compatible sequences with the minimum ply number grouping are retained.

This process is demonstrated in Table 4 for single ply terminations for ply number groupings with between \((n = 3)\) and \(12\) plies. The number of stacking sequences from the definitive listing are presented in column 2 of Table 4; noting that stacking sequences with only a single ply orientation, i.e.: sequences of the form \([+/[+1/0]+]\), have been removed. Column 3 lists the total number of compatible single-ply terminations following the top down process of the first stage. Also given is an indication of which ply orientations are terminated, representing the number of cross-ply, positive and negative angle-ply (\(\bigcirc\) or \(\bullet\), +, –), respectively. Column 4 lists the number of individual sequences used, given that compatible sequences may result from the termination of different plies (or ply combinations). Column 5 presents the number of compatible solutions in each ply number grouping, \(n\), leading to a tapered sequence from \(n = 3\) to \(n = 12\). By contrast, column 7 presents the number of compatible solutions in each ply number grouping, \(n\), leading to a tapered sequence from \(n = 8\) to \(n = 12\). Columns 6 and 8 represent a reduced data set, with respect to the original data set in column 3, due to the bottom up approach of stage 2.

A recent study on bi-angle non-crimp-fabric (NCF) designs has now led to commercially available forms, with the claim that wet lay-up times will be reduced by half; in comparison to tradition UD tape. Here single ply terminations, , can be achieved by hybrid designs, i.e., using a combination of bi-angle \([0/0]\) NCF and UD plies, as illustrated in the following example of a \((n = 14)\) ply laminate, tapered to \((n = 8)\) plies:

\[
\begin{align*}
&[0/0/0/0/0/0/0/0/0/0/0/0/0/0/0]_T \\
&[0/0/0/0/0/0/0/0/0/0/0/0/0/0/0]_T \\
&[0/0/0/0/0/0/0/0/0/0/0/0/0/0/0]_T \\
&[0/0/0/0/0/0/0/0/0/0/0/0/0/0/0]_T \\
&[0/0/0/0/0/0/0/0/0/0/0/0/0/0/0]_T \\
&[0/0/0/0/0/0/0/0/0/0/0/0/0/0/0]_T
\end{align*}
\]

This single ply termination design highlights a simple pattern that can be developed for higher ply number grouping, e.g. with up to \((n = 16)\) plies, by the introduction of consecutive pairs of cross- or angle-plies at the laminate mid-plane to maintain symmetry within the central ply block, e.g.:

\[
\begin{align*}
&[0/0/0/0/0/0/0/0/0/0/0/0/0/0/0]_T \\
&[0/0/0/0/0/0/0/0/0/0/0/0/0/0/0]_T
\end{align*}
\]

or

\[
\begin{align*}
&[0/0/0/0/0/0/0/0/0/0/0/0/0/0/0]_T \\
&[0/0/0/0/0/0/0/0/0/0/0/0/0/0/0]_T \\
&[0/0/0/0/0/0/0/0/0/0/0/0/0/0/0]_T \\
&[0/0/0/0/0/0/0/0/0/0/0/0/0/0/0]_T
\end{align*}
\]

All tapered results with single ply terminations appear to be restricted to a mid-plane symmetric ply block for all ply number groupings up to and including \((n = 16)\) plies. However, Tapering from other ply number grouping may reveal different forms of sub-sequence symmetries.

A 4-ply NCF: \([0/0/0/0]_T\) offers similar tapering possibilities, where multiple terminations are permissible, e.g., a \((n = 16)\) ply laminate tapered to \((n = 8)\) plies:

\[
\begin{align*}
&[0/0/0/0/0/0/0/0/0/0/0/0/0/0/0]_T \\
&[0/0/0/0/0/0/0/0/0/0/0/0/0/0/0]_T \\
&[0/0/0/0/0/0/0/0/0/0/0/0/0/0/0]_T \\
&[0/0/0/0/0/0/0/0/0/0/0/0/0/0/0]_T
\end{align*}
\]

whereby the 16 ply stacking sequence is the 8-ply stacking sequence repeated; both are quasi-homogeneous anisotropic laminates and with matching \(A* = (A_H/\rho) = D* = (12D_H/\rho)^3\). This relationship is however broken by the intermediate 12 ply laminate.

The recent emergence of thin-ply ‘spread-tow’ technology offers the prospect of an 8-ply NCF stacking sequence \([0/0/0/0/0/0/0/0]_T\) with the same thickness as a single ply of traditional UD material. This design also follows the bi-angle philosophy proposed by Tsai [4], but with immunity to thermal warping distortions irrespective of the number of NCF layers. However, the buckling strength of such designs, with standard \((0^\circ, 45^\circ)\) or non-standard shallow angles \((0^\circ, 22.5^\circ)\), are less favourable than laminates containing a more traditional mixture of fibre orientations, see Fig. 3(c).

Examples of two ply terminations follow, which assume standard UD materials and standard ply angle combinations; many produce rather different forms of non-symmetric tapered laminates.
In this first example, angle-ply layers are terminated in symmetric and non-symmetric pairs, including the lower surface angle-ply layer:

\[
\begin{align*}
&\{+/+\}T/\{0\}T/\{+\}T/\{+/+\}T/\{0\}T/\{+/+\}T/\{0\}T/\{+/+\}T \\
&\{+/+\}T/\{0\}T/\{+\}T/\{+/+\}T/\{0\}T/\{+/+\}T/\{0\}T/\{+/+\}T
\end{align*}
\]

A second example introduces blend ratio changes by terminating a symmetric angle-ply pair (at the laminate mid-plane) and a non-symmetric angle-ply pair (one ply from the lower surface):

\[
\begin{align*}
&\{+/+\}T/\{0\}T/\{+\}T/\{+/+\}T/\{0\}T/\{+/+\}T/\{0\}T/\{+/+\}T \\
&\{+/+\}T/\{0\}T/\{+\}T/\{+/+\}T/\{0\}T/\{+/+\}T/\{0\}T/\{+/+\}T
\end{align*}
\]

A final example involves the termination of cross plies in a non-symmetric pattern, with angle-ply terminated symmetrically to introduce a blend ratio change:

\[
\begin{align*}
&\{+/0\}T/\{0\}T/\{+\}T/\{+/0\}T/\{0\}T/\{+/0\}T/\{0\}T/\{+/0\}T \\
&\{+/0\}T/\{0\}T/\{+\}T/\{+/0\}T/\{0\}T/\{+/0\}T/\{0\}T/\{+/0\}T
\end{align*}
\]

5 Conclusions

- Thin laminates must exploit non-symmetric and unbalanced stacking sequence configurations to fully exploit the available design space, particularly where tapered designs are required.
- Tapered laminate solutions have been demonstrated in non-symmetric laminates, with immunity to thermal warping and consistent mechanical coupling properties.
- A simple design rule has been identified in which consistent mechanical extension-shear and bending-twisting coupling can be preserved by modifying a mid-plane symmetric ply block, within an otherwise non-symmetric laminate configuration.
- Non-crimp fabric designs and hybrid bi-angle and UD layers have also been demonstrated. However, the buckling strength of such designs, with standard, and especially shallow shapes, are less favourable than laminates containing a more traditional mixture of fibre orientations.
- Tapered designs with 8-ply NCF have been proposed, but exploiting thin-ply technologies; which will facilitate a significant reduction in overall laminate thickness, and therefore allow an exponential increase in tailoring opportunities by bringing design flexibilities found only in traditionally thick laminate construction into the thin laminate domain.

References

THE 19\textsuperscript{th} INTERNATIONAL CONFERENCE ON COMPOSITE MATERIALS

Uncoupled in Extension

Uncoupled in Bending

Bending-Twisting

Extension-Shearing

Uncoupled in Bending

Bending-Twisting

\[ [+/\pm/O/\pm/O]_T \]

\[ B-T \text{ coupled laminate} \]

\[ E-S \text{ coupled laminate} \]

\[ E-S;B-T \text{ coupled laminate} \]

\[ [+/+]_T \]

\[ \text{Simple laminate} \]

\[ \text{B-T coupled laminate} \]

\[ \text{E-S coupled laminate} \]

\[ \text{E-S;B-T coupled laminate} \]

Figure 1 – In-plane thermal contraction responses (exaggerated) resulting from a typical high temperature curing process. All examples shown are square, initially flat, composite laminates. The stacking sequences provided, in symbolic form, are representative of the minimum ply number grouping for each laminate class, with standard ply orientations \( \pm 45, 0 \) and \( 90^\circ \) in place of symbols +, −, \( O \) and \( \bullet \), respectively.

Figure 2 – Illustration of (a) extension-shearing coupling, as a result of fully populated extensional stiffness matrix (\( A_F \)), producing (b) bend-twist deformation in aircraft wing-box structures when top and bottom skins have identical alignment of bias angle plies.

Table 1 – Development of non-dimensional parameters for a Extension-Shearing and Bending-Twisting coupled (\( A_F B_D D_F \)) laminates.

<table>
<thead>
<tr>
<th>Ply</th>
<th>( \theta )</th>
<th>( \Sigma^2 \sum_{-z_{k-1}}^{z_k} )</th>
<th>( \Sigma^2 \sum_{-z_{k+1}}^{z_k} )</th>
<th>( \Sigma^2 \sum_{-z_{k+1}}^{z_k} )</th>
<th>( [A] )</th>
<th>( [B] )</th>
<th>( [D] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+</td>
<td>147.25</td>
<td>147.25</td>
<td>147.25</td>
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<tr>
<td>2</td>
<td>−</td>
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<td>108.25</td>
<td>108.25</td>
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<tr>
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<td>48.25</td>
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</table>
Table 2 – Number of \( E\cdot S\cdot B\cdot T \) coupled laminate stacking sequences for each ply number grouping, \( n \), arranged by sub-sequence symmetry using a double prefix notation: 1\(^{st} \) prefix for Angle-ply sub-sequence; 2\(^{nd} \) prefix for Cross-ply sub-sequence. Example stacking sequences, representing the minimum number of plies within each symmetry grouping, are given in the footnote. Symmetric laminates of the form \([+/...+]/\) have been disregarded.

\[
\begin{array}{cccccccccccccccccc}
 n & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \\
 NN & - & - & - & 8 & 2 & 66 & 32 & 676 & 280 & 6,184 & 2,744 & 56,738 & 24,622 & -
 NS & - & - & - & - & 16 & 8 & 168 & 74 & 1,176 & 582 & 7,960 & 4,120 & - & -
 SS & 3 & 2 & 13 & 11 & 55 & 48 & 225 & 199 & 911 & 813 & 3,675 & 3,303 & 14,799 & 13,380 & -
 \hline
 \Sigma & 3 & 2 & 13 & 11 & 65 & 50 & 321 & 239 & 1,811 & 1,191 & 11,651 & 6,847 & 83,573 & 43,830 & -
\end{array}
\]

A – Anti-symmetric; C – Cross-symmetric; N – Non-symmetric; S – Symmetric

AC: \([+/O/+/O/+/O/+/O/+/O/+/O/+/O/+/O/+/O/+/O/+/O]_T\)
AN: \([+/O/+/O/+/O/+/O/+/O/+/O/+/O/+/O/+/O/+/O/+/O]_T\)
SC: \([+/O/+/O/+/O/+/O/+/O/+/O/+/O/+/O/+/O/+/O/+/O]_T\)
SN: \([+/O/+/O/+/O/+/O/+/O/+/O/+/O/+/O/+/O/+/O/+/O]_T\)
AS: \([+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/_T\)
NS: \([+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/_T\)
SS: \([+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/+/_T\)

Table 3 – Non-dimensional stiffness parameters for laminate comparators of: (a) Fig. 3(a); (b) Fig. 3(b) and; (c) Fig. 3(c), including compression \( (k_{x,x}) \) and shear \( (k_{xy,x}) \) buckling factors (and associated buckling half-wavelength, \( \lambda \)) for the simply supported, infinitely long plate.

### (a)

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( \delta )</th>
<th>( k_{x,x} )</th>
<th>( k_{xy,x} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.50</td>
<td>0.29</td>
<td>0.44</td>
<td>3.16(( \lambda = 1.21b ))</td>
<td>6.00(( \lambda = 1.83b ))</td>
</tr>
<tr>
<td>1.50</td>
<td>0.13</td>
<td>0.20</td>
<td>3.99(( \lambda = 1.23b ))</td>
<td>5.06(( \lambda = 1.80b ))</td>
</tr>
</tbody>
</table>

### (b)

| \( A_{2}B_{2}D_{2} \) | 1.00 | - | - | 4.00(\( \lambda = b \)) | 5.34(\( \lambda = 1.25b \)) | -5.34(\( \lambda = 1.25b \)) |

### (c)

| \( A_{2}B_{2}D_{2} \) | 1.87 | - | - | 4.64(\( \lambda = 1.21b \)) | 4.89(\( \lambda = 1.65b \)) | -4.89(\( \lambda = 1.65b \)) |
| 1.87 | 0.14 | 0.18 | 3.99(\( \lambda = 1.23b \)) | 5.70(\( \lambda = 1.73b \)) | -4.03(\( \lambda = 1.55b \)) |
| 1.87 | - | - | 4.64(\( \lambda = 1.12b \)) | 4.89(\( \lambda = 1.65b \)) | -4.89(\( \lambda = 1.65b \)) |
| \( A_{2}B_{2}D_{2} \) | 1.20 | 0.24 | 0.45 | 2.67(\( \lambda = 1.33b \)) | 5.28(\( \lambda = 1.94b \)) | -1.60(\( \lambda = 1.45b \)) |
| 0.98 | 0.28 | 0.19 | 2.24(\( \lambda = 1.74b \)) | 2.31(\( \lambda = 2.51b \)) | -1.29(\( \lambda = 1.9b \)) |
Figure 3 – Normalised buckling interaction envelopes for combined shear \( (N_x) \) and compression \( (N_y) \) loaded infinitely long plates for:

(a) 18 ply laminates with matching stiffness properties, including uncoupled datum laminate \( A_1B_1D_1 \): 
\[
\pm\frac{1}{\sqrt{5}}\bigg\{N_x-\frac{1}{\sqrt{5}}N_y+\frac{2}{\sqrt{5}}N_z\bigg\}_{1T}
\] 
and \textbf{bending-twisting coupled laminates} \( A_1B_1D_2 \): 
\[
\pm\bigg\{N_x-\frac{1}{\sqrt{5}}N_y+\frac{2}{\sqrt{5}}N_z\bigg\}_{1T}
\]
and \( A_1B_1D_3 \): 
\[
\pm\bigg\{N_x+\frac{1}{\sqrt{5}}N_y-\frac{2}{\sqrt{5}}N_z\bigg\}_{1T}
\]
(b) 18 ply laminates with matching stiffness properties, including \textbf{Simple (uncoupled)} laminate \( A_1B_1D_1 \): 
\[
\pm\frac{1}{\sqrt{5}}\bigg\{N_x+\frac{2}{\sqrt{5}}N_y-\frac{1}{\sqrt{5}}N_z\bigg\}_{1T}
\] 
\textbf{extension-shearing coupled laminate} \( A_1B_1D_2 \): 
\[
\pm\bigg\{N_x-\frac{1}{\sqrt{5}}N_y+\frac{2}{\sqrt{5}}N_z\bigg\}_{1T}
\]
and \textbf{extension-shearing and bending-twisting coupled laminate} \( A_1B_1D_3 \): 
\[
\pm\bigg\{N_x+\frac{1}{\sqrt{5}}N_y+\frac{2}{\sqrt{5}}N_z\bigg\}_{1T}
\]
(c) 16 ply repeating NCF bi-angle design: 
\[
\pm\frac{1}{\sqrt{2}}\bigg\{N_x-\frac{1}{\sqrt{2}}N_y\bigg\}_{1T}
\] 
with standard \((\theta = 45^\circ)\) and shallow \((\theta = 22.5^\circ)\) angle plies.
Table 4 – Single ply terminations in $E-S-B-T$ coupled laminates.

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>1,191</td>
<td>2,856 (691,843,631)</td>
<td>1,105</td>
<td>1,260 (364,344,188)</td>
<td>469</td>
<td>1,684 (481,444,278)</td>
<td>626</td>
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<tr>
<td>11</td>
<td>1,843</td>
<td>1,546 (380,416,370)</td>
<td>944</td>
<td>872 (234,226,178)</td>
<td>520</td>
<td>1,166 (307,298,254)</td>
<td>692</td>
</tr>
<tr>
<td>10</td>
<td>241</td>
<td>602 (145,175,137)</td>
<td>231</td>
<td>352 (104,96,48)</td>
<td>130</td>
<td>470 (133,124,80)</td>
<td>174</td>
</tr>
<tr>
<td>9</td>
<td>321</td>
<td>330 (83,86,78)</td>
<td>200</td>
<td>248 (67,63,51)</td>
<td>148</td>
<td>330 (83,86,78)</td>
<td>200</td>
</tr>
<tr>
<td>8</td>
<td>50</td>
<td>130 (33,36,28)</td>
<td>50</td>
<td>100 (30,26,14)</td>
<td>37</td>
<td>–</td>
<td>50</td>
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<tr>
<td>7</td>
<td>65</td>
<td>72 (19,17,17)</td>
<td>44</td>
<td>40 (12,10,6)</td>
<td>4</td>
<td>–</td>
<td>4</td>
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<tr>
<td>6</td>
<td>11</td>
<td>28 (8,6,6)</td>
<td>11</td>
<td>16 (6,4,0)</td>
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<td>–</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>13</td>
<td>12 (4,2,2)</td>
<td>8</td>
<td>12 (4,2,2)</td>
<td>8</td>
<td>–</td>
<td>8</td>
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<tr>
<td>4</td>
<td>2</td>
<td>4 (2,0,0)</td>
<td>2</td>
<td>4 (2,0,0)</td>
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<td>–</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>–</td>
<td>2</td>
<td>–</td>
<td>2</td>
<td>–</td>
<td>2</td>
</tr>
</tbody>
</table>

Column:
(1) Ply number grouping, $n$.
(2) Number of stacking sequences; laminates with single-ply-orientation, i.e.: [+/$\ldots$/+]$_T$, have been removed.
(3) Number of compatible single-ply terminations and corresponding ply orientations (O or ●,+,-).
(4) Number of individual sequences used in (3).
(5) Number of compatible solutions in each $n$, leading to a tapered sequence from $n = 12$ to $n = 3$.
(6) Number of individual sequences used in (5).
(7) Number of compatible solutions in each $n$, leading to a tapered sequence from $n = 12$ to $n = 8$.
(8) Number of individual sequences used in (7).
Table 5 – Abridged stacking sequence listings for 15-ply \( E-S-B-T \) coupled laminates, grouped according to angle- and cross-ply sub-laminate symmetries and ordered by blend ratio, \( n_s/n_n \), then by increasing bending stiffness \( \zeta_s \) and \( \zeta_n \), respectively.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>( n )</th>
<th>( n_s )</th>
<th>( n_n )</th>
<th>( \zeta_s )</th>
<th>( \zeta_n )</th>
<th>( \zeta_{\theta} )</th>
<th>( \zeta_{\phi} )</th>
<th>( n_s/n_n ) (%)</th>
</tr>
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<td>15-AC-1</td>
<td>+○● – – ● ○ – ● ○ + ○ ● –</td>
<td>15 7 4 4 3375 1783 796 796 892</td>
<td>42.9</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>15-AC-24</td>
<td>– ○ ○ ○ ● – + ○ ● ○ ○ +</td>
<td>15 7 4 4 3375 2071 652 652 1035</td>
<td>57.1</td>
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<td></td>
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<tr>
<td>15-AN-1</td>
<td>○ ○ ● – – ● ● – ○ ○ + ○ ● –</td>
<td>15 7 3 5 3375 1783 747 845 892</td>
<td>42.9</td>
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<tr>
<td>15-AN-24</td>
<td>+ ○ ○ ○ ○ ○ + ○ ○ ○ ○ +</td>
<td>15 7 5 3 3375 2071 845 459 1035</td>
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<td>15 7 0 8 3375 1783 0 1592 892</td>
<td>42.9</td>
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<tr>
<td>15-AS-288</td>
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<td>57.1</td>
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<tr>
<td>15-NC-1</td>
<td>○ ○ ● – – ● ○ – ○ + ○ ● –</td>
<td>15 9 3 3 3375 1881 747 747 990</td>
<td>33.3</td>
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<td>15-NC-48</td>
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<tr>
<td>15-SN-1</td>
<td>+ ● ○ – – ○ – ○ – ○ ○ +</td>
<td>15 7 3 5 3375 1423 675 1277 245</td>
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